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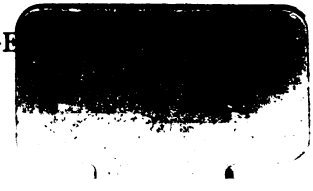

EXAMPLES IN ALGEBRA


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EXAMPLES IN ALGEBRA

EIGHT THOUSAND EXERCISES AND PROBLEMS CAREFULLY GRADED FROM THE EASIEST TO THE MOST DIFFICULT

BY

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PREFACE.

For more than twenty years the writer taught algebra and, he has reason to believe, with a fair measure of success. During most of that time the only text book in the hands of the class was a set of examples, the nucleus of this book. Each topic was developed in the class, not always in the same way, but suited, in his judgment, to the varying needs and capacities of the different divisions.

To him it was a labor of love, yet it had serious drawbacks. One of the most serious was the time and labor consumed in dictating to the class or writing upon the board each succeeding lesson. And when, year after year, every new book was tried, only to find the same defects, the result was disheartening.

No book offered a sufficient number and variety of examples, and, apparently, no attempt was made to grade them. Perhaps, out of ten examples assigned, the first three would prove easy, but the fourth so difficult as to consume all the pupil's remaining time and discourage him from trying further.

The first need of beginners in algebra is drill, drill, drill, until the fundamental processes become as familiar to him as in arithmetic. Too much time spent upon problems teaches him to think rather than to act; too much upon examples teaches him to act rather than to think. A judicious mixture is the only wise course. But the make-up of different classes causes this admixture to be an ever-varying one. How necessary, then, that each teacher should have at command material that is ample to supply the diverse needs of his different classes! This book is an attempt to supply these needs.

It begins each topic with its simplest possible application to arithmetic; it furnishes a superabundance of easy examples before introducing difficult ones; it proceeds gradually, step by step, to the most difficult; this grading is the result of many years test with many different classes, under different teachers; most of the examples are now printed for the first time.

In practice it has been found possible to use these examples, at the same time, with several large divisions of between thirty and forty pupils each, to assign to each division a different set of examples, and to avoid the danger of a set of answers being handed down from class to class.

An attempt has been made to approach each subject from as many standpoints and to offer as great a variety as possible.

Some special features are the equations called for, pages 20–28, and, also, 206, and the double set of problems, pages 201–212, and 236–243.

As an example of thoroughness of treatment attention is called to "The Theory of Exponents," which usually presents so much difficulty to pupils. It is hoped that the suggestions and hints given here and there throughout the text, and the final chapter on methods of solving equations will prove helpful to pupils.

The writer believes that those teachers of algebra who put a copy of this book into the hands of each pupil will be saved thereby the necessity of dictating or copying examples, or of consulting other text books for work enough for adequate practice.

The thanks of the author are due the publishers for their unsparing efforts to present the work in the most acceptable form, and especially to Miss Maud G. Leadbetter, of Roxbury High School, whose painstaking trial of the examples

and thorough test with her classes, and whose suggestive and scholarly criticism have rendered this book possible.

The author will be grateful for criticisms and suggestions and particularly for information as to any mistakes that may have escaped his notice.

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ORAL EXERCISES: PROBLEMS.

NOTE: Solve these problems, first, in the simplest way, that is, without making use of fractions. Afterwards try to solve them by letting x equal each of the unknown quantities required, in turn.

1. The sum of the ages of a boy and a girl is 27 years, and the boy is twice as old as the girl. What is the age of each?

2. A bat and a ball together cost 90 cents, and the ball costs five times as much as the bat. What is the cost of each?

3. The sum of two numbers is 96, and the greater is seven times the smaller. What are the numbers?

4. A girl bought a melon and an orange for 10 cents, paying for the melon four times as much as for the orange. What was the cost of each?

5. John bought a blank-book and a pencil for 20 cents, paying three times as much for the book as for the pencil. What was the cost of each?

6. Divide 77 cents between two boys so as to give one boy six times as much as the other.

7. If this class numbered forty pupils, nine times as many boys as girls, how many of each would there be?

8. James walked a certain distance, then rode three times as far on his wheel, and finished the journey on horseback, riding five times as far as he had walked. The whole journey was 36 miles. How far did he go each way?

9. If I buy pears at two cents each and the same number of peaches at five cents each, how many of each can I purchase for 91 cents?

10. A farmer bought an equal number of ducks and hens for 60 cents each and turkeys for 90 cents each. How many of each did he buy for $63?

11. I need the same number of two-cent revenue stamps and five-cent postage stamps. How many of each can I purchase for 63 cents?

12. A, B, and C have $56. A has twice as much as B, and C twice as much as A. How much has each?

13. Joe has twice as much money as Charles, and Sam has three times as much as they both, while all three together have $6. How much has each?

14. I have three cows, together worth $150, and their values are as the numbers 1, 2 and 3. What are their respective values?

15. A school numbers 720 pupils. The number of pupils in the four classes are as the numbers 1, 2, 4, and 5. How many pupils in each class?

16. Divide 99 into three parts in the proportion of 2, 3, and 4.

17. Three times a number added to five times the double of the number gives the sum of 78. What is the number?

18. A fish weighs 12 lbs. The body weighs as much as the head and the tail together, and the head weighs twice as

much as the tail. What is the weight of each part separately?

19. Three bins hold 320 bushels of potatoes. The first holds three times as much as the second and the third holds as much as the other two. How many bushels does each contain?

20. Mary bought cloth at 5 cents a yard and three times as much ribbon at 8 cents a yard, paying 58 cents. How much of each did she buy?

21. William bought apples at 2 cents each and four times as many oranges at 3 cents each, paying 42 cents. How many of each did he buy?

22. If I have to ride the bicycle a certain time at 12 miles an hour, and walk five times as long at 3 miles an hour, to accomplish 81 miles, how long do I ride and how long walk?

23. If, by walking 6 hours and riding 4, at three times the speed, I accomplish 72 miles, at what rate do I walk and ride?

24. A and B set out at the same time from towns 63 miles apart and walk toward each other, A at the rate of 3 miles per hour, and B, 4. How long before they meet?

25. A and B start from the same point and travel in opposite directions. A travels twice as fast as B. How far has each traveled when they are 54 miles apart?

26. The sum of the three angles of a triangle always equals 180°. If two angles are equal and the third is four times their sum, how many degrees in each?

27. If one angle of a triangle is three times the second, and the third is twice the first, how many degrees in each?

28. The sum of the angles of a quadrilateral is always 360°. If they are to each other as 1, 2, 3 and 4, how many degrees in each?

29. A man earned a certain sum on Monday, twice as much on Tuesday, three times as much on Wednesday, and so on until Sunday. His week's earnings were $84. How much did he earn Monday?

30. A hired a man and two boys to do a piece of work. To the man he paid $3 per day, to one boy $2, and to the other, $1. They received $108. How many weeks did they work?

31. A father is five times as old as his son, and the difference between their age is 36 years. How old is each?

32. Divide 96 into two parts such that one shall be one seventh of the other.

33. A horse and a harness are together worth $120, but the harness is worth only one third as much as the horse. What is each worth?

34. Divide 70 into two parts such that one shall be 20 larger than the other.

35. Divide 91 into two parts such that one shall be 11 smaller than the other.

36. Divide 81 into three parts such that the first is one half the second and one sixth the third.

37. Divide 85 into two such parts that the greater divided by 3 shall equal the smaller divided by 2.

38. Divide 96 into two such parts that the greater divided by 7 shall equal the smaller divided by 5.

39. What two numbers, whose difference is 7, are to each other as 4 : 5?

40. If a certain number be multiplied by 12, and the product divided by 4, and 7 added to the quotient, the sum will be 31. What is the number?

41. Cut a ribbon, 54 inches long, into three pieces such that the first shall be three times the second and the third one half the second.

42. Divide the number 65 into two such parts that the first shall equal two thirds the second.

43. A's age is three halves of B's, and their united ages are 70 years. How old is each?

44. Five times the smaller of two numbers is four times the greater, and their sum is 72. Required, the numbers.

45. Divide $480 among three persons so that A shall have as much as B and C together, and B $5 as often as C $7.

WRITTEN EXERCISES: PROBLEMS.

TO THE STUDENT: Try first to solve each problem mentally. Avoid using fractions.

1. There are three brothers whose ages together amount to 30 years and their birthdays are two years apart. What is the age of each?

2. A man sold 19 bushels of oats at a certain price, and afterwards 11 bushels at the same rate. The second time he received 96 shillings less than the first. What was the price per bushel?

3. Four men, A, B, C and D, contributed $8,800 to found a library. B gives twice as much as A, C as much as

A and B together, and D as much as B and C together. How much did each give?

4. If it takes $5,600 to pay a debt, principal and interest, how much is each, the principal being six times the interest?

5. A grocer sold tea at 60 cents a pound and coffee at 30 cents, of each the same quantity, getting for the whole $8.10. How much of each did he sell?

6. If a certain number be divided by 9, the sum of the divisor, dividend and quotient will be 89. What is the number?

7. What number is as much greater than 89 as its third part is greater than 25?

8. What number is that whose fifth part exceeds its sixth part by 5?

9. The leaning tower of Pisa is 178 feet high, which is 9 feet more than 13 times the distance it leans over. How far does it lean?

10. The planet Jupiter has two more moons than the planet Mars is known to have, and Saturn has two more than the other two together. In all they have 14 moons. How many moons has each?

11. At a certain election 670 persons voted for two candidates, the successful candidate having a majority of 120. How many votes had each?

12. A, B and C have together $145. A's share is two thirds and B's three fourths as great as C's. What is the share of each?

13. From a cask one third full of oil there leaked out 21 gallons, when there was found to be just half the oil left. Required the capacity of the cask.

14. A father is four times as old as his son, but 5 years hence he will be only three times as old. What are their ages now?

15. There are three numbers, the second of which is four times the first, and the third is twice as much as the first and second together. The difference between the second and third is 48. What are the numbers?

16. What number is that which, being multiplied by 7, and the product added to the number, the sum, product, and given number will together equal 80?

17. If $3x - 5$ stands for 34, for what number will $12 + x$ stand?

18. For what number will $2x + 7$ stand, if $5x + 8$ stands for 48?

19. A banker paid \$120 in five-dollar, two-dollar, and one-dollar bills, using the same number of each kind. How many bills of each kind did he use?

20. A boy has three times as many dimes as quarters and twice as many nickels as dimes, and in all he has 80 pieces of money. How many pieces of each kind has he? How much money has he in all?

21. Find three consecutive odd numbers whose sum is 69.

22. The sum of three consecutive even numbers exceeds the least by 54. What are the numbers?

23. A sum of \$6 is made up of 30 coins, which are either quarters or dimes. How many are there of each?

24. A man is hired for 45 days on condition that for each day he works he is to receive \$2 and his board, but for each day he is idle he is to pay \$1 for board. At the end he received \$60. How many days did he work?

25. Suppose that your teacher assigns you these 50 problems to solve, giving you 3 marks in your favor for every problem solved correctly, and one mark against you for each failure. How many must you solve correctly to get a credit of 90 marks?

26. A girl was engaged to wash a gross of lamp chimneys for 2 cents each, but she was to forfeit 20 cents for each one she broke. She received $2. How many did she break?

27. A bankrupt owed A four times as much as he owed B, and C three times as much as he owed A. He owed D the difference between his debts to C and B. The amount of his debts to all four was $56,000. How much did he owe each man?

28. John has 6 more nuts than James, but John eats 10 while James finds 14 more. They now have together 44 nuts. How many has each now? How many had each at first?

29. Sam is 6 years older than Charles. In five years three times Sam's age will equal four times Charles'. How many years old is each?

30. A man's boat broke loose and drifted down stream at the rate of 2 miles an hour. Six hours afterwards he secured another boat and started after it at the rate of 5 miles an hour. In how many hours did he overtake it?

31. What number is to 20 increased by one fourth the number as 2 : 3?

32. A man spent one third of his life in studying, one fifth in traveling and the rest, which was 28 years, in working. To what age did he live?

33. A post is one fifth in the mud, three sevenths in the water, and 13 feet above the water. How long is the post?

34. What two numbers are in the ratio 3 : 4, while, if 9 be added to each, the sums will be to each other as 6 : 7?

35. Four persons, A, B, C and D, entered into partnership with a capital of $96,000. B put in three times as much as A, C as much as both A and B, and D as much as A, B and C. How much did each put in?

36. Twenty-seven times a certain number added to seven times the double of that number equals 2,993. What is the number?

37. The difference between thirty-seven times a number and nine times the double of that number is 1,007. What is the number?

38. I buy apples at 1 cent each, oranges at 3 cents each, and bananas at 9 cents each, paying for the whole $1.92. There were three times as many apples as oranges, and half as many bananas as apples and oranges together. How many were there of each?

39. A mixture made of corn worth 80 cents a bushel, oats worth 45 cents a bushel, and barley worth $1.10 a bushel, is worth $97.20 in all. There is twice as much corn as barley, and twice as much oats as corn and barley together. How much is there of each?

40. A gains in trade $300. B gains one half as much as A plus one third as much as C and C gains as much as A and B. What is the gain of B and C?

41. Two men owning a flock of sheep together agree to divide its value equally. A takes 72 sheep, and B takes 92 sheep and pays A $35. What is the value of a sheep?

42. The fore and hind wheels of a carriage are 10 feet and 12 feet, respectively, in circumference. How many feet will

the carriage have passed over when the fore wheel has made 100 more revolutions than the hind wheel?

43. Find two numbers whose difference is 6, and such that seven times the greater minus five times the smaller shall equal 156 minus nine times the sum of the numbers?

44. There are two numbers which are to each other as $\frac{1}{2} : \frac{1}{3}$, but if 9 be added to each, they will be as $\frac{1}{5}$ to $\frac{1}{6}$. What are the numbers?

45. Four towns, A, B, C, and D, are on a straight road. The distance from A to D is 108 miles. The distance from B to C is two sevenths the distance from A to B, and the distance from C to D is three times the distance from A to C. Find the distance from A to B, B to C, and C to D.

FORCE OF SIGNS.

NOTE: The operations of multiplication and division must be performed before those of addition and subtraction, unless parentheses indicate otherwise.

Find the value of each of the following expressions.

1. $(7 + 2) \times 3$.
2. $7 + 2 \times 3$.
3. $(7 - 2) \times 3$.
4. $(5 + 3) \times 4$.
5. $(6 + 5 + 4) \times 3$.
6. $6 + 5 + 4 \times 3$.
7. $(10 + 6 - 2) \times 5$.
8. $10 + 6 - 2 \times 5$.
9. $(8 - 5 + 3) \times 2$.
10. $8 - 5 + 3 \times 2$.
11. $9 \times (5 - 3)$.
12. $4 \times (6 + 2 - 5)$.
13. $4 \times 6 + 2 - 5$.
14. $6 \times (3 + 2 + 4)$.
15. $6 \times 3 + 2 + 4$.
16. $(5 + 2) \times (7 - 4)$.
17. $(5 + 2) \times 7 - 4$.
18. $5 + 2 \times 7 - 4$.

19. $(7 + 3) \div 5$.
20. $7 + 3 \div 5$.
21. $(3 + 4 - 2) \div 5 + 5$.
22. $(3 + 4 - 2) \div (5 + 5)$.
23. $3 + 4 - 2 \div 5 + 5$.
24. $4 + 2 \times (-6 + 9) \div 5$.
25. $(4 + 2) \times (-6 + 9) \div 6$.
26. $7 + 19 - 3 \times 4 - 3$.
27. $7 + (19 - 3) \times 4 - 3$.
28. $7 + (19 - 3) \times (4 - 3)$.
29. $(7 + 19 - 3) \times 4 - 3$.
30. $(7 + 19) - 3 \times (4 - 3)$.
31. $2 + 3 - 4 + 5 \times 6$.
32. $2 + 3 - 4 \div 5 + 5$.
33. $(2 + 3 - 4) \div 5 + 5$.
34. $(2 + 3 - 4) \div (5 + 5)$.
35. $2 + 3 - 4 \div (5 + 5)$.
36. $25 + 5 \times 4 - 10 \div 5$.
37. $24 - 5 \times 4 \div 10 + 3$.
38. $(24 - 5) + (4 \div 10 + 3)$.
39. $18 - 2 \times 4 \div 2 + 10$.
40. $3(13 - 7)(19 - 14)$.
41. $(1 - 2 + 3)(4 - 5 + 6)$.
42. $9(9 - 3) - 8(8 - 3) + 7(7 - 3)$.
43. $(8 - 2)(7 - 3) - (6 - 3)(5 - 2)$.
44. $(6 \cdot 8 - 5 \cdot 9)(2 \cdot 3 \cdot 4 - 4 \cdot 5)$.
45. $(7 \cdot 8 - 2 \cdot 12)(5 \cdot 6 - 4 \cdot 7)$.
46. $(2 \cdot 3 \cdot 4 - 20)(4 \cdot 5 \cdot 6 - 7 \cdot 5 \cdot 3)$.
47. $2(2 \cdot 3 - 3 \cdot 1)(4 \cdot 5 - 2 \cdot 7)5$.

48. $\dfrac{13-4+9}{3\times 2+3}$.

49. $\dfrac{1\cdot 3\cdot 5\cdot 7}{2\cdot 4\cdot 6\cdot 8}$.

50. $\dfrac{2\cdot 4(5+2)}{(5-3)(3+1)}$.

51. $\dfrac{5+11-(7-3)}{2\cdot 3\cdot 4}$.

52. $5\times\dfrac{(6-4)8}{2+1+(10\div 2)}$.

53. $\dfrac{14-(14-4)+8}{3\times 3+3}$.

54. $\dfrac{13\cdot 13+31}{2\cdot 5\cdot 5\cdot 2}$.

55. $\dfrac{4+7\cdot 4(7+4)(7-4)}{7+2(4+7)}$.

56. $\dfrac{(2\cdot 4)+(3\cdot 5)+(5\cdot 7)}{(1\cdot 3)+(4\cdot 6)+(6\cdot 8)}$.

57. $\dfrac{3(14-8)(16-11)}{(8-2)(5-1)-(5-2)(6-3)}$.

58. $\dfrac{(3+4-5+6)\times 6}{(4\cdot 12-3\cdot 15)(2\cdot 3\cdot 4-4\cdot 5)}$.

59. $\dfrac{4\times 5+18\div 2}{2\cdot 3\cdot 4+5}$.

60. $\dfrac{(5+4-6)\div 3+3}{5+4-6\div 3+3}$.

61. $\dfrac{7+2-18\div 3+2\times 5}{1\cdot 2\cdot 3\cdot 4+10\div(2+3)}$.

62. $\dfrac{(7+2-18\div 3+2)\times 5}{1\cdot 2\cdot 3\cdot 4+10\div(2+3)-1}$.

63. $\dfrac{2\cdot 3+3\cdot 4+4\cdot 5}{12\div 3+8\cdot 3-3\cdot 3}$.

64. $6\cdot 3^2$; $(6\cdot 3)^2$; $6^2\cdot 3$.

65. $(6-4)^{6-4}$.

66. 6^2-4^{6-4}.

67. $\dfrac{(10+5)^2}{5}$.

68. $\dfrac{10+5^2}{5}$.

69. $\dfrac{10+2\cdot 5\cdot 10+5^2}{5}$.

70. $2^2\cdot 3^2\cdot(2+3)^2$.

71. $\frac{(2+4)^3}{6^2}$; $\frac{2^3(2+4)^3}{12}$.

72. $(1+2)(2+3)^2[(1+2)(2+3)]^2$.

73. $75-2\cdot5^2$.

74. $(75-2\cdot5)^2$.

75. $75-2^2\cdot5$.

76. $\frac{1+2+2^2+2^3+2^4+2^5}{2^6-1}$.

77. $\frac{1+3+3^2+3^3+3^4+3^5}{3^6-1}$.

78. $\frac{2^4\cdot4^2-200}{7\cdot2^3}$.

79. $\frac{6(7-1)(8-2)(9-3)(10-4)}{6^6}$.

80. $\frac{(4+7)^2+11(7+4)^2-(13-2)}{(19-9)^3+2^5-5^2-7}$.

NUMERICAL VALUES.

If $a=1$, $b=2$, $c=3$, $d=4$, $e=5$, $f=6$, etc., find the values of the following expressions:

1. ab; abc; $abcd$; $abcde$; $abcdef$.

2. $ab+abc+abcd+abcde+abcdef$.

3. $\frac{b}{a}+\frac{f}{c}$.

4. $\frac{e+d}{e-d}+\frac{c+b}{c-b}$.

5. $\frac{1}{3}f(f-2)$.

6. $\frac{abc}{def}$; $\frac{efg}{cde}$.

7. $(a+b)(c+d)(e+f)$.

8. $(c-a)(d-b)(e-c)$.

9. $(a + b + c + d)e + f$.

10. $(a + b + c + d)\,(e + f)$.

11. $a + b + c + d(e + f)$.

12. $\dfrac{cd + ab}{cd - ab}$.

13. $(f - e)\,(e - d)\,(d - c)\,(c + d + e + f)$.

14. $8e + bcd - \dfrac{4bde}{ad}$.

15. $4c + ef \div bc - 3d$.

16. $(c + d)^2$; $c^2 + 2cd + d^2$.

17. $(e - c)^2$; $e^2 - 2ec + c^2$.

18. $e - a - (d - b)$.

19. $e - a + d - b$.

20. $(e - a)\,(d - b)$.

21. $(e - a)d - b$.

22. $(c^2 - bc)c$.

23. $\dfrac{d^2 - ab}{g}$.

24. $\dfrac{ab + a^2c + bc}{e + f}$.

25. $\dfrac{(d - a + c - b)e^2}{d^2 - (b + c) - a}$.

26. $\dfrac{bd + ce - b}{af + a} + \dfrac{de}{cd - bd}$.

27. $\dfrac{3c(b^3 - a^3)}{c^3 + d^2 + de}$.

28. $6\left(\dfrac{2e}{5} + \dfrac{bc}{3}\right) - bc$.

29. $5(a^2 + b^2 + c^2) - a^2b^2c^2$.

30. $\dfrac{ab}{b + c} + \dfrac{bc}{2} + \dfrac{a + c}{b}$.

31. $(c^2 + d^2 + e^2) \div \dfrac{c^2 + d^2}{e}$.

32. $(e - c)^2 - (b - a)^2$.

33. $\sqrt{2ab}$; $\sqrt{4bcf}$.

34. $\sqrt{3bc^2f}$; $\sqrt{2bc^2e^2}$.

35. $\sqrt{4c + df}$; $\sqrt{c^2 + d^2}$.

36. $\sqrt{3b^2 + 4a^2}$; $\sqrt{12de - 11ab^2}$.

37. $\sqrt{\frac{b^3f}{c}}$; $\sqrt{\frac{3bde}{5f}}$.

38. $\sqrt{\frac{2(bc+de)}{b^2d-a^2c}}$.

39. $\sqrt{\frac{2de^2+6f^3}{2(2b^3+4bc^3-ce^2)}}$.

40. $\sqrt{\frac{d^2e-bcf+3a^2b}{f^2-2e^2+d^2}}$.

41. $\sqrt[3]{2bd^2}$; $\sqrt[3]{2b^2d^2f^2c}$.

42. $\sqrt[3]{2b^2e^3+4a^3d}$.

43. $\sqrt[3]{2d^3-3e^3+7ae+d^3}$.

44. $\frac{8a^2+3b^2}{a^2b^2}+\frac{4c^2+6b^2}{c^2-b^2}-\frac{c^2+d^2}{e^2}$.

45. $\sqrt{4c^2+5d^2+e}-\sqrt{e^2+d^2+c^2-a^2}$.

46. $\sqrt[5]{\frac{b^2-2bc+c^2}{a^2-2ab+b^2}}$.

47. $\frac{a^2f+2ab+b^2}{a}-\frac{b^2+2bf+4e}{d}+\frac{c^2+2d+2ad}{b+c}$.

48. $\frac{a^2+2ab+b^2}{a+b}-\frac{b^2+2bc+c^2}{b+c}+\frac{c^2+2cd+d^2}{c+d}$.

49. $\frac{a^4-4a^3c+6a^2c^2-4ac^3+c^4}{b^4-4b^3c+6b^2c^2-4bc^3+c^4}$.

50. $\frac{d^6}{b^6}$; $\frac{e^6+b^6}{c^6-b^6}$.

51. $\frac{b^6+d^6}{b^2+d^2-bd}$.

52. $\frac{e^6-d^6}{e^2+ed+d^2}$.

53. $\sqrt{d}+\sqrt[3]{4ce+d}-\sqrt{5e}$.

54. $be+bd\sqrt{cd+d}-(f-d)\sqrt[3]{cd-d}$.

55. $\frac{5f^2d^3}{8b^2c^2d^2}-\sqrt[3]{\frac{3bdc^4}{f^2d^2}}+\sqrt{\frac{bdf^3}{3d}}$.

56. $\sqrt{2d+1}-\left(d+\frac{6}{\sqrt{d}}\right)-3-\frac{d^2}{4-\sqrt[3]{2d}}$.

57. $\dfrac{(6ab^2d^2 - 4c^3b)(c^3 - 7a)}{ac} \div bd\left(\dfrac{5b^3d - 3d^2}{b^2d}\right)$.

58. $\dfrac{2b^c + c^b}{\sqrt[3]{3c - a + e}} - \dfrac{\sqrt{4d^c} + 3\sqrt{b^d}}{3^i - 2^a}$.

If $p = 5$; $q = 8$; $s = 2$; $t = 3$.

59. $s^t t^s$; $(p - t)^{2t}$; $(q - p)^s$.

60. $(s + t)^{s+t}$; $(q - p)^2 s^2$. **61.** $(q - s)^{q-p}$; $(q - t)^{t-s}$.

62. $\left(\dfrac{s + q}{p}\right)^t \times \left(\dfrac{p - s}{q}\right)^s$.

If $a = 0$; $y = 4$; $x = 6$; $m = 8$; $n = 10$.

63. $m\sqrt{x^3 + ny} + \sqrt{4ax + 4n}$.

64. $\dfrac{2n - 4y}{\sqrt{8m}} + \dfrac{mx}{y}$. **65.** $\dfrac{2x + 3m}{y - a} - \sqrt{\dfrac{2m + \frac{1}{8}nx}{m^2}}$.

66. $\dfrac{\sqrt{8m + 4y + 2n}}{\sqrt{mn + ny + yx}} - \dfrac{n + y + x - 2m}{yx}$.

If $k = 6$; $l = 5$; $m = 4$; $n = 1$; $s = 0$.

67. $k\sqrt{k^2 + l^2 + lm} + 3lm\sqrt{k^2 - (l + k)}$.

68. $\dfrac{2l + m}{3k - m} - \dfrac{\sqrt{5l} + 3\sqrt{m} + n}{2k + m}$.

69. $4k^2n^m + \sqrt[3]{m^2 - \sqrt{2km + m^2}}$.

70. $\dfrac{3\sqrt{m^m} + 2k\sqrt{2k + l - n}}{k^n + m^n - 2n}$.

If $s = 8$; $t = 4$; $x = 2$; $y = \frac{1}{2}$.

71. $\sqrt[3]{s^2} + 4st^3 + \frac{x^3}{sy}$.

72. $(2t^3 - \sqrt[3]{s^3}) \times \frac{2\sqrt{t}}{2x} - 2y^4$.

73. $\frac{(s^2 - t^2)(x^2 - y^2)}{5stxy} - \frac{x^2 + (4y)^2}{stx}$.

74. $\frac{s}{t^2} - 3\sqrt{4xy} + 5tx^3 - \frac{s^2}{2y^3}$.

75. $\frac{t\sqrt[3]{s}}{y^2} + \frac{\sqrt[3]{stx}}{ty} - \sqrt[3]{\frac{sx^3}{y}}$.

76. $\sqrt[3]{\frac{s}{x} \times \frac{x}{t} \times \frac{2y}{t^2}}$.

77. $\left(\frac{\sqrt[3]{s}}{\sqrt{t}} + \frac{\sqrt{2s}}{\sqrt[3]{2t}}\right)^t$.

78. $\sqrt[4]{y^5 + y^4 + y^3 + y^2 + y + \frac{1}{32}}$.

79. $\left(\frac{\sqrt{st \cdot tx \cdot xy}}{\sqrt[3]{yxt \cdot yst}}\right)^y$.

ALGEBRAIC EXPRESSIONS.

Learn the definition of *coefficient; exponent.*

In $7x$ what is the coefficient of x? the exponent? In $8y^3$ what is the coefficient of y? the exponent? In the quantity $5xy$ name the coefficient; the exponents. In $4ax$ name the coefficient; the exponent. In the expression $9a^4b^3c^2x$ what is the coefficient of x? of c^2x? of b^3c^2x? of $a^4b^3c^2x$?

Translate the following algebraic expressions into words:

4, $4'$, 4^2, 4^3, 4^4, 4^n, 4^{n+1}, $4^{\frac{1}{2}}$, $4^{\frac{3}{2}}$, $\sqrt[2]{4}$, $\sqrt{4}$, $\sqrt[3]{4}$, $\sqrt[4]{4}$, $\sqrt[5]{4}$, $\sqrt[n]{4}$, $\sqrt[n-1]{4}$, $\sqrt{4 \cdot 25}$, $\sqrt{4a}$, $\sqrt{ab}$, $x + y$, $x - y$, xy, $x \div y$, $\frac{x}{y}$, $x = y$, $x > y$, $x < y$, $\sqrt[2]{25}$, $\sqrt{25}$, $\sqrt{a}$, $\sqrt[2]{b}$, $\sqrt[3]{8}$, $\sqrt[3]{x}$, $\sqrt[3]{8x}$, $\sqrt[3]{x^3}$, $\sqrt[3]{8x^3}$, $\sqrt[3]{a - b}$, $\sqrt{\frac{a}{b}}$, $\frac{\sqrt{a}}{b}$, $\frac{a}{\sqrt{b}}$, $\frac{\sqrt{a}}{\sqrt[3]{b}}$, $\frac{\sqrt[3]{a}}{\sqrt[5]{b}}$,

$5\sqrt{xy}$, $6\sqrt{5a^3b^3}$, $\sqrt[3]{\frac{2a^2}{3b^4}}$, $\sqrt{a}+\sqrt[3]{5x}$, $\sqrt[3]{x+2y^2}$, $\sqrt[3]{x}+\sqrt[3]{2y^2}$,
$\sqrt[2]{x^2y^3}$, $\sqrt[3]{a^3b^5}$, $\sqrt[4]{a^2+b^2}$, $\sqrt[3]{a^2-b^2}=(a^2-b^2)^{\frac{1}{3}}$, a^n, b^n,
a^2+x^2, $(a+x)^2$, a^3-c^3, $(a-c)^3$, $(a+b)(a-b$,
$5x^2(a+b)^3$, $(x+y)+(x-y)=2x$, $(x+y)-(x-y)=2y$,
$(x+y)(x-y)=x^2-y^2$, $(x+y)^2-(x-y)^2=4xy$,
$(a+1)x+(a-1)y>95$, $3a^2+bc-\frac{d}{4}$, $x:\frac{c}{d}=ab:\sqrt[3]{u^2}$,
$(c-d)y=(e+f)x$, $\sqrt[3]{a^2-b}=\sqrt{3^3+b^5}$, $\sqrt[3]{pq}<\sqrt{rs}$,
$\sqrt{\frac{2a}{a-b}}<(p-q)\left(a+\frac{b}{3}\right)$, a^{n+1}, b^{m-1}, $x^{\frac{1}{n}}$, $y^{\frac{1}{m}}$, x^ny^c,
a^m-y^n, $\frac{x^{n+1}}{y^{n-1}}$, $\sqrt{a^n}$, $\sqrt[3]{x^m}$, $\sqrt[4]{b^{n-1}}$, $\sqrt[5]{y^{m+n}}$, $\left(\frac{a^{n-1}}{b^{n+1}}\right)^{\frac{n}{m}}$, $\sqrt[n]{a^2}$,
$\sqrt[m]{x^3}$, $\sqrt[n-1]{a^{x+1}}$, $\sqrt[n]{x^5}=x^{5/n}$, $\sqrt[3]{y^m}=y^{m/3}$, x^{-1}, y^{-3}, $\sqrt[n]{x^{-1}}$, $\sqrt[3]{a^{-2}}$,
$\sqrt[m]{a^{-3}}=a^{-\frac{3}{m}}$, $\sqrt[3]{x^{-n}}=x^{-\frac{n}{3}}$, $\left(a+\frac{1}{a}\right)\left(b-\frac{1}{b}\right)=c+\frac{1}{c}$,
$\frac{x+y}{m-n}-\frac{m+n}{x-y}=\sqrt[3]{\frac{a+b}{c-d}}$,
$a^2c^3x^4\times b^5d^4y^3\div e^3f^4z^5=a^3-3a^2b+3ab^2-b^3$,
$\sqrt[4]{a-b}+\sqrt[3]{8n}-(c-d)^n$.

Put into the form of algebraic expressions the following:

1. How much will 5 knives cost at \$2 each?

2. What will $3\frac{1}{2}$ lbs. of tea cost at \$2 a pound?

3. If I buy 5 sheep at \$4 each and 3 calves at \$9 each, what is the total cost?

4. If I have \$25 and spend \$11, how much have I left?

5. If I have a cents in one pocket and b cents in another, how much have I?

6. If I have x dollars and pay out y dollars, how much have I left?

Express in algebraic form:

7. Three times x, added to 4 times y.

8. Four times x, subtracted from 8 times y.

9. The sum of x and y, diminished by z.

10. The sum of c and three times d, diminished by a.

11. x plus the product of y and z diminished by t.

12. The sum of a and b multiplied by the difference of x and y.

13. Three times x divided by six times y.

14. a plus b multiplied by x into y.

15. x diminished by y, divided by k multiplied by l.

16. Three times s plus the quotient of x divided by k.

17. Four times x square into y cube, plus seven times r square into s cube.

18. x third power minus a fourth power, divided by c minus e square.

19. x third power, minus a fifth power divided by c square, minus e third power.

20. The addition of a, b and c.

21. Of x, two times y, and three times z.

22. The subtraction of a from b.

23. Of the square of x from y cube.

24. Of a times x from c times z.

25. Express the multiplication of a, b and c in three different ways.

26. Write without the signs $\times$ or $\cdot$ the product of 3, x and y square.

27. Of 12, ab and c cube.

28. Express the division of a by c in two ways.

29. Of xy by $3ab$.

30. By how much does 7 exceed 4? a exceed b? x exceed 9?

Learn the definition of an "EQUATION."

Express No. 30 in the form of equations.

31. By how much is a less than 12? 5 less than n? c less than b? Express each of these in the form of an equation.

32. What must be added to 4 to make 11? to a to make 9? to c to make d? Equations?

33. By what must 29 be multiplied to make 783? 4, to make a? x, to make y? Equations?

34. What is the quotient when 1426 is divided by 46? when 15 is divided by a? when x is divided by 4? by k? Equations?

35. By what must 17 be multiplied to obtain 323? to obtain d? c, to obtain x? Equations?

36. By how much does $7x$ exceed $3x$? $5c$ exceed $4d$? a square exceed the product of m and n? Equations?

37. If x represent a whole number, what represents the next whole number above it? below it?

38. Write five consecutive numbers, the middle one being m.

39. How far can a man walk in x hours at the rate of 3 miles an hour? in s hours at the rate of t miles an hour?

40. If he walks a distance of k miles in the previous question, write the equations.

41. If a is one factor of 24, what is the other factor? Equation?

42. What dividend gives x as a quotient when 4 is the divisor? Equation?

43. If 12 be separated into two parts and one part is s, what is the other? Equation?

44. The difference of two numbers is 7 and one of the numbers is b, what is the other? Equation?

45. If 64 contains x four times, what is the value of x?

46. If the product is xyz and the multiplier y, what is the multiplicand? Equation?

47. What is the sum of $x + x + x +$ etc., written n times?

48. What is the shortest way of expressing the sum of eight y's? The product? Of x added m times? Used m times as a factor?

49. In the number 75 how many tens? units? Add 3 tens and 4 units so as to give the right sum. Equation?

50. What is the number whose ten's figure is x and unit's figure y? If the number equal 89, write the equation.

51. Bought two pounds of tea for y cents. What did it cost a pound?

52. If 7 bushels of oats cost a dollars, what is the cost per bushel in dollars? in cents?

53. Bought s dollars worth of flour at t dollars per barrel. How many did I buy?

54. What is the value of k books at x dollars each?

55. How many cents are there in x dollars? How many dollars are there in x cents?

56. Four men rode at the cost of p dollars. What did each pay?

57. If I buy 5 cows at 65 dollars each, and 4 horses at 125 dollars each, express the total cost. Equation?

58. Sold a sheep at k dollars each and lost x dollars. How much had I originally? Equation, if I have \$50 left? Equation, if they cost me \$50?

59. One boy has x cents and another a cents. If they divide equally, how much will each have? Equation, if they both together had 50 cents?

60. If I have m dollars and pay out n cents, how much have I left?

61. What will $m + n$ horses cost at x dollars each? Equation?

62. If I have y dollars in one purse and y cents in another, how many dollars have I? How many cents?

63. What will $a + b$ houses cost at $x + y$ dollars each? If the total cost is \$57,000, write the equation.

64. How many dollars will a pounds of tea cost at m cents a pound? Equation?

65. If $5m$ represents a man's age, what represents it three times x years ago? Equation, if he was fifteen then?

66. If I have x dollars and lose one fifth of it, how much have I left? Equation?

67. If I have x sheep and sell two thirds of them, represent what is left. Equation?

68. If I have $\frac{3}{4}(x - 10)$ dollars, and spend one third of it, how much have I left? Equation?

69. If 8 people hire a boat for $3x$ dollars, represent the share of each. Equation?

70. If x people hire a horse for n dollars, represent the share of each. Equation?

71. If the difference in the two preceding cases is \$2, what equation results?

72. Bought m pounds of sugar at n cents a pound, a pounds of rice at b cents a pound, and x pounds of soap at y cents a pound. How many cents was the bill? How many dollars? Equation?

73. Bought n pounds of coffee at x cents a pound and handed the grocer a y dollar bill in payment. How many cents and how many dollars in change? Equations?

74. There are 5 baskets, each with q mackerel, and 7 baskets, each with p mackerel. Divide the mackerel equally among the baskets.

75. A man left x bonds worth y dollars each, and m acres of land worth n dollars each, but he owed a dollars to each of b creditors. What was the value of the estate?

76. If a man can do a piece of work in 5 days, what part can he do in one day? In 3 days?

77. If he can do it in m days, what part can he do in one day? In seven days? In x days?

78. If I can do a piece of work in x days and work 12, what part of the work do I accomplish?

79. If you can do a piece of work in p days and work 4, and I in q days and work 6, what part will each do? Both together?

80. If you perform the x/y part of a piece of work, and I the a/n part, what will the sum of x/y and a/n represent?

81. Two numbers, a and b are to be added, their sum multiplied by x and the product divided by $p + q$. Express the quotient. Equation?

82. In the preceding question express the product resulting from multiplying the quotient by $p + q$.

83. The quotient of a divided by b is to be subtracted from the quotient of m divided by n, and the remainder multiplied by the sum of p and q divided by the difference between x and y.

84. The number a is to be increased by k, the sum to be multiplied by $c + d$, and s to be added to the product, and this sum divided by the product of x and y.

85. Express in three different ways, using signs only, the quotient of m divided by n, divided by the quotient of p divided by s.

86. The quotient of c divided by d is to be added to the quotient of x divided by z and the sum divided by the sum of a and u.

87. If $r + s$ houses each had $u + v$ rooms and each room had $k + l$ pieces of furniture, how many pieces were there?

88. Express four times the cube root of the sum of x, y square and z fourth power.

89. The difference of the squares of a and b, divided by the square of their difference.

90. Three times a, into the cube root of the difference of the squares of m and n, minus half their sum.

91. The product of the sum and difference of x and y.

92. The half sum plus the half difference of a and b, divided by the sum of their squares.

93. The ratio of six x divided by y, to m square divided by n, equals the ratio of 4 into the cube root of a, to y square z cube.

94. The square of the sum of any two quantities equals the sum of their squares plus twice their product.

95. The square of the difference of any two quantities equals the sum of their squares minus twice their product.

96. The product of the sum and difference of any two quantities equals the difference of their squares.

97. If p denote the principal, r the rate per cent., t the time in years, i the interest and a the amount, how would you express the interest? the time? the rate? the principal? the amount?

98. What are such expressions called? Translate each into words. What do these translations become?

99. A man works v days out of 24 and is idle the remainder. At seventy-five cents a day, what represents the board he must pay while idle?

100. If a man has $c - d$ horses and buys five times as many more, represent how many he will then have.

101. If a rectangular piece of land is m rods wide and 6 rods longer than wide, represent its length and the equation for its contents.

102. A rectangular lot is m rods long and n rods wide. What will it cost at x dollars per square foot? Equation?

103. How many rolls of paper a feet long by b feet wide will be required to paper a room x feet long, by y feet wide, by z feet high, if no allowance be made for windows? Equation?

104. What will be the cost at 30 cents per square yard?

105. If apples are sold x for a dime how many can be bought for y cents?

106. If lemons are sold m for a nickel, what will be the cost in cents of n lemons?

107. How many days must I work to earn \$8 at the rate of x dimes a day?

108. If it costs x persons y quarters for a dinner, how many dollars does each pay?

109. How many dimes must I pay out of q dollars so as to have left $10x$ cents?

110. Write a number which, when divided by x, gives a quotient of y and a remainder of z. Equation?

111. What is the remainder, if m divided by n gives a quotient of q? Equation?

112. What is the quotient, if a divided by b gives a remainder of r? Equation?

113. What is the divisor, if you have a remainder of r, a dividend of s, and a quotient of t? Equation?

114. A product is p and the multiplier $x + a$. What is the multiplicand? Equation?

115. A locomotive passes over k yards a second. How long will it be in traveling a mile? Equation?

116. A man has oats enough for x horses for y days. How long will they last z horses?

117. Into how many parts will a line be divided, if it be first divided into a parts, and afterwards each of these a parts be divided into a new parts, and so on n times?

118. A house cost a dollars and rents for n dollars per month. What per cent. does it pay?

119. If x represents a certain number of dollars, express half its value diminished by 50 cents.

120. The number of dimes in one third of it.

121. The number of quarters left after paying out 50 cents.

122. One man's share, if twice the amount diminished by \$7 be divided among 12 men.

123. A boy's share, if the amount increased by five nickels be divided among four boys.

124. The number of hats, if n hats cost \$3.

125. The charge per mile, if the amount buys a railway ticket for 24 miles.

126. If x represents the number of years in a man's age, express the age of his wife, whose age was three fifths of his twelve years ago.

127. The age of his wife 8 years hence.

128. If x represents the number of yards in the length of a line, express its fourth increased by an inch.

129. 129 feet diminished by twice its length.

130. The result obtained by taking from the length one half of it, one inch, and one fifth of the remainder.

131. The time required to traverse it at the rate of n rods a minute.

132. The time required to traverse it by a man walking s yards in t hours.

133. The time that a man who walks p miles per hour takes to walk it?

134. Express d per cent. of s.

135. What must I charge for a house which cost me u dollars in order to make fifteen per cent. on it?

136. If x per cent. of p is q, what is x per cent. of t?

137. If m gallons of wine be drawn from a cask holding n gallons, and the cask be then filled with water, how many gallons of wine will there be in x gallons of the mixture?

ADDITION.

There may be four cases in the addition of MONOMIALS and a fifth case, the addition of POLYNOMIALS.

CASE I.

The addition of like quantities with the same sign.

Add the following:

1. 7, 5, 3.
2. 4, 6, 9.
3. 17, 21, 35.
4. $-3, -4, -5$.
5. $-9, -8, -4$.
6. $-15, -25, -24$.
7. $6x, 9x, x, 2x$.
8. $-4y, -y, -5y, -3y$.
9. $3x^2, 4x^2, 5x^2, 2x^2$.
10. $6ab, 7ab, 5ab, 3ab$.
11. $-4x^3, -3x^3, -9x^3, -x^3$.
12. $-11ax, -5ax, -4ax, -7ax$.
13. $-3yz, -5yz, -8yz, -6yz$.
14. $7m^2n^2, 4m^2n^2, m^2n^2, 2m^2n^2$.
15. $-4pqx^2, -3pqx^2, -8pqx^2, -12pqx^2$.
16. $4ab^2c, 7acb^2, 12b^2ac, cab^2, 10acb^2, 2b^2ca$.
17. $-7cd^2, -9cd^2, -6cd^2, -8cd^2, -10cd^2, -5cd^2$.
18. $-8x^2y, -9x^2y, -4x^2y, -11x^2y, -x^2y, -5x^2y$.
19. $-4a^3c^5x^7, -9a^3c^5x^7, -a^3c^5x^7, -2a^3c^5x^7, 8a^3c^5x^7$.
20. $3(c-x), 5(c-x), 12(c-x), (c-x)$.
21. $3(a+b-c), 5(a+b-c), 2(a+b-c), (a+b-c)$.
22. $-4x^n, -7x^n, -5x^n, -x^n$.
23. $3x^ay^c, 6x^ay^c, 8x^ay^c, 9x^ay^c$.

24. $-2(x^3-y^3-z^3)$, $-10(x^3-y^3-z^3)$, $-21(x^3-y^3-z^3)$, $-(x^3-y^3-z^3)$.

25. $-3a^2b^3c^4$, $-9a^2b^3c^4$, $-5a^2b^3c^4$, $-17a^2b^3c^4$.

26. $7\sqrt{x}$, $19\sqrt{x}$, $3\sqrt{x}$, $\sqrt{x}$.

27. $-5a\sqrt{y}$, $-4a\sqrt{y}$, $-a\sqrt{y}$, $-6a\sqrt{y}$.

28. $4p^{\frac{1}{2}}$, $9p^{\frac{1}{2}}$, $8p^{\frac{1}{2}}$, $p^{\frac{1}{2}}$.

29. $-3c^{\frac{3}{2}}$, $-5c^{\frac{3}{2}}$, $-8c^{\frac{3}{2}}$, $-16c^{\frac{3}{2}}$.

30. $-3x\sqrt{a-b}$, $-4x\sqrt{a-b}$, $-9x\sqrt{a-b}$, $-15x\sqrt{a-b}$.

31. $5q^{m-n}$, $11q^{m-n}$, $10q^{m-n}$, q^{m-n}.

32. $-8x^{n/2}$, $-10x^{n/2}$, $-18x^{n/2}$, $-36x^{n/2}$.

33. $3(a-x)\sqrt[4]{y^3}$, $7(a-x)\sqrt[4]{y^3}$, $12\sqrt[4]{y^3}(a-x)$, $(a-x)\sqrt[4]{y^3}$,

34. $\frac{1}{2}a^2x^3$, $\frac{3}{2}a^2x^3$, $\frac{5}{2}a^2x^3$, $\frac{4}{2}a^2x^3$.

35. $-\frac{2}{3}(c^2-d^2)$, $-\frac{1}{3}(c^2-d^2)$, $-\frac{5}{3}(c^2-d^2)$, $-\frac{7}{3}(c^2-d^2)$,

36. $\frac{2}{a}(x^3-2x+1)$, $\frac{3}{a}(x^2-2x+1)$, $\frac{7}{a}(x^3-2x+1)$, $\frac{1}{a}(x^3-2x+1)$.

37. $-7a^{\frac{4}{3}}b^{\frac{1}{2}}c^n$, $-9a^{\frac{4}{3}}b^{\frac{1}{2}}c^n$, $-a^{\frac{4}{3}}b^{\frac{1}{2}}c^n$, $-20a^{\frac{4}{3}}b^{\frac{1}{2}}c^n$.

38. $3c^2d-a^5c$, c^2d-a^5c, $4c^2d-a^5c$, $7c^2d-a^5c$, c^2d-a^5c, $16c^2d-a^5c$.

39. $-x^ny^2+8s^2t^3$, $-x^ny^2+3s^2t^3$, $-x^ny^2+s^2t^3$, $-x^ny^2+5s^2t^3$, $-x^ny^2+4s^2t^3$, $-x^ny^2+15s^2t^3$.

40. $-4p^2q^3-a^5b$, $-p^2q^3-a^5b$, $-6p^2q^3-a^5b$, $-11p^2q^3-a^5b$, $-1p^2q^3-a^5b$, $-2p^2q^3-a^5b$.

41. $5\sqrt{x}(a^2-b^2)(c+d)^3$, $8\sqrt{x}(a^2-b^2)(c+d)^3$, $7\sqrt{x}(c+d)^3(a^2-b^2)$, $3(a^2-b^2)\sqrt{x}(c+d)^3$,

Case II.

The addition of like quantities with unlike signs.

Add the following :

42. $+6, -5.$

43. $-6, +5.$

44. $+9, -5.$

45. $-9, +5.$

46. $-19, +30.$

47. $+8, -7, -6.$

48. $+8, -7, +6.$

49. $-8, +7, +6.$

50. $-8, -7, -6.$

51. $-8, -7, +6.$

52. $5, -10, +4, +7.$

53. $-6, -8, -9, +25.$

54. $-25, +24, -23, +21.$

55. $3z, -8z.$

56. $-9x^3, +4x^3.$

57. $7d^n, -2d^n.$

58. $-6a^2c, -7a^2c.$

59. $-11\sqrt{x}, 8\sqrt{x}.$

60. $3a^{-1}, 5a^{-1}.$

61. $-4a^m z^n, -5a^m z^n, 13a^m z^n.$

62. $5x^{\frac{1}{2}}y^{\frac{1}{3}}, 8x^{\frac{1}{2}}y^{\frac{1}{3}}, -15x^{\frac{1}{2}}y^{\frac{1}{3}}.$

63. $-c^2x^n, -3c^2x^n, 2c^2x^n.$

64. $8w^{n-1}, -4w^{n-1}, -3w^{n-1}.$

65. $8k, -7k, -6k, 3k.$

66. $4wx, +4wx, -2wx, -7wx.$

67. $6b^2c, -5b^2c, -4b^2c, -2b^2c.$

68. $u^2, 4u^2, -3u^2, 5u^2.$

69. $7ac, -3ac, -5ac, 8ac, -ac.$

70. $-2pq, -pq, -3pq, -9pq, pq, 12pq.$

71. $-7abc, -4abc, -3abc, -5abc, 20abc, -abc.$

72. $4(x+y), -7(x+y), -11(x+y), 8(x+y), 9(x+y), (x+y).$

73. $9\sqrt{x^3}$, $\sqrt{x^3}$, $-7\sqrt{x^3}$, $-3\sqrt{x^3}$, $-13\sqrt{x^3}$, $-\sqrt{x^3}$.

74. $3c\sqrt{ab}$, $9c\sqrt{ab}$, $4c\sqrt{ab}$, $-11c\sqrt{ab}$, $-5c\sqrt{ab}$, $c\sqrt{ab}$.

75. $ac\sqrt{x-u}$, $-3ac\sqrt{x-u}$, $5ac\sqrt{x-u}$, $-11ac\sqrt{x-u}$.

76. $\frac{2}{3}ab$, $-\frac{5}{8}ab$, $\frac{1}{8}ab$, $\frac{7}{8}ab$, ab, $-\frac{2}{3}ab$.

77. $-\frac{1}{4}\sqrt[3]{v^n}$, $2\sqrt[3]{v^n}$, $\frac{3}{4}\sqrt[3]{v^n}$, $\frac{1}{2}\sqrt[3]{v^n}$, $\frac{5}{4}\sqrt[3]{v^n}$, $-\sqrt[3]{v^n}$.

78. $s^3\sqrt{a^3k^5}$, $-2s^3\sqrt{a^3k^5}$, $-5s^3\sqrt{a^3k^5}$, $-7s^3\sqrt{a^3k^5}$, $s^3\sqrt{a^3k^5}$, $4s^3\sqrt{a^3k^5}$.

79. $a^cx^{\frac{1}{2}}$, $-5a^cx^{\frac{1}{2}}$, $9a^cx^{\frac{1}{2}}$, $11a^cx^{\frac{1}{2}}$, $-19a^cx^{\frac{1}{2}}$, $5a^cx^{\frac{1}{2}}$.

80. $-5p^t\sqrt[3]{x^a}$, $3p^t\sqrt[3]{x^a}$, $-11p^t\sqrt[3]{x^a}$, $16p^t\sqrt[3]{x^a}$, $-p^t\sqrt[3]{x^a}$, $21p^t\sqrt[3]{x^a}$.

81. $-3a^{-n}x^{-2}$, $-7a^{-n}x^{-2}$, $8a^{-n}x^{-2}$, $5a^{-n}x^{-2}$, $-4a^{-n}x^{-2}$, $a^{-n}x^{-2}$.

82. $8x^{-n/3}$, $-5x^{-n/3}$, $-11x^{-n/3}$, $6x^{-n/3}$, $-13x^{-n/3}$, $12x^{-n/3}$.

Find the algebraic sum of

83. $8u - 11u + 13u - 15u + 10u$.

84. $11 + (-5) - (+3) + (+8) + (-7)$.

85. $-12 - (+8) + (-5) + (+6) + (-7)$.

86. $-5x^2 + (+8x^2) - (+3x^2) + (-4x^2) - x^2$.

87. $7ab + (-5ab) - (+2ab) + (+4ab) + ab$.

88. $-3a^2x^3 + (-2a^2x^3) - (+7a^2x^3) + (+a^2x^3) - a^2x^3$.

89. $ax + 3ax - 7ax + 9ax - ax$.

90. $7cd - 5cd + 6cd - cd - 7cd$.

91. $13pqn^2 - pqn^2 + 5pqn^2 - 8pqn^2 + 3pqn^2$.

92. $8(a-b) - 4(a-b) + 3(a-b) - (a-b) - 13(a-b)$.

93. $-a^2c^3+7a^2c^3-5a^2c^3-9a^2c^3+10a^2c^3$.

94. $7\sqrt{x}-9\sqrt{x}-\sqrt{x}+4\sqrt{x}+12\sqrt{x}$.

95. Add $x-y$, $-2x-7y$, $7x-2y$, $-3x+3y$, $-8x+y$.

96. Add $2ab+3bc$, $3ab-2bc$, $4ab-bc$, $-5ab+4bc$.

97. Add $3m^2-4mn^2$, $-4m^2-3mn^2$, m^2+mn^2, $-m^2-mn^2$.

98. Add $5ax^2+7a^2y$, $-3ax^2-5a^2y$, $9ax^2-10a^2y$, $-4ax^2+a^2y$.

99. $7abcd-9abcd-13abcd+18abcd$.

100. $\frac{1}{2}x-\frac{1}{3}x+x+\frac{2}{3}x+\frac{1}{6}x$. **101.** $\frac{2}{3}c-\frac{1}{2}c+\frac{3}{4}c+\frac{1}{6}c-c$.

102. $-3x+\frac{1}{4}x-\frac{3}{2}x+x-\frac{1}{2}x$.

103. $-\frac{5}{8}a^2-a^2-\frac{1}{4}a^2-\frac{4}{3}a^2-\frac{3}{4}a^2$.

104. $3(a-z)-12(a-z)+5(a-z)+10(a-z)$.

105. $-4(x+2y)^2+5(x+2y)^2-12(x+2y)^2+20(x+2y)^2$.

106. $12ac^{\frac{1}{2}}-5ac^{\frac{1}{2}}-4ac^{\frac{1}{2}}+3ac^{\frac{1}{2}}-5ac^{\frac{1}{2}}$.

107. $-5c^{\frac{3}{4}}d^2-2c^{\frac{3}{4}}d^2+8c^{\frac{3}{4}}d^2+3c^{\frac{3}{4}}d^2-4c^{\frac{3}{4}}d^2$.

Case III.

The addition of unlike numbers, *i. e.*, no two alike.

Add the following:

108. $2a$, $+3b$, $-4c$. **109.** a^2, $-2ab$, $+b^2$.

110. $4c^3$, $4c^3y$, $-2y^3$, $-2cy^3$. **111.** ax, $-bx$, $+cx$.

112. $z^{\frac{1}{2}}$, $-z^{\frac{1}{3}}$, $z^{\frac{3}{4}}$, $-z^{\frac{3}{5}}$.

Case IV.

The addition of quantities, some like and others unlike.

Add the following:

113. $4ac,\ 3a^2c,\ -7a^2c,\ -a^2c,\ 5ac,\ 2ac^2.$

114. $-4pq,\ -5pq^2,\ 6pq,\ -3pq,\ 9pq^2,\ -p^2q^2.$

115. $10xy,\ -6yz,\ -4xy,\ 12yz,\ -5xy,\ 3xyz.$

116. $c^3,\ -3c^2d,\ -cd^2,\ 4c^2d,\ 5cd^2,\ d^3.$

117. $6\sqrt{st},\ -8s^2t^2,\ -9\sqrt{st},\ 3\sqrt{st},\ 11s^2t,\ -s^2t.$

118. $5uvx^2,\ -9vx^2,\ -5ux^2,\ 7uvx^2,\ 4v^2x^2,\ 19ux^2.$

119. $5qr - 7fg + 3n - 2qr + 4an + 5fg - 13 + 7n - 3 - qr - 5an + fg.$

120. $3a^2b^3 - 7ab^4 + 5axy - 7a^2b^3 - 2ab^4 - axy + ab^4 - 7axy + 8a^2b^3 - 10ab^4 + a^2b^3 + 3axy - 5a^2b^3 + 18ab^4.$

121. $7a - 5y^3 + 8\sqrt{x} + 2a + 5y^3 - \sqrt{x} - 9a + 7\sqrt{x}.$

122. $3x^{\frac{1}{2}} + 4yz - u^{-2} + 10 - 5x^{\frac{1}{2}} + 6yz - 15 + 2u^{-2} - 9yz - 4x^{\frac{1}{2}} - 10u^{-2} + 21.$

123. $4x^3 - 5a^3 - 5ax^2 + 6a^2x + 6a^3 + 3x^3 + 4ax^2 + 2a^2x - 17x^3 - 39 + 19ax^2 - 15a^2x + 13ax^2 - 27a^2x + 18a^3 + 3a^2x - 20a^3 + 12x^3 + 31a^2x - 2x^3 - 31ax^2 - 7x^3.$

Addition of Polynomials.

Add the following:

124. $a - b,\ b - c,\ c - d,\ d - c.$

125. $p - q - s,\ q - s - p,\ s - p - q.$

126. $x^3 + 1,\ x^2 + 1,\ x + 1,\ 1 - x,\ 1 - x^2.$

127. $7(x - y) - 6(x - y) + 2(x - y) - 5(x + y).$

128. $8\sqrt{u-x} - 9\sqrt{u-x} + 3\sqrt{u+x} - \sqrt{u-x}$.

129. $3(x^2-a) + 2(x^2-a) - (a-x^2) + 6(x^2-a) + (x^2-a)$.

130. $7y - 4(a+b)$, $6y + 2(a+b)$, $-2y + (a+b)$, $y - 3(a+b)$.

131. $x^2 - 2y^2$, $3x^2 + 5y^2$, $4x^2 - 6y^2 + z^2$, $8y^2 - 2z^2$, $-7x^2 + 5y^2 - 6z^2$.

132. $4abc - 2yz^2 + 3y^3$, $5abc - 2y^3 + 6yz^2$, $yz^2 - 4abc - 9y^3$, $-7abc - 3yz^2 + y^3$, $-9y^3 + 6abc + yz^2$.

133. $21ax + 13abc + 2ay - abc$, $13abc + 14ax - ax - 5ay$, $3ay - 27abc + 3ax$.

134. $5a^3b^3 - 8a^2b^3 + x^2y + xy^2$, $4a^2b^3 - 3xy^2 - 7a^3b^2 + 6x^2y$, $5xy^2 + 3a^2b^3 - 3x^2y + 3a^3b^2$, $-3x^2y - a^3b^2 + 2a^2b^3 - 3xy^2$.

135. $\frac{1}{2}x - \frac{1}{3}y$, $\frac{2}{3}x - y$, $x - \frac{3}{4}y$, $x + y$,

136. $\frac{2}{3}a^2 + \frac{1}{3}ac - \frac{1}{4}c^2$, $-a^2 - \frac{2}{3}ac + 2c^2$, $\frac{1}{3}a^2 - ac - \frac{5}{4}c^2$.

137. $\frac{3}{2}k^2l - \frac{3}{4}kl^2 + 2l^3$, $\frac{1}{2}k^3 - 2k^2l - \frac{3}{2}l^3$, $-\frac{3}{2}k^3 + kl^2 + \frac{1}{2}l^3$.

138. $3s^3 - \frac{2}{3}st - \frac{1}{2}t^3$, $-\frac{3}{2}s^3 + 2st - \frac{2}{3}t^3$, $-\frac{2}{3}s^3 - st + t^3$.

139. $a + b + c$, $3b - x + y$, $5(a+b) + 3x$, $7c - 3m^2n$, $5ab + 6b - 3y$, $3(x+y) - 8c$.

140. $10a^2b - 12a^3bc - 15b^3c^4 + 10$, $-4a^2b + 8a^3bc - 9$, $-10b^3c^4$, $-3a^3bc - 3a^2b - 3 + 20b^3c^4$, $+2 + 5b^3c^4 + 12a^3bc + 2a^2b$.

141. $4\sqrt{x} + 2a^2x - 3 - \sqrt{x} - a^2x + 10y - 18 + 4\sqrt{x} - a^2x - 3\sqrt{x} - 5y + 16 - 5y + 5 - 4\sqrt{x}$.

142. $3(x+y) - 4c + 6 - 3ax - 5x - 8 + 5(x+y) + 5d - 8y + ax + 2$.

143. $5(p+q) - 3st - 4k + 24 - 8st + 3(p+q) + 15st - 4q + 2(st-k) - 3p$.

144. $2l^3m + 9l^2 - 6ln + 8 + 2l^2 - 4 + 16l^3m - 10l^2 + 7ln + 4l + 8l^2 - 20 + 6(l - m) - 8l - 20l^3m - 7l + 7m + 8 + 2l^3m - 9l^2 + 4l + (l - m)$.

145. $4w + 2\sqrt{u-1} + 8w^2 - 2\sqrt{u^2-1} - 12wu - 6w + 7\sqrt{u-1} + 2w + 3\sqrt{u^2-1} - 9\sqrt{u-1} + 8wu - 8\sqrt{u^2-1}$.

146. $a^2 - 2ab + b^2 - 2bc + 2ca + c^2$, $b^2 - 2cb + c^2 - 2ac + 2ba + a^2$, $c^2 - 2ac + a^2 - 2ba + 2bc + b^2$.

147. $18a^4bcd + 21a^2 - 3c - 3a^2 + 2c + 19a^4bcd - 3a^2 + 24c - 2a^4bcd + 5a^4cdb - 3a^2 - 2c + 12a^4dcb$.

148. $9a^2b^3c^4 + 8a^3b^2c^4 - 7a^4b^2c^3 + 21a^3b^2c^4 - 5a^2b^3c^4 - 11a^2b^3c^4 + 2a^3b^3c^4 + 21a^4b^2c^3 + 5a^3b^3c^4 - 2a^3b^2c^4$.

149. $7x^{\frac{1}{2}}y - 2x\sqrt{y}$, $\sqrt{xy} + 3xy^{\frac{1}{2}} + 2$, $3y\sqrt{x} - \sqrt{yx} - 6$, $9y\sqrt{x} - 4y^{\frac{1}{2}}x - 3$, $1 + 7xy^{\frac{1}{2}} - 2yx^{\frac{1}{2}}$.

150. Show that, if $a = s + 2t - 3r$, $b = t + 2r - 3s$, and $c = r + 2s - 3t$, then will $a + b + c = 0$.

151. Show that, if $x = 5m - 3n - 2p$, $y = 5n - 3p - 2m$, and $z = 5p - 3m - 2n$, then will $x + y + z = 0$.

152. $\frac{1}{4}mz + mx - \frac{1}{2}my - \frac{1}{3}mu + my + \frac{5}{6}mu - \frac{1}{2}mz + \frac{3}{4}mx + mz + 2my + \frac{1}{2}mu + \frac{1}{4}mx + mx + my + mz + mu$.

153. $\frac{1}{4}cg - \frac{2}{3}ck + \frac{1}{3}c^2g - \frac{1}{2}cg^2$, $c^2g + 2ck - cg$, $\frac{1}{2}cg - \frac{1}{2}c^2g + cg^2$, $2c^2g - \frac{1}{3}cg^2 - \frac{5}{6}ck$.

154. $(x + y) + 2x(z + y)$, $z(z - y) + x(x + y) - (z + y)$, $(x + y) - (z - y) - x(z + y)$, $4(z - y) + (x + y)$.

155. $\frac{7}{12}a^3b^2 + a^2b^3 + \frac{1}{2}ac - \frac{2}{3}ab + ab - a^3b^2 - ac + \frac{3}{4}ac - \frac{3}{4}ab - \frac{2}{3}a^2b^3 + ab + \frac{3}{4}a^3b^2 + \frac{5}{6}a^2b^3$.

156. $17c^3d^3e^4 - 8cd^2e^3 + 4c^4d^5e^6 - 3c^3d^4e^5 + 4cd^2e^3 - 3c^4d^5e^6 + 14c^2d^3e^4 + 5c^3d^4e^5 - c^4d^5e^6 + 19c^2d^3e^4 - 2c^3d^4e^5 + 4cd^2e^3 + 9c^2d^3e^4 + 2c^3d^4e^5 + 5cd^2e^3 - 7c^4d^5e^6 + 4c^4d^5e^6 - 12cd^2e^3 - c^3d^4e^5 - 15c^2d^3e^4 - c^3d^4e^5 + 7cd^2e^3 + 41c^2d^3e^4 + 3c^4d^5e^6$.

157. $z^3 + \sqrt{w} - 10 + 2\sqrt{zw}$, $2z^3 - 3\sqrt{w} + 20 + \sqrt{zw}$, $-5z^3 - 3\sqrt{w} - 5\sqrt{zw} + 15\sqrt{4}$, $2\sqrt{w} + \sqrt{1} + 12\sqrt{zw} - 4z^3$, $-25z^3 - 25\sqrt{w} - 25\sqrt{zw} - 5\sqrt{25}$.

158. $a^m - 2b^n + 3c^p - 4d^q$, $23a^m - 13c^p + 12d^q + 16b^n$, $-14a^m + 15b^n + 19d^q - 17c^p$, $-b^n + 18a^m + c^p - d^q$, $-3c^p + 11d^q - 13a^m + 5b^n$.

Grouping similar terms into single terms with compound (polynomial) coefficients.

Group the following:

159. ax, bx. **160.** ez, z. **161.** ex, $-dx$.

162. $3x$, $-3y$. **163.** $-w$, cw. **164.** $-5x$, ax, $-x$.

165. $3st$, $-4rs$, $2st$. **166.** mn, $8m$, $-9m$.

167. $8a^2b^3x$, $4b^3c^4x$, $-9b^3c^4x$.

168. $4(k-l)$, $-2(k-l)$. **169.** $3(p-q)$, $(p-q)$.

170. $x(a-1)$, $-(a-1)$. **171.** $-8(1-x)$, $a(1-x)$.

172. $4(a+1)$, $c(a+1)$, $-3(a+1)$, $-3c(a+1)$.

173. $8(m+n)$, $-7a(m+n)$, $-11(m+n)$, $5(m+n)$.

174. $mnx + 2by + pqx - 4by$.

175. $7c(x^2-y^2)$, (x^2-y^2), $5(x^2-y^2)$, $-4c(x^2-y^2)$.

176. $ax + bx + cx$. **177.** $x^3 - 3ax + 4cx$.

178. $ay^3 - ac + ad^2$. **179.** $4axy + cxy - axy$.

180. $fz^3 + 3gz^2 + z^3$. **181.** $3x$, bx, $(a+b)x$.

182. $(a+b)x$, $(a-c)x$. **183.** $a\sqrt{x}$, $-a\sqrt{y}$, $+a\sqrt{z}$.

184. $x^3 - 3x^2y + 5x^3y^2$. **185.** $3x^2y^3 - 5x^3y^3 - x^3y^3$.

186. $ax + by - cx - dy$. **187.** $bcxy + adxy - acxy - xy$.

188. $7x^2 - 12y + 5x + 14y^2$.

189. $3ax^3 - 3ax^2 - 3axy^2 - 6ay^4$.

190. $bz - 4dz + 6az + dz + 4bz - 2az$.

191. $2pq - apq + 6pq - 4apq$.

192. $aw + s - bw + t$.

193. $cx^3 + dx + ex^2 - fx$.

194. $cu + dv + su - tv$.

195. $5ax + 5bx$, $by + 3x - 4y$, $x - 5y$.

196. $ax^2 + by^2 + cxy$, $mx^2 - ny^2 - pxy$.

197. $ax^3 + bx^2 + cx$, $a^2x^3 - b^3x^2 - c^4x$.

198. $6a - 6(x-y) + 7$, $3a + 9(x-y) - 8$, $2(x-y) - 5a - 15$.

199. $(a - b + c)\sqrt{x}$, $(a + b - c)\sqrt{x}$, $(b + c - a\sqrt{x}$.

SUBTRACTION.

Subtraction of Monomials.

Subtract the following:

1. From 7 take 4.
2. $7, -4$.
3. $-7, 4$.
4. $-7, -4$.
5. $4, 7$.
6. $4, -7$.
7. $-4, 7$.
8. $-4, -7$.
9. $9a, 3a$.
10. $9a, -3a$.
11. $-9a, 3a$.
12. $-9a, -3a$.
13. $2x, 7x$.
14. $2x, -7x$.
15. $-2x, 7x$.
16. $-2x, -7x$.
17. $5x^2, 9x^2$.
18. $11c^3, -8c^3$.
19. $5x^2y^2, -7x^2y^2$.
20. $a^3c, 6a^3c$.
21. $2\sqrt{x}, 3\sqrt{x}$.
22. $p^3, -p^3$.
23. x^2, x^3.
24. a, k.
25. $-s, -t$.
26. $-7c, +7c^3$.
27. $-8x, x^3$.
28. $\sqrt{x}, \sqrt[3]{x}$.
29. $2x^{\frac{1}{2}}, -3x^{\frac{1}{2}}$.
30. $3x^{\frac{1}{3}}, 2x^{\frac{1}{3}}$.
31. $3\sqrt[5]{k}, 9\sqrt[5]{k}$.
32. $a^n, 3a^n$.
33. $7a^1, -4a$.
34. $3x^{n-1}, 4x^{n-1}$.
35. x^p, x^q.
36. $2a^{n+1}, a^{n-1}$.
37. $5e^2, 3e^{-2}$.
38. $7c^{-n}, 3c^{-n}$.
39. $c^{-3}, 4c^{-3}$.
40. $y^{-a}, -y^a$.
41. $9y^{\frac{3}{2}}, 5y^{\frac{3}{2}}$.
42. $3\sqrt{x^3}, -5^3\sqrt{x^3}$.
43. $3x^{a'}, x^{/a}$.
44. $8c^{n+1}, 5c^{1+n}$.
45. $4z^{n-1}, 2z^{1-n}$.
46. $4x^3\sqrt[5]{8a}, -x^3\sqrt[5]{8-a}$.
47. $2^3a^4, 3^2a^3$.
48. $7x^{\frac{2}{3}}, 5x^{\frac{3}{3}}$.
49. $3^3x^3y^3z^3, -2^3x^3y^3z^3$.

50. $3a^n x^{\frac{2}{3}}$, $-7a^n x^{\frac{2}{3}}$.

51. $5x^2$, $y^{-3}z^m$, $12z^m y^{-3}x^2$.

52. $14\sqrt[3]{xy^{\frac{1}{2}}}$, $5x^{\frac{1}{2}}\sqrt{y}$.

53. $-x^{-\frac{1}{2}}$, $-3x^{-\frac{2}{3}}$.

54. $8c^{\frac{4}{5}}\sqrt[n]{d^2}$, $2\sqrt[5]{c^4 d^{2/n}}$.

55. $4x^{c-d}$, $-3x^{c+d}$.

56. $-\sqrt[n]{x^2}$, $-\sqrt[m]{x^3}$. **57.** $a^{n/2}$, $3a^{n/2}$. **58.** $v^{a/2}$, $-5v^{3/a}$.

59. $3b^{m-n}$, $3b^{n-m}$.

60. $7c$, $(-3c)$.

61. $-2xy$, $(+3xy)$.

62. $2a^2x^2$, $(+8a^2x^2)$.

63. $-5ef$, $(-3ef)$.

64. $17\sqrt{x}$, $(-20\sqrt{x})$.

65. $3x^2y^3z^4$, $(+x^4y^3z^2)$.

66. $7a^2c - (-13a^2c) + (-10a^2c)$.

67. $15p^2q^3 - (-p^2q^3) - (+p^2q^3)$.

68. $11a^{\frac{1}{2}}b^{\frac{1}{3}} - (+5a^{\frac{1}{2}}b^{\frac{1}{3}})$.

69. $-5a^n\sqrt[3]{x} + (-a^n\sqrt[3]{x})$.

70. $-7\sqrt[3]{x-4} - [-5\sqrt[3]{x-4}]$.

71. $6x^{-3}y^{-2} - (-7x^{-3}y^{-2})$

72. $x^{-m}y^{-n} - (+x^{-n}y^{-m})$.

73. a, $\frac{1}{2}a$.

74. $4x$, $\frac{2}{3}x$.

75. $\frac{1}{3}y$, $-\frac{2}{3}y$.

76. $\frac{1}{2}z$, $-\frac{1}{3}z$.

77. $\frac{1}{2}a^2x$, $\frac{2}{3}a^2x$.

78. $\frac{1}{2}c^3d^3$, $\frac{1}{4}c^3d^3$.

79. $-\frac{1}{3}cde$, $-\frac{1}{2}cde$.

80. $-\frac{3}{4}x^2y^3z^4$, $-\frac{5}{4}x^2y^3z^4$.

81. $a(c-d)$, $-2a(c-d)$.

82. $9x^n(s-t)^2$, $-3a^n(s-t)^2$.

83. $-\frac{2}{3}a\sqrt[3]{x^2}$, $\frac{1}{2}a\sqrt[3]{x^2}$.

84. $\frac{1}{2}a^{\frac{1}{2}}$, $-\frac{1}{3}a^{\frac{1}{2}}$.

85. $\frac{2}{3}\sqrt{xy}$, $-\frac{1}{2}\sqrt{xy}$.

86. $\frac{3}{n}x^2$, $-\frac{4}{n}x^2$.

87. $\frac{7}{a}\sqrt{x^3}$, $\frac{3}{a}\sqrt{x^3}$.

88. $\frac{3}{4}x^{1/n}y^{\frac{1}{2}}$, $-\frac{1}{2}\sqrt[n]{x}\cdot\sqrt{y}$.

89. $-c^m d^n$, $-3c^n d^n$.

90. $\frac{1}{2}a^{n-1}(3x-2y)$, $\frac{3}{4}a^{n-1}(3x-2y)$.

91. $7a(s^2-t^2)$, $-11a(s^3-t^2)$.

92. $\frac{1}{6}a^n(\sqrt{x} - \sqrt[3]{y})$, $a^n(\sqrt{x} - \sqrt[3]{y})$.

93. $a^{\frac{1}{2}}b^{\frac{1}{3}}\sqrt{c+u}$, $-\frac{2}{3}\sqrt{a} \cdot \sqrt[3]{b}(u+c)$.

94. $x^{2m}y^{3n}(c-d)^s$, $\frac{1}{3}y^{3n}x^{2m}\sqrt[s]{c-d}$.

95. $\frac{1}{4}s^{\frac{3}{2}}t^{\frac{3}{4}}(x-y)^{1/n}$, $-\frac{3}{4}\sqrt[3]{s^2} \cdot \sqrt[4]{t^3}\sqrt[n]{x-y}$.

96. $\frac{2}{a}x^{\frac{1}{2}}z^{\frac{3}{4}}(e-f)^{2/n}$, $\frac{1}{a}\sqrt{x} \cdot \sqrt[3]{z^3}\sqrt[n]{(-f+e)^2}$.

Express the answers to the following in two different ways:

97. ac, bc.

98. cx, x.

99. d, $-ad$,

100. $-e$, $-de$.

101. ax, $-7x$.

102. $-4s$, $-rs$.

103. st, rs.

104. mn, n.

105. mn, $-m$.

106. $-ax$, $-3x$.

107. $-4y$, $-zy$.

108. $7st$, rst.

109. $5a$, $6ab$.

110. $3\sqrt{x}$, $-a\sqrt{x}$.

111. $a^2b^3c^2$, $-a^2b^3d^2$.

112. $-7a^4y^3z^2$, $-9x^4y^3z^2$.

113. $7ax\sqrt[3]{y}$, $5az\sqrt[3]{y}$.

114. $-a^2$, $-5a^2y^2$.

115. x^my^n, $5x^my^nz$.

116. $\frac{1}{3}x\sqrt{a}$, $-\frac{2}{3}y\sqrt{a}$.

117. $r^3s^3t^4$, $r^3s^4t^5$.

118. $a^2b^3c^4$, $-a^4b^3c^2$.

119. $6u^3v^4w^5$, $3u^5v^4w^3$.

120. $7c^2d^3e^4$, $-9c^4d^3e^2$.

121. $3x^{n-1}y^2$, $3x^{n-1}z^2$.

122. $4y^{\frac{3}{2}}z^{\frac{3}{4}}$, $-7y^{\frac{3}{2}}z^{\frac{3}{4}}u^{\frac{3}{4}}$.

123. $7x^{n+1}\sqrt[3]{a}$, $-5x^{n+1}\sqrt{a^3}$.

124. $12a^{n-1}\sqrt{x^3}$, $-5a^{n+1}\sqrt{x^3}$.

125. $\frac{2}{3}a^{\frac{1}{2}}x^2$, $-\frac{1}{2}a^{\frac{1}{2}}$.

126. $\frac{3}{5}x^my^nz$, $\frac{2}{5}x^my^nw$.

SUBTRACTION OF POLYNOMIALS.

Subtract the following:

(Solve examples 127–141 both arithmetically and algebraically and compare the results.)

127. From $9 + 5$ take $6 + 4$.

128. $9 + 5,\ 6 - 4.$

129. $9 + 5,\ 6 + 5.$

130. $9 + 5,\ 6 - 5.$

131. $9 + 5,\ 6 - 6.$

132. $18 - 4,\ 5 + 3.$

133. $18 - 4,\ 5 - 3.$

134. $18 - 4,\ 5 + 4.$

135. $18 - 4,\ 5 - 4.$

136. $18 - 4,\ 5 + 5.$

137. $18 - 4,\ 5 - 5.$

138. $27 + 5,\ 13 - 8.$

139. $19 + 12,\ 15 + 14.$

140. $24 - 9,\ 13 - 11.$

141. $28 - 6,\ 16 + 3.$

142. $a + b,\ a.$

143. $a + b,\ -b.$

144. $a + b,\ a - b.$

145. $a + b,\ -a + b.$

146. $a + b,\ -a - b.$

147. $x - y,\ y.$

148. $x - y,\ -x.$

149. $x - y,\ -y.$

150. $x - y,\ x + y.$

151. $x - y,\ -x + y.$

152. $x - y,\ -x - y.$

153. $c,\ c - d.$

154. $c,\ d.$

155. $d - 7,\ d - 9.$

156. $c + 4,\ d - 7.$

157. $p - q,\ q - p.$

158. $e^2 f,\ ef^2 + 1.$

159. $8x - y,\ 3z + y.$

160. $3\sqrt{x} + 1,\ 5\sqrt{x} - 5.$

161. $a \cdot b - 3,\ b \cdot a + 3.$

162. $7 \cdot 4x^n,\ 4 \cdot 7x^n - 9.$

163. $-9 - 6y,\ -10 + y.$

164. $-f^2g^3,\ 8\sqrt[3]{x^3} - f^2g^3.$

165. $5a^2x^3z,\ 25a^2x^3z.$

166. $25abx,\ 100abcx.$

167. $5x - 4,\ 8y - 3.$

168. $a^2 - b^2,\ a^2 - 2ab + b^2.$

169. $-a^n,\ a^n - b^n.$

170. $-y^q,\ x^p - y^q.$

171. $c^n - d^m,\ -c^n.$

172. $z^n,\ -w^n - z^n.$

173. $\frac{2}{3}x - \frac{3}{4}y,\ \frac{3}{4}y - \frac{2}{3}x.$

174. $\frac{2}{3}s - \frac{4}{5}t,\ \frac{5}{6}s - \frac{1}{10}t.$

175. $\frac{3}{5}m - \frac{7}{8}n,\ -\frac{3}{10}m.$

176. $\frac{5}{6}a - \frac{5}{6}c,\ \frac{2}{3}c.$

177. $x + y,\ \frac{7}{12}y.$

178. $a - b,\ 3\frac{1}{2}b.$

179. $7x^{\frac{1}{2}} - y^n,\ -y^n + 7z^{\frac{1}{2}}$.

180. $-1 + 4a\sqrt{x},\ -6 - 3b\sqrt{x}$.

181. $a^{n-1}b^2 + x,\ a^2b^{n-1} + x$.

182. $2\sqrt{x} - 3\sqrt[3]{y},\ -3\sqrt[3]{y} + 4\sqrt[4]{z}$.

183. $-3x^{m/n} - 2a^{p/q},\ -5x^{n/m} - 2a^{p/q}$.

184. From $x^2 - 2xy + y^2$ take $x^2 + 2xy + y^2$.

185. From $a^2 + 2ab + b^2$ take $a^2 - 2ab + b^2$.

186. $a^2 - ab + b^2,\ b^2 - 3ab + a^2$.

187. $abx,\ cx + dx - zw$.

188. $abx,\ 3cx + dx + ex$.

189. $8xy,\ abxy - cdxy - efxy$.

190. $7cd + efd,\ 4cd - pq$.

191. $2u^2 - 6u + 1,\ 7u^2 - 4u + 4$.

192. $7x - 3y - z,\ 2x - 3y - 3z$.

193. $-14x^3 - 4x + 4,\ 9x^3$.

194. $5hk + 3k^2 - 36,\ 3hk + 21$.

195. $26ab,\ a^2 - 5ab - b^2$.

196. $150m^2n - c^2d + 7xy,\ 115m^2n + 8xy - c^2d$.

197. $x^2 + qx - pv,\ x^2 + pv + qx$.

198. $15x^2 - 7xy + z,\ 12x^2 + 3xy + 4z$.

199. $-4c^3n - 4cn^3 - 4^3c^3n,\ -7c^3n + 8cn^3 - 9c^3n^3$.

200. $6abx + 12 - 3xy,\ 3abx - 7$.

201. $bx^2 + cx - 12d,\ ax^2 - bx^2 + cx$.

202. $3a + b - 18\sqrt{x},\ b - 3a - 16\sqrt{x}$.

203. $7m^2 - 4ab - c,\ 2m^2 - 3c + s$.

204. $5x^2y - 3bx + c,\ 3x^2y + 2bx + c^2$.

205. $300x^3 - 70y^3 - 30z^3$, $118x^3 - 48y^3 - 8z^3$.

206. $6\sqrt{mn} + 2p - 7q$, $3\sqrt{mn} - p + 3q$.

207. $3x^3 + 5x^2y + 4xy^2$, $4x^2y + 6xy^2 + 7y^3$.

208. $2x^3 - y^3 + z^3 - 2xy$, $x^3 - 2xy - y^3 - z^3$.

209. $-p^2mn - pm^2n + pmn^2 - pmn$, $p^2mn + pm^2n - pmn^2 + pmn$.

210. $4x^4 - 3x^3 - 2x^2 - 7x + 9$, $x^4 - 2x^3 - 2x^2 + 7x - 9$.

211. $9(m+n) - 6(p+q)$, $5(m+n) - 8(p+q) + 3(x+y)$.

212. $5a(a - x) + 4cz + a^3$, $2a(a - x) - 7cz + 4a^3$.

213. From $x - y$ take $x + y$ and $-x + y$.

214. From $5a - 3b$, take $9a - b$, and $-7a + b$, and $6a + 3b$.

215. From $x - y - z$ take $x + y + z$ and $-x - y - z$.

216. $6(m^2 + n^2) + 4(p^2 - q^2)$, $2(m^2 + n^2) - 5(p^2 - q^2)$.

217. $8\sqrt{x^2 + y^2} + \sqrt{x^2 - y^2} + 5\sqrt{7}$, $\sqrt{x^2 + y^2} - \sqrt{7}$.

218. $\frac{5}{8}m^2 + 3mn - \frac{5}{2}n^2$, $2m^2 - mn - \frac{1}{2}n^2$.

219. $\frac{5}{2}c - 14 - 3fg + \frac{1}{2}g$, $\frac{3}{4}e - 4\frac{1}{8} + \frac{1}{4}g - 3fg$.

220. $\frac{1}{2}y - \frac{5}{2}a - \frac{3}{4}x + \frac{1}{8}b$, $3y + \frac{7}{4}a - \frac{2}{3}x$.

221. $ay^3 + by^2 - cy + d$, $ey^3 - fy^2 + gy - 6$.

222. What must be added to $4m - 5n$ in order that the sum may be $2m - 3n$? Subtracted from $4m - 5n$ in order that the difference may be $2m - 3n$?

223. From $2x^2 - xy$, that the difference may be $x^2 + y^2$?

224. Added to $7pq + 2st$, that the sum may be $5pq - 2qt + 3ts$?

225. From $3a - 4b$ take the sum of $2a + 7b$, $-4a - 6b$, $6a - 5b$.

226. From $2a^2+2ab+2b^2$, take the sum of $a^2-4ab-3b^2$, $ab-4b^2-3a^2$, and b^2-4a^2-3ab.

227. From $19^{a/b}+23^{c/d}-18^{x/y}+k$ take $19^{a/b}-23^{c/d}+x/y-k$.

228. If $5a^3-4a^2+3^n$ be subtracted from zero, what will be the remainder?

229. If $5a^3-4a^2+3^n$ be subtracted from 4^n, what will be the remainder?

230. To what must $3x^2y^2-4xy^3+z^{-2a}$ be added in order to produce zero?

231. From the sum of $ax^3-b^2x+a^2b^2x$ and $3a^2b^2x-ax^3+b^2x$ take the sum of $2ax^3-2b^2x-2a^2b^2x$ and $3a^2b^2x-2ax^3+2b^2x$.

232. From the sum of $-4a-3b-2c$ and $5a+2b-c$ take the difference between $3a-2b+c$ and $8a+5b-3c$.

233. From the sum of $\frac{1}{2}a-\frac{1}{3}b+\frac{1}{4}c$ and $\frac{2}{3}a+\frac{5}{6}b-\frac{3}{4}c$ take the sum of $\frac{5}{2}a-\frac{4}{3}b-\frac{5}{4}c$ and $\frac{2}{3}a-\frac{2}{3}b-\frac{2}{3}c$.

234. From $m-\frac{1}{2}n+\frac{3}{4}q$ take $3m-\frac{1}{4}n+\frac{1}{2}q$ and to the remainder add the difference between $\frac{1}{3}m-2n+5q$ and $-\frac{2}{3}m-\frac{3}{2}n+4\frac{1}{2}q$.

235. Verify the results in number 232 and 233, if $a=12$, $b=18$ and $c=24$.

If $x=a^2+2ab+b^2$, $y=a^2-2ab+b^2$, $z=3a^2+4ab+5b^2$, $u=4a^2-3ab-2b^2$, find the value of

236. $x+y+z+u$. **237.** $x-y+z+u$.

238. $x-y-z+u$. **239.** $x-y-z-u$.

240. From $x^{\frac{2}{3}}+2x^{\frac{1}{3}}y^{\frac{1}{3}}+y^{\frac{2}{3}}$ take $x^{\frac{2}{3}}-2x^{\frac{1}{3}}y^{\frac{1}{3}}+y^{\frac{2}{3}}$.

241. From $6\sqrt{a+b} + 5c^3d^{-3}$ take $3(x+y)^{\frac{1}{2}} - 4c^3d^{-3}$.

242. From $7r + \frac{1}{2}s - \frac{2}{3}t - 1$ take $3r - \frac{1}{2}s + \frac{1}{3}t - 2$.

243. From $8x - 5\sqrt{xy} + 19(a+b)^n$ take $5x + 3\sqrt{xy} + 14(a+b)^n$.

244. From $4a^2x^{\frac{1}{2}} - 4w + 10c^{\frac{3}{2}}y^{\frac{1}{2}}$ take $8c^{\frac{3}{2}}y^{\frac{1}{2}} + 4a^2\sqrt{x} - 3w$.

245. From $9k^{\frac{1}{2}}l^{-3} - 3xy + 12 - 2\sqrt{xy}$ take $11 + 10xy - 5k^{\frac{1}{2}}l^{-3} + 2x^{\frac{1}{2}}y^{\frac{1}{2}}$.

246. From $ax^{-3} - bx^{-2} + cx - d$ take $ax^3 - bx^{-2} - ex - d$.

247. From the sum of $5cv - 90 + 4v^{\frac{1}{2}}$, $5v^2 + 3cv + 8\sqrt{v}$, and $60 - 2cv - 12\sqrt{v}$ take the sum of $2cv - 75 + 6v^2$, $4v^{\frac{1}{2}} - 8cv - 50$, and $20 - 3v^{\frac{1}{2}} - 2v^2 + 9c^2v^2$.

248. To $5ax^2 - 7bc + 8m^{\frac{1}{2}} - 2c^{-n}$, add $3cb - 4c^{-n} + 2ax^2$, then subtract $10m^{\frac{1}{2}} - 5xy + 3bc - 12c^{-n}$, add $6m^{\frac{1}{2}} - 11cb + 3n - 4ax^2$, subtract $-16ax^2 - 2m^{\frac{1}{2}} + 4bc$, add $5m^{\frac{1}{2}} + 6c^{-n}$, and subtract $4xy - 4n$.

249. To $7cy^{-\frac{2}{3}} + 8ax - 5b$, add $4b - 2cy^{-\frac{2}{3}} + m$, then subtract $5ax - m + 3$ and $-3ax + 5cy^{-\frac{2}{3}} - 6$, add $10ax - 2b + 8m - 3$, and subtract $3m - 10cy^{-\frac{2}{3}} - 2m$.

PARENTHESES.

Free from parentheses and combine:

1. $19 + (8 - 14 + 11)$.
2. $34 - 8 - (-5 + 9 - 2 + 6)$.
3. $x - 1 - \overline{y - z - w + u}$.
4. $(p+q) + (q+r) - (p+r)$.
5. $(2m - n - p) - (m - 2n + p)$.
6. $(2a - b) - (2b - c) - (2c - a)$.

7. $a - (b + c - d + e - f + g)$.
8. $(e - y - z) - (f - y + z) + (g + 2z)$.
9. $1 - (1 - v) + (1 - v + v^2) - (1 - v + v^2 - v^3)$.
10. $4a - b + 3c + (-3a + 8b - 4c) + 9a$.
11. $36m + p - q + (24m - 5q) - (42m - 8p)$.
12. $x - 3y - (2y - 5x) + (z + 3x - y) - (x - z)$.
13. $6k + 9l - (k - l) + (l - k) - (2k + 2l)$.
14. $x - (-x + y) - (-x + z) - (-x + u)$.
15. $x + y - 2z + (x - 2y + z) - (2x - y - z)$
16. $r + s - 2t - (t - 2s + r) - (s - 2r + t)$.
17. $(3x - y + 7z) - (2x + 3y) - (5y - 4z) + (3z - x)$.
18. $x + y + z - (-y + x - z) - (y - x - z)$.
19. $m - n - (a - b) - (c + d) - (e + f)$.
20. $3px - qy - (-qy - rz) - (rz + 2px)$.
21. $3x^2 - 2x + 1 - (x^2 + 2x + 4) - (2x^2 - 6x - 7)$.
22. $a - (b - c) - (a - c) + c - (a - b)$.
23. $\frac{x}{y} - \frac{c}{d} + \left(2\frac{c}{d} - \frac{x}{y}\right)$.
24. $\frac{a}{b} - \frac{c}{d} + \left(\frac{c}{d} - \frac{e}{f}\right) - \left(\frac{a}{b} - \frac{e}{f}\right)$.
25. $a - (2b - c) - (-2d + 3e - f)$.
26. Prove that $\frac{1}{2}x + \frac{1}{2}y - (\frac{1}{2}x - \frac{1}{2}y) = y$.
27. $\frac{1}{2}c^2 - \frac{3}{2}d^2 - \frac{1}{8}cd - (cd - d^2 - \frac{3}{2}c^2)$.
28. $\frac{3}{4}u^2 - 2 - \frac{5}{2}u - (-\frac{1}{2} - \frac{1}{4}u^2 + u)$.
29. $\frac{2}{3}a^2 - \frac{1}{3}a + \frac{5}{9} - (\frac{2}{3}a + a - 1)$.

30. $\frac{3}{4}p^2 - \frac{2}{3}pq - (\frac{1}{4} - \frac{1}{4}p^2 - \frac{5}{3}pq)$.

Free from parentheses and add

31. $x - (y - z + w)$, $y - \overline{z - w + x}$, $z - [w - x + y]$.

32. $e - (f - g)$, $f - (g - e)$, $g - (e - f)$.

33. From $a - c - (c - 2a) + 2a - c$ take $a - 2c - (2a - c) + (-2a + c)$.

34. From the sum of $x - (p - q)$ and $p - (q - x)$ take $q - (p - x)$.

35. Subtract $a - (b - c)$ from $c - (a - b)$ and to the remainder add $b - (a - c)$, subtracted from $a + (b + c)$.

36. Add $x - (2y + 3z)$ and $2y - (3x - 4z)$ and from the sum subtract the sum of $3z - (4x + 5y)$ and $6x + 7z - 8y$.

37. From the sum of $1 - (\frac{1}{2}x + \frac{1}{3}y)$ and $-3 - (\frac{1}{3}x - \frac{1}{2}y)$ take $-5 - (\frac{1}{6}x - \frac{5}{6}y)$.

38. To the difference between $x - y - z$, minuend, and $z - x - y$ add the sum of $-y - x - z$ and $y - (x + z)$ and from that sum subtract $\frac{1}{3}x + \frac{1}{2}y - (\frac{1}{2}y - \frac{2}{3}x)$.

Brackets Within Brackets.

(Note: To free from nests of brackets, begin with the inner one. With care, one may free in the order in which they occur by observing the following rule: If the number of *minus* parentheses within which the numbers are included be odd, change all the signs, otherwise make no change; this method, however, should be used by advanced students only. Thus in

$$x - [\overset{1}{5}\overline{y} - \{\overset{2}{x} - (\overset{3}{3}\overline{z} - \overset{3}{3}\overline{y}) + \overset{2}{2}\overline{z} - (\overset{3}{x} - \overset{3}{2}\overline{y} - \overset{3}{z})\}],$$

the numbers over the quantities show within how many brackets preceded by the sign *minus* each quantity is included and we can free by inspection as follows: $x - 5y + x - 3z + 3y + 2z - x + 2y + z = x$.)

Free from brackets and combine:

39. $32 - 5 - [8 - \{3 - (9 + 4) - 2\} + 5]$.

40. $20 - \{9 - (7 - 4)\}$.

41. $9 - \{9 - [9 - (9 - 5)]\}$.

42. $2 - [4 - (6 - 8) - 10]$.

43. $1 - \{1 - (1 - [1 - 2] - 1) - 1\}$.

44. $2 - \{3 - [4 - (5 - 6)]\}$.

45. $9 - [3 - \{4 - (-5 - 2)\}]$.

46. $7 + (1 - 3) - \{(3 - 2) + 1\} + 5 - (4 - 3)$.

47. $52 - 5 - [8 - \{3 - (9 + 4) - 2\} + 6]$.

48. $x - [y - (z + w)]$. **49.** $c - [d - (e - f)]$.

50. $2x - \{z - (x - 2z)\}$. **51.** $e - \{2f - (3g + 2f) - e\}$.

52. $3m - [n + (2m - n) - (m - n)]$.

53. $c - (d - c) - [d - (c - d)]$.

54. $x - \{y - [2y + m] + [y - (m - 2y)]\}$.

55. $e - [e - f - (f - h) - (h - e)]$.

56. $3a - \{3b - [4a - (6b - 2a)]\}$.

57. $5x - \{3x - [4x - (5x - 2x)]\}$.

58. $10y - \{5u - [4v + 2w - (6v - 5w + 2z)]\}$.

59. $s - \{s - [s - (s + t)]\}$.

60. $9c - [3c - \{4c - (5c - 2c)\}]$.

61. $e - \{-(-f - g) + h\}$.

62. $2a - [7x - \{4a - (5x - 6a)\}]$.

63. $1 - [1 - (1 - e)] + [2e - (3 - e)]$.

64. $a + (b - 3d) - \{(3a - 2b) + d\} + 2a - (4b - 3d)$.

65. $7x - [x + 5z - \{x - z - \overline{3x - 2z}\}]$.

66. $6k^2 - (2 - 3k + k^2) + [-7 + (5k - \overline{8k - 3}) - (3 - \overline{2 - 4k})]$.

67. $x - \{y - \overline{z + u}\} + [y - \overline{u - 2y}]$.

68. $\{6a - [3b + (8x - \overline{3 + by} - x) + 4a] - 3b\}$.

69. $4 - [3 - (2 - y)] + [5y - (3 - 6y)] + [7 - (-5 + 8y)]$.

70. $12 - q - [7q - \{8q - (9q - \overline{3q - 6q})\}]$.

71. $12x - \{x - [x - (x - 3) + 3] - 3\} - \{-x - [-x - (-x + 3) + 3] - 3\}$.

72. $\{(5a - 3x) + (6b - a)\} - \{a - (2x - 3a) - b\} + \{a - (x - 5b - a)\}$.

73. $h - [2h + (3h - 4h) - 5h - \{6h - [(7h + 8h) - 9h]\}]$.

74. $14a - (3b + 2c) - [5b - (6c - 6b) + 5c - \{2a - (14c + 2b)\}]$.

75. $5p - [3q + (2q - x) - 4x + \{5p - (3q - \overline{x - 2q})\}]$.

76. $4u - [5v - \{4u - (3w - 3v) + 2v - (4u - 2v - w)\}]$.

77. $x - [2y + \{3k - 3x - (x + y)\} + \{2x - (y + k)\}]$.

78. $-[-\{-(-x + y - 2z)\} - x] + [-\{-(y + z - 5x)\}]$.

79. $-[-(-\{-4x\})] - (-(-(-7y)))$.

80. $3x - (2x + 1) + [x - (2 - x)] - \{-1 - (-x - \{-2 - x + (-1)\} - 2x)\}$.

Introduction of Parentheses.

Prove your work in the first twelve examples by solving each result.

$$24 - 5 + 4 - 6 - 7 + 3 - 2 + 1 - 8.$$

Use those numbers in the order given and without changing the value of the expression.

81. Bracket as trinomials.

82. Bracket as binomials.

83. Bracket as trinomials, each preceded by the sign $+$.

84. Bracket as trinomials, each preceded by the sign $-$.

85. Bracket as binomials, each preceded by the sign $+$.

86. Bracket as binomials, each preceded by the sign $-$.

87. Begin with 5 to bracket as trinomials.

88. Begin with 5 to bracket as binomials.

89. Bracket as trinomials, beginning with each following number, in turn.

90. Bracket as binomials, beginning with each following number, in turn.

91. Bracket as trinomials, placing the last two numbers of each trinomial within parentheses.

92. Bracket as polynomials of four terms each, and inclose the second and third terms of each polynomial within parentheses.

Without changing the value of the following expressions write within brackets :

93. The last three terms of $x + y - z + u$.

94. The last two terms of $a - b + c$.

95. The last three terms of $e - f - g + h$.

96. The second and third terms of $3p - k + q - 4r$.

97. The third and fourth terms of $x^3 + 2xy - y^3 + z^3$.

98. The second and third, also the fifth and sixth terms of $3c^5 - 4c^4d + 5c^3d^2 - 6c^2d^4 + 7cd^5 - 8$.

Express as binomials, also as trinomials:

99. $6x - 5y - 4z + a + 2b - 3c$.

100. $p - 2q + 4s - 6t - 8m + n$.

101. $x^5 + 4x^4 - 3a^3 - 2a^2 + x - 1$.

102. $-4k - 5m - 6n - 3p + 2q + s$.

103. $6a^5 - 5a^4c + 4a^3c^2 - 3a^2c^3 + 2ac^4 - c^5$.

In the following examples obtain two results, first with the sign + before each bracket, and second with the sign −:

$$abx - acx - bcx - aby - acy + bcy.$$

104. Bracket terms containing ab.

105. Bracket terms containing ac.

106. Bracket terms containing bc.

107. Bracket terms containing x and y, separately.

In the following, bracket the like powers of x twice, the first time so that each bracket shall be preceded by the sign +, and the second time by the sign −:

108. $4x^3 + cx^2 + 8 + 3cx - 7x^2 + ax^3 - 2x$.

109. $6dx^3 - 9 - 4x + bd + 7bx^3 + ex - 5x^2 - dx^3$.

In the following examples inclose the last three terms within parentheses preceded by the sign −, and the second and third terms of each trinomial within brackets preceded by the sign −:

110. $a^4 - 3a^2 - 6a + 4$.

111. $p^3 + p^2q + pq^2 + q^3$.

112. $m - n - p + q + t$.

113. $a^2 - c^2 - 2cd - d^2$.

114. $x^2 - y^2 - 2yz - z^2$.

115. $c^3 + d^3 + e - 3cde$.

116. $x^5 + 7x^4 + x^3 - 5x + 8$.

117. $-5 - 6a - 7b - 8c - 9d$.

118. $9abcd + 7a^2bc - 5ab^2c + 3abc^2 - 21.$

Collect within parentheses the coefficients of x, y and z in the following examples:

119. $4ax - 7ay + 3bz - 5bx - cx - 2cy.$

120. $cx - dx + 3cy + 4y + 5cz - 6dz - z.$

121. $ex - 3fx + 9gz - 4fx - 7gy + ez - 2gx - ey + 5fz.$

122. $9mx + 10my + 8ny - 12nz - 15px + 9py + 4pz.$

123. $6dx - 4ey - 5fz - 2dx + fx + 8fz - 3fx - fy - fz.$

Simplify the following and regroup them according to the powers of y and x:

124. $ey^2 - 3gy - (fy^2 - [gy - hy - \{fy^2 + 4gy^2\}]) - [gy^2 - fy].$

125. $6mx^2 - (5nx - 4px^2) - [8nx^2 - \{2mx^2 + 3mx\} - 7px^2].$

MULTIPLICATION.

1. Monomials.

$+3 \times (+4) = (+4) + (+4) + (+4) = +12.$

$+3 \times (-4) = (-4) + (-4) + (-4) = -12.$

$-3 \times (+4) = -(+4) - (+4) - (+4) = -12.$

$-3 \times (-4) = -(-4) - (-4) - (-4) = +12.$

Multiply the following:

1. $+6$ by $+4$.
2. $+6$ by -4.
3. -6 by $+4$.
4. -6 by -4.
5. -9 by 5.
6. x by 6.
7. a by -5.
8. $2b$ by 7.
9. $3c$ by -4.
10. $-x^2$ by 8.
11. $-y^2$ by -2.
12. 6 by $-x$.
13. -9 by $3a^2$.
14. -13 by $-2c$.
15. $+x$ by $+y$.
16. $+x$ by $-y$.
17. $-x$ by $+z$.
18. $-x$ by $-z$.
19. $5a$ by $7b$.
20. $-3p$ by $6q$.
21. $9c$ by $-2c$.
22. $-5a$ by $-3a$.
23. $-8m$ by $-3m$.
24. $-n$ by n.
25. $-5cd$ by d.
26. $3ef$ by $4ef$.
27. $5ab$ by $-n$.
28. $-3m^2n$ by $-7xy$.
29. $5mn$ by $9m$.
30. $2cd$ by $-5ef$.
31. $-7abx$ by cy.
32. $-4pq$ by $-2px$.
33. $3c^2x^3$ by $4c^3d$.
34. $-9mn^2p^3$ by $-3mnp$.
35. $20abcdef$ by $2ghi$.
36. $5m^2x^3z$ by $-5x^4y^3z^2$.
37. $-11x^2z^2$ by $-11w^3s^2$.
38. $9aaa$ by $4aa$.

39. $12aabb$ by $-3ab$.

40. $-14xxyy$ by $2yyyzz$.

41. $5a^3c^3e^3$ by $-6a^5c^3f^4$.

42. $-3x^3y^5z^7$ by $12x^6y^4z^2$.

43. $17a^7x^9y^{11}$ by $3a^2x^3y$.

44. $4c^{21}d^{31}e^{41}$ by $3d^{12}e^{13}f^{14}$.

45. $-12ap^6q^9rs$ by $3a^5pqrs^9$.

46. $0x^3y^4z^5$ by $25x^3y^5z^7$.

47. $-6a^3b^9c^{13}$ by $0abc^{14}$.

48. $a(x+y)$ by 3.

49. $c(u-w)$ by d.

50. $(a-x)$ by $-6y^3$.

51. $-7(c+z)$ by $-5z$.

52. $-2e(p-q)$ by $-3ef$.

53. $-5z^3(a^3+a)$ by $6xyz$.

54. $4\sqrt{x+1}$ by $-3x^3$.

55. $-5a^3\sqrt{a^3-x^3}$ by $-8a^2b^2c^2$.

56. $-5x(c-d)$ by $4x(c-d)$.

57. $-ax(p+s)$ by $-3ax(p+s)$.

58. $9a\sqrt[3]{a^2+1}$ by $a^2\sqrt[3]{a^2+1}$.

59. $a(a+b)$ by $a(a+b)^2$.

60. $c(c-d)$ by $-3c(c-d)^3$.

61. $-2a^2z^2(e-f)^3$ by $-5a^3z^4(e-f)^4$.

62. $-20(x^3+x^2+x)$ by $-a^3(x^3+x^2+x)$.

63. $-5a\sqrt{x^2+2x+1}$ by $3a^2\sqrt{x^2+2x+1}$.

Multiply together the following:

64. ab, ac, ad.

65. xy, xz, zx.

66. mn^2, np^3, pm^2.

67. a, ab, abc, $abcd$.

68. $4a$, $5b$, $-6c$.

69. $6xy^2z^3$, $2y^3z^3$, $-5x^2z$.

70. $-2pq$, $4p^3q$, $8p^2q^3$.

71. $3m^2$, $-3mn$, $3n^2$.

72. $-xy$, yz, $-zw$, wx.

73. abc, bcd, cde, $-dea$.

74. $-hk$, $-lk$, $-hl$.

75. $(-x^3)$, $(-x^2)$, $(-x)$.

76. $(-x)^3$, $(-x)^2$, $(-x)$.

77. $3p^2$, $-4p^3$, $-5p^4$.

78. $(4mn), (-cm), (-5bn)$.

79. $-27abx$, $39mp$, $18ap$.

80. $10a^{11}z^3$, $-2a^3z$, $6a^3x$.

81. $5m^2n^2p^2q^3$, $-3m^3n^2$, $-7p^5q^8$.

82. $-12a^3b^2x^4y^2$, $-5a^4b^3x^4y^2$, $2a^9b^3x^6y^4$.

83. $9a^4c^3d^3$, $3a^{13}c^{16}$, $-2a^{10}cd^{15}$.

84. $12a^3b^4c^2d^3e^5$, $-9a^2b^4c^3e^4$.

85. $-12m^9n^9x$, $-4m^9nx^9$, $-3mn^4x^8z^7$.

86. $-8p^9q^4r^2$, $-12p^2q^3r^4x^{11}y^{12}z^{13}$.

87. $-a$, $-a$, $-a$, $-a$, $-a$, $-a$.

88. $-x$, $-x$, $-x$, $-x$, $-x$.

89. $-dw$, dw, $-dw$, dw, $-dw$.

90. $-2a$, $-3a^2$, $-4a^3$, $-5a^4$, $-6a^5$.

91. $-p^2$, $-3p^2$, $-0p^2$, $-8p^2$, $-17p^2$.

92. x^n by x.

93. z^m by $-2z$.

94. $-3c^m$ by $-5c^2$.

95. d^n by d^m.

96. $-3a^mb^n$ by $4ab$.

97. $2c^m$ by $-3c^m$.

98. $-5y^n$ by $-4y^n$.

99. $-7a^5$ by $4a^s$.

100. x^n by x^{-n}.

101. $3x^{-n}$ by $2x^{4n}$.

102. $-4a^{-n}$ by $-6a^{-n}$.

103. $-5x^{-1}$ by $-x^n$.

104. a^{-2} by a^3.

105. $(a+b)^n$ bv $(a+b)$.

106. $(c+d)^m$ by $(c+d)^3$.

107. $5a(e-x)^{-1}$ by $-3a(e-x)^{-3}$.

108. $(6xy)^3$.

109. $(4a^2b)^3$.

110. $(-2a^3y^2)^2$.

111. $(-3a^2y^3)$.

112. $(-2xy)^4$.

113. $(-2xy)^4$, $(-3xy)^2$.

114. $(-2xy)^4$, $(-2am)^3$.

115. $(-am^2)^3$, $(2x^2y)^2$.

116. $(-m)^3$, $(-2m)^2$, $(-3m)$.

117. $(ab)^3$, $(-ab)^3$, $-(ab)^2$.

118. $(a^m b^n)^3$. 119. $(-a^n c^n)^2$. 120. $-5(a^r z^r)^4$.

121. $(-5a^m c^n)^2$. 122. a^{n-1}, a. 123. x^{p-1}, x^3.

124. $2a^x$, $-3b^y$, $5c^z$, $-abc$. 125. $8^{\frac{1}{3}}$, $8^{\frac{2}{3}}$.

126. $5^{\frac{1}{2}}$, $5^{\frac{3}{2}}$. 127. $4^{-\frac{1}{2}}$, $4^{\frac{3}{2}}$. 128. $7^{-\frac{2}{3}}$, $7^{\frac{5}{3}}$.

129. $4^{-\frac{1}{3}}$, $4^{-\frac{1}{2}}$. 130. $6^{\frac{3}{2}}$, $6^{\frac{1}{2}}$. 131. $9^{-\frac{1}{2}}$, $9^{-\frac{1}{3}}$.

132. $3x^{\frac{1}{2}}$, $2x^{\frac{1}{4}}$. 133. $29^{\frac{1}{2}}$, $-29^{-\frac{1}{2}}$. 134. x^5, $3x^{-\frac{2}{3}}$.

135. $7a$, $-5a^{-\frac{2}{3}}$. 136. $-5c$, $4c^{-\frac{7}{4}}$. 137. $10a^2$, $3a^{-2}$.

138. $3a^{\frac{1}{2}}b^{\frac{1}{3}}$, $-7a^{\frac{1}{4}}b^{\frac{1}{3}}$. 139. $4x^{-\frac{1}{2}}$, $-4x^{-\frac{1}{4}}$.

140. $6a^{-\frac{2}{3}}$, $8a^{-\frac{1}{2}}$. 141. $2a^{\frac{1}{3}}b^2$, $6a^{\frac{1}{3}}b^{-2}c^n$.

142. $9a^{1/x}$, $-4a^{5/x}$. 143. $-5x^{-1}y^{-2}$, $3x^2y^3z^{x/y}$.

144. $3a^{\frac{2}{3}}b^{\frac{2}{3}}$, $9a^{\frac{1}{2}}b^{\frac{1}{3}}$. 145. $8a^{\frac{3}{4}}b^{\frac{1}{2}}c^{\frac{5}{12}}$, $-5a^{\frac{1}{2}}b^{\frac{3}{10}}c^{\frac{1}{4}}$.

146. $-3x^2y^{\frac{2}{3}}z^{-3}$, $-14x^{\frac{1}{4}}y^{\frac{7}{6}}z^{\frac{1}{2}}$. 147. $a^{\frac{2}{3}}b^{\frac{5}{6}}c^{\frac{1}{15}}$, $-a^{\frac{1}{2}}b^2cd^{\frac{1}{5}}$.

148. $-4a^3b^{\frac{5}{12}}d^{\frac{2}{3}}e$, $11a^{\frac{4}{3}}c^{\frac{2}{3}}d^{\frac{7}{3}}e^{\frac{7}{3}}$. 149. $3^n a^{-7}b^{-1}$, $2^n a^{-2}b^{-4}$.

150. $4^q a^{-\frac{3}{5}}c^{-\frac{3}{7}}$, $-4^{2q}a^{-1}c^3$. 151. $5^x x^{-6}y^3z^{-7}$, $3^x x^4 y^{-2}z^1$.

152. $-1^n x^7 y^{-2}z^2$, $-1^{-n}x^{-6}y^3z^{-2}$.

153. $-4^m a^{-1}b^{-7}c^{-\frac{3}{5}}$, $2^n a^6 b^{\frac{3}{5}}c^{-\frac{2}{5}}$.

154. $3^{\frac{1}{2}}a^{-\frac{4}{5}}b^{\frac{6}{7}}c^{-\frac{1}{5}}$, $3^{-\frac{1}{2}}a^{-\frac{6}{5}}b^{-\frac{6}{7}}c^{-\frac{6}{5}}$.

155. $-2^s x^{\frac{4}{5}}y^{-\frac{7}{3}}z^{-\frac{1}{5}}$, $2^{-t}x^{-\frac{1}{5}}y^{-\frac{2}{3}}z^{\frac{4}{5}}$.

156. $7^{-n}a^{-5}b^{\frac{3}{5}}c^{-\frac{3}{5}}d^{-\frac{6}{5}}$, $-7a^3b^{-\frac{6}{5}}c^{\frac{3}{5}}d^{-\frac{7}{5}}$.

Multiplication of Polynomials.

The number 74 may be read in three different ways, each equally correct, as follows: (1) Seventy-four; (2) seventy units plus four units; (3) seven tens plus four units. The first is rather the method of arithmetic, the last, that of algebra, the second is occasionally used in both.

To illustrate this, let us multiply 2112 by 321. This is the regular method of arithmetical multiplication, familiar to everyone, and calls for no explanation.

For convenience we will abbreviate the names of the places occupied by the different figures as follows: *th* for thousands, *h* for hundreds, *t* for tens, and *u* for units, and write the example thus, and multiply as in algebra.

$$\begin{array}{r} 2112 \\ 321 \\ \hline 2112 \\ 4224 \\ 6336 \\ \hline 677952 \end{array}$$

$$\begin{array}{l} 2th + 1h + 1t + 2u \\ 3h + 2t + 1u \\ \hline 6hth + 3hh + 3th + 6uh \\ \quad + 4tth + 2th + 2tt + 4ut \\ \quad\quad + 2uth + 1uh + 1ut + 2uu \end{array}$$

In examining these partial products we should remember that, when no sign is expressed between letters in algebra, multiplication is always understood.

Bearing in mind that any quantity multiplied by unity, or 1, remains unchanged and that tens times tens, or ten tens, is the same as hundreds, and hundreds times hundreds, or hundred hundreds, is the same as ten thousands, we may re-write the partial products thus:

$$\begin{array}{l} 2th + 1h + 1t + 2u \\ 3h + 2t + 1u \\ \hline 6hth + 3tth + 3th + 6h \\ \quad + 4tth + 2th + 2h + 4t \\ \quad\quad + 2th + 1h + 1t + 2u \\ \hline 6hth + 7tth + 7th + 9h + 5t + 2u \end{array}$$

Which of the two products is correct? What is the difference between them? Carefully examine and compare the two results and we find this difference; the denominations and the relations of the several figures in arithmetic are shown by their positions, while in algebra they are shown by letters (which stand for words) and signs. Each method has its peculiar advantages. Arithmetic obtains the product by the shortest, most concise method, without showing of what the product is composed. Algebra goes into details and allows us to analyze every step of the work. For example, 25 squared arithmetically gives 625, and that is all we know about it, and so with any number. But, if we wish to square any number algebraically, consisting of tens and units, we proceed as follows:

$$\begin{array}{r} t+u \\ t+u \\ \hline t^2+tu \\ \quad tu+u^2 \\ \hline t^2+2tu+u^2 \end{array}$$

Let t represent any number of tens and u any number of units, and from this result learn that the square of any number consisting of tens and units is always made up of the same three parts, namely, the square of the tens, plus twice the product of the tens and units, plus the square of the units.

Multiply

157. $5+3$ by 2. **158.** $7-2$ by 4. **159.** $-3+9$ by 8.

160. $6+5$ by -3. **161.** $-3-7$ by -5.

162. $-29+305$ by 23. **163.** $x+4$ by 2.

164. $x-3$ by 7. **165.** $-a-4$ by 12.

166. $b+7$ by -2. **167.** $a+b$ by 3.

168. $a-c$ by 7. **169.** $c-d$ by -15.

170. $e-4$ by a. **171.** $7-11$ by d.

172. $15-8$ by $-y$. **173.** $x-3a$ by $3x$.

174. $-5c-4d$ by $3e$.

175. $-8y-9z$ by $-2x$.

176. $-4cd+ef$ by $3de$.

177. $a^2x^2-b^2y^2$ by ay.

178. $-3+4x^2y^2z$ by $2axz$.

179. $3p^2q+2pq^2$ by $-abpq$.

180. $-abc+def$ by $9ghk$.

181. $3x^2y-4xy^2$ by $-5x$.

182. $5x^2y^2+3ab^3$ by $2z^4$.

183. $3\sqrt{x}-7\sqrt{y}$ by xy.

184. $25a^2b^3c^4-30xy^2z^9$ by $3acx^4$.

185. $-6y\sqrt[3]{a}-8z\sqrt[5]{b}$ by $9c$.

186. a^n-b^m by -14.

187. x^m-y^3 by $4t^2$.

188. p^n-q^3 by $-5pq$.

189. $3c^{\frac{1}{2}}+8y^n$ by $7c^2y^3$.

190. $4a+3x-2y$ by x.

191. $4a^3b-2a^2b^3+5ab^3$ by $-2ab$.

192. $6c^4d^2e+4c^3de^3-2e^5$ by $-7d^2e^2$.

193. $-19mnq-14dq+16q^3$ by $0a^{10}b^{11}x^{13}z^{18}$.

194. $4c^3d^2z^2-4d^2e^3z^2-4e^2f^2z^2$ by $9efz$.

195. $-5x^{11}y^{17}+13x^7y^9z^{19}$ by $6x^{14}y^{15}z^{31}$.

196. $-16-9a^5c^6+12cdy$ by $-acdy$.

197. $12p^2q^3-7a^9p^2c^4x^6d^{19}q^{21}$ by $3a^{17}p^{22}cxy$.

198. $-6p^2+2pq^3-8q$ by $\frac{1}{2}p^2q^3$.

199. $\frac{2}{3}c-\frac{1}{6}d^5-e$ by $\frac{3}{5}cy$.

200. $-\frac{5}{8}a^3x^2$ by $-\frac{3}{2}a^2+ax-\frac{8}{5}x^2$.

Multiply together

201. $\frac{1}{3}a^2b-\frac{1}{4}ab^2$, $6a^2$, and $3y^3$.

202. $\frac{5}{4}x^3y-\frac{4}{5}xy^2+1$, $-x^2y^2$, and $\frac{1}{2}xy$.

203. $xy+\frac{1}{2}yz-zu$, $\frac{2}{3}xz$, and $-\frac{1}{4}xw$.

204. $\frac{1}{2}a^2x^2+\frac{3}{4}a^2x-\frac{1}{2}ax^2$, $\frac{2}{3}ax$, and $-\frac{3}{4}a^2x^2$.

205. $mn^2 - m^2n^3 + \frac{2}{3}m^3n^4$, $-\frac{1}{2}m^2n$, and $\frac{2}{3}mn^2$.

206. $\frac{1}{12}k^2l - \frac{1}{15}kl - \frac{1}{9}$, $-15kl^2$, and $-6lm$.

Simplify

207. $3x(x-y) + 4y(y-x)$.

208. $p(h+k) - k(-2k-3h)$.

209. $7c(x-2y) - 2x(3c-3y)$.

210. $\frac{1}{2}(x-2p) - \frac{3}{4}(p-2x)$.

211. $x^2y^2(z^2-w^2) + z^2w^2(x^2-y^2) + y^2z^2(w^2-x^2)$.

212. $3[4cd - 5c(e-6d)]$.

213. $4m - 2\{n - (k - 6m - 3[n-2k]) - 2(n-3k)\}$.

214. $7x^3 - [(3y^3 - 4x^3) - 6yz(y+z) + 3z(z^2+2y^2)]$.

215. $6rt - 4[2t(r-3s) - 3(4t-2s)r]$.

216. $z^4 - 2a(a-b)z^2 + (a^2+b^2)z - [z^3 - (a-b)z + b^2]$.

Multiply

217. $9x^{\frac{3}{4}} + 2 + 6x^{-\frac{3}{4}}$ by $x^{\frac{4}{3}}$.

218. $3m^{\frac{7}{4}} - 2m^{\frac{7}{6}} - 1$ by m^{-1}.

219. $c^{\frac{7}{8}} - 2c^{\frac{4}{3}}d^{\frac{3}{5}}$ by $-2d^{-\frac{1}{6}}$.

220. $4a^{\frac{6}{5}} + 2a^{\frac{1}{2}}b - a^{\frac{4}{5}}b^2$ by $-9a^{-\frac{1}{2}}b^{-1}$.

221. $12p^2q^{\frac{2}{3}} - 2q + p^{-2}q^{\frac{4}{3}}$ by $p^{-\frac{1}{2}}q^2$.

222. $7kl^{-2}s^3 - 4k^{\frac{1}{2}}l^{-1} + 6s^{-3}$ by $12k^{\frac{3}{2}}l^2$.

223. $5x^{-4} + 4x^{-2}y^{-1} + y^{-2}$ by $-3x^2y^{-2}$.

224. $a^{-\frac{5}{2}} + a^{-\frac{1}{2}} + a^{-\frac{3}{2}}$ by $-a^{-\frac{1}{2}}$.

225. $c^{\frac{5}{4}} - 2c^{\frac{1}{2}} + 3c^{-\frac{1}{4}}$ by $-7c^{-\frac{1}{4}}$.

226. $-6x^{\frac{1}{2}}z^{-\frac{1}{3}} + y^{-2} - 2y^{-1}z^{-\frac{1}{3}}$ by $3x^{\frac{1}{2}}y^{-\frac{1}{3}}z^{-1}$.

Multiply

227. $4 + 5x$ by $2a + 3$.

228. $x^7 - x^5 + x^3 - x$ by $x^3 + 1$.

229. $c^6 + c^4 + c^2 + 1$ by $c^2 - 1$.

230. $x^3 + x^2y + xy^2 + y^3$ by $x - y$.

231. $8 - 3a + a^2$ by $3 + a$.

232. $x^4 + 2cx^3 + 4c^4$ by $x^4 + 2cx^2 - 4c^4$.

233. $a + 2b - 3c$ by $a - 2b + 3c$.

234. $p^2 - px + qx + q^2$ by $p + q + x$.

235. $x^2 + y^2 + z^2 - yz - xz - xy$ by $x + y + z$.

236. $c^2 - 2cd + e^2 + f^2$ by $c^2 + 2cd + e^2 - f^2$.

237. $y^3 - 4y^2 + 11y - 24$ by $y^2 + 4y + 5$.

238. $n^3 + 4n^2 + 5n - 24$ by $n^2 - 4n + 11$.

239. $u^3 + 6u^2 + 24u + 60$ by $u^3 - 6u^2 + 12u + 12$.

240. $a^3 - 2a^2 + 3a - 4$ by $4a^3 + 3a^2 + 2a + 1$.

241. $2m^2 + 3mn + 4n^2$ by $3m^2 + 4mn + n^2$.

242. $x^2 + 4ax + 4a^2y^2$ by $x^2 - 4ax + 4a^2y^2$.

243. $k^2 + 4kl + 5l^2$ by $k^3 - 3k^2l - 2kl^2 + 3l^3$.

244. $4x + 6y + 10z$ by $2x - 3y + 5z$.

245. $a^3 - 3 + 2a^2 - a$ by $3 - a + a^3 - 2a^2$.

246. $6x + 4x^2 - 8$ by $-4 - 3x + 2x^2$.

247. $w^3 - 2w^2 + w - 2$ by $w^2 + 2w + 3$.

248. $x^3 - 3x^2y + 3xy^2 - y^3$ by $x^2 - 2xy + y^2$.

249. $16x^4 - 24x^3 + 36x^2 - 54x + 81$ by $2x + 3$.

250. $q^3 - 6q^2 + 12q - 8$ by $q^3 + 6q^2 + 12q + 8$.

251. $a^5b^5 + a^4b^4 + ab + 1$ by $-a^3b^3 + a^2b^2 - ab + 1$.

252. $c^3 + 2c^2d + 2cd^2 + d^3$ by $c^3 - 2c^2d + 2cd^2 - d^3$.

253. $1 - 3m + 3m^2 - m^3$ by $1 - 2m + m^2$.

254. $25x^3 + 6x^4y + 7x^5y^2$ by $2x - 3y$.

255. $3c^2 - 4cz + 5z^2$ by $7c^2 - 2cz - 3z^2$.

256. $81m^4 + 27m^3n + 9m^2n^2 + 3mn^3 + n^4$ by $3m - n$.

257. $32p^5 + 16p^4q + 8p^3q^2 + 4p^2q^3 + 2pq^4 + q^5$ by $2p - q$.

258. $1024a^5 - 256a^4x + 64a^3x^2 - 16a^2x^3 + 4ax^4 - x^5$ by $4a + x$.

259. $81a^4 + 54a^3b + 36a^2b^2 + 24ab^3 + 16b^4$ by $3a - 2b$.

260. $3e^4 - 12e^3f + 2ef^3 - 5f^4$ by $7e - 2f$.

261. $3g^4 + 5g^2h + h^3$ by $3g^4 - 5g^2h + h^3$.

262. $7m^5n - 2p^3q + kn$ by $5m^2 + 3p^4 - q^2$.

263. $16c^4d^2 - 5c^2y + u^2$ by $c - u + 3y^2$.

264. $11b^2c - 5a^3y^2 + 2z$ by $14a^3m + yz$.

265. $13u^2v - 5uv^2 + 3uv$ by $6u^2v + 2uv^2 - uv$.

266. $\frac{1}{2}x^2 + \frac{1}{3}x + \frac{1}{4}$ by $\frac{1}{2}x - \frac{1}{3}$.

267. $\frac{1}{2}c^2 - 2c + \frac{3}{2}$ by $\frac{1}{2}c + \frac{1}{3}$.

268. $\frac{2}{3}a^2 + am + \frac{3}{2}m^2$ by $\frac{1}{3}a + \frac{1}{2}m$.

269. $\frac{3}{4}p^2 - pq - \frac{2}{3}q^2$ by $\frac{3}{4}p^2 - \frac{1}{2}pq + \frac{1}{3}q^2$.

270. $\frac{1}{2}k^2 - \frac{2}{3}k - \frac{3}{4}$ by $\frac{1}{2}k^2 + \frac{2}{3}k - \frac{3}{4}$.

271. $\frac{2}{3}st + \frac{2}{3}t^2 + \frac{1}{3}s^2$ by $\frac{2}{3}s^2 + \frac{3}{2}t^2 - \frac{3}{2}st$.

272. $\frac{1}{9} + \frac{2}{3}x + x^2$ by $\frac{1}{9} - \frac{2}{3}x + x^2$.

273. $\frac{3}{2}a^2 - \frac{5}{4}ac + c^2$ by $\frac{1}{2}a^2 - \frac{1}{4}ac - c^2$.

274. $\frac{1}{4}a^2 - \frac{2}{3}ax + 12x^2$ by $12a^2 + \frac{2}{3}ax - \frac{1}{4}x^2$.

275. $3\frac{1}{3}x^4y + 2\frac{3}{10}x^2y^3 - 4\frac{1}{2}x^3y^2$ by $2\frac{1}{3}x^3 - 2\frac{1}{2}x^2y + xy^2$.

Expand

276. $(1 - 2x^2)^3$. 277. $(4ab^3 + 7)^2$. 278. $(2x^2 - \frac{1}{3})^2$.

279. $(2 + 5ay)^2$.

280. $(3a - 4x^2y^2)^2$.

281. $(x^2 + 2x - 3)^2$.

282. $(x^2 - 6x + 7)^2$.

283. $(x^3 - 2y^3 - 3z^3)^2$.

284. $(2x - 3y)^3$.

285. $(2c - 3d)^3$.

286. $(3c - 2x)^3$.

287. $(x + y + z)^3$.

288. $(f - g + y)^3$.

289. $(e - n - s)^3$.

290. $(8a^n + 6b^2 - a^2)^2$.

291. $(a + x)^2(a - x)^2$.

292. $(c - d)^2(c + d) + (c + d)^2(c - d)$.

293. $(a + b + c)^2 - a(b + c - a) - b(a + c - b) - c(a + b - c)$.

294. $(a + c + x)^2 + (a + c - x)^2 + (a - c + x)^2 + (-a + c + x)^2$.

295. $(m - n - p)^2 + (n - p - m)^2 + (p - m - n)^2$.

296. $(a + x)(a - x)(a^2 + x^2)$.

297. $xy(x^2 + y^2)(x + y)(x - y)$.

298. $(c^2 - d^2)(c^2 + cd + d^2)(c^2 - cd + d^2)$.

299. $(x - a)(x - b)(x - c)$.

300. $(k^5 - m^5)(k + m)(k^4 - k^3m + k^2m^2 - km^3 + m^4)$.

301. $(c + 1)(c - 1)(c^2 + c + 1)(c^2 - c + 1)$.

302. $(a^2 + a + 1)(a^2 - a + 1)(a^4 - a^2 + 1)$.

303. $(x + z)(x - z)(2x - 3z)(2x + 3z)$.

304. $(x + 1)(x - 2)(x + 3)(x - 4)$.

305. $(3e - 5n)^2 - 5(e - n)(e - 5n)$.

306. $(a + 1)(a + 2)(a + 3) - (a - 1)(a - 2)(a - 3)$.

307. $(a - b)(b - c) - (a - c)(b - c) - (a - b)(a - c)$.

308. $(4y - 3)(y + 1)(2y + 5) + (y - 1)(2y - 5)(4y + 3)$.

Show that

309. $(a - 1)^2 + (2a + 1)^2 = 4a^2 + (a + 1)^2 + 1$.

310. $(c+2)^2+(2c+1)^2=(c-2)^2+4c(c+3)+1.$

311. $(m^2+m+1)^2+(m^2-m+1)^2=2(m^4+3m^2+1).$

312. $x^3+y^3+z^3+3(y+z)(x+y)(x+z)=(x+y+z)^3.$

313. $(y-z)^3+(z-x)^3+(x-y)^3-3(y-z)(z-x)(x-y)=0.$

314. $(e+f)(f+g)(g+e)+efg=(e+f+g)(ef+fg+eg).$

315. $(a+b)(a+c)+(b+c)(a+b)+(a+c)(b+c)-(a+b+c)^2=ab+bc+ac.$

316. $x^3-y^3=(x-y)(x^2+xy+y^2)=(x-y)^3+3xy(x-y)=(x+y)^3-3xy(x+y)-2y^3.$

317. If $x=2a+5c$, that $x^3=8a^3+125c^3+30acx.$

318. If $x=l+m-2n$, $y=m+n-2l$, and $z=l+n-2m$, that $x^2+y^2+z^2+2xy+2xz+2yz=0.$

319. Substitute $a+1$ for x in $x^3-3x.$

320. $n-1$ for y in $3y^2-5y.$

321. $p+2$ for a in $a^3-3a^2+2a.$

322. $q-1$ for x, and $1-q$ for y, in $x^2+2xy+y^2.$

323. In $3x^2-4xy+3y^2.$

Free from parentheses and combine :

324. $[1+(x-y)][1-(x-y)].$

325. $[(a+x)+(b-y)][(a+x)-(b-y)].$

326. $[(a+c)(c+x)]-[c\{c-(a-x)\}].$

327. $(a+b)^2-[(a+b)(a-b)]-(2b^2-ab).$

328. $(a+b+c)^2-(a+b)^2-c(2a+2b+c).$

329. $(m-n)(m^3+m^2n+mn^2+n^3)[(m^3+n^3)^2-2m^3n^3],$

330. $2(p+2q)(p-2q)[(p+2q)^2+(p-2q)^2].$

331. $[(c+y)^2+(c-y)^2][(2c+y)^2-(c-2y)^2]$.

Multiply

332. $(x^{\frac{1}{2}}-b^{\frac{3}{5}})(x^2+b)$.

333. $(2m^{-\frac{1}{2}}-3k^{\frac{2}{3}})(2m^{-\frac{1}{2}}+3k^{\frac{2}{3}})$.

334. $(a^{\frac{2}{4}}+a^{\frac{1}{2}}c^{-\frac{1}{4}}+a^{\frac{1}{4}}c^{-\frac{1}{2}}+c^{-\frac{2}{4}})(a^{\frac{1}{4}}-c^{-\frac{1}{4}})$.

335. $(b^{\frac{7}{5}}+2b^{\frac{4}{5}}c^{\frac{3}{5}})(b^{\frac{3}{5}}-2c^{\frac{3}{5}})$

336. $(3a^{\frac{7}{6}}-2a^{\frac{7}{6}}-1)(a^{\frac{1}{6}}+a^{-1})$.

337. $(2x^{\frac{4}{5}}-4x^{\frac{3}{5}}y^{\frac{1}{5}}+6x^{\frac{2}{5}}y^{\frac{2}{5}})(6a^{\frac{6}{5}}+8ay^{\frac{1}{5}})$.

338. $(12n^2x^{\frac{3}{5}}-2x+n^{-2}x^{\frac{4}{5}})(2n^{\frac{3}{2}}x^{-\frac{3}{5}}+n^{-\frac{1}{2}}x^{-\frac{1}{5}})$.

339. $(4rs^{-2}t^3-3r^{\frac{1}{2}}s^{-1}+5t^{-3})(12r^{\frac{3}{2}}s^2+9rs^3t^{-3})$.

340. $(4c^{\frac{5}{2}}x^{-\frac{3}{2}}+8c^{\frac{3}{2}}x^{-\frac{1}{2}}+16c^{\frac{1}{2}}x^{\frac{1}{2}}+32c^{-\frac{1}{2}}x^{\frac{3}{2}})(c^{\frac{3}{2}}x^{-\frac{3}{2}}-2c^{\frac{1}{2}}x^{-\frac{1}{2}})$.

341. $(a^{-\frac{7}{2}}-6a^{-\frac{5}{2}}+3a^{-\frac{3}{2}})(a^{\frac{5}{2}}+4a^{\frac{7}{2}}+2a^{\frac{9}{2}})$.

342. $(k^{\frac{5}{2}}+k^{\frac{3}{2}}+k^{\frac{1}{2}}+k^{-\frac{1}{2}}+k^{-\frac{3}{2}})(k^{\frac{1}{2}}-k^{-\frac{1}{2}})$.

343. $(m^{\frac{3}{3}}-4+4m^{-\frac{2}{3}})(m^{\frac{3}{3}}+4+4m^{-\frac{2}{3}})$.

344. $x^{\frac{5}{4}}-2x^{\frac{1}{2}}+3x^{-\frac{1}{4}})(2x^{\frac{1}{2}}-3x^{-\frac{1}{4}}-x^{-1})$.

345. $(3p^{-1}q^{\frac{3}{4}}+2p^{-\frac{3}{2}}q^{\frac{1}{2}}-p^{-2}q^{\frac{1}{4}}-2p^{-\frac{5}{2}})(4p-p^{\frac{1}{2}}q^{-\frac{1}{4}}+2q^{-\frac{1}{2}})$.

DIVISION.

Division of Monomials by Monomials.

1. Divide a by 1; by -1; $-a$ by 1; by -1.

What is the result of dividing any quantity by 1? by -1?

Perform the following divisions:

2. $4)12$.
3. $-4)12$.
4. $4)-12$.
5. $-4)-12$.
6. $-3)384$.
7. $+12)-264$.
8. $3\cdot4)7\cdot3\cdot4$.
9. $8)7\cdot8\cdot9$.
10. $-6\cdot-3)-3\cdot-6\cdot-9$.
11. $2)8x$.
12. $-3)12x$.
13. $3)-6a$.
14. $-4)-12y$.
15. $-13)13x$.
16. $14)-14y$.
17. $ab)abc$.
18. $-xz)xyz$.
19. $-7ef)-21efh$.
20. $cde)-edc$.
21. $8aa)-16aaax$.
22. $-3aaxx)12aaaxx$.
23. $-x)14xxx$.
24. $6cc)-18ccccxx$.
25. $-17ppqq)17apppqq$.
26. $2)2^3$.
27. $-4^2)4^5$.
28. $7^3)-7^9$.
29. $-4^{13})-4^{18}$.
30. $6x)12x^3$.
31. $-5a^2)15a^6$.
32. $4x^3)-24x^7$.
33. $-7x^5)-35x^5$.
34. $-a^2c^3)6a^3c^4$.
35. $-e^5f^7x)-e^5f^8x$.
36. $-x^3y^5z^3)-x^5y^7z^9$.
37. $7m^2np^3)-14m^3n^3p^3q^3$.
38. $-4l^3m^3n^2)16l^3m^3s^4n^2$.
39. $9ap^2t^2)18a^5p^4t^2x^3$.

40. $-17h^{11}k^{12}m^4n^5) - 51h^{11}m^4k^{12}n^5$.

41. $-8uuvvw^6) - 32u^2v^3w^9x$.

42. $-31m^3ax)62m^2ya^2xb$.

43. $12xy^5z^9) - 36y^5xz^9k$.

44. $-7hp^2q^3)49hkp^4q^5$.

45. $9xe^3zf)81e^5x^2z^7f^3$.

46. $2a^3x^3)8a^3x^3$.

47. $-3c^4d^3)18c^3d^4$.

48. $7a^5x^4) - 21a^3x^5$.

49. $-13x^3y^5z^7) - 39x^3y^3z^3$.

50. $4a^4b^3x) - 8a^3b^3x^3$.

51. $-20acx^3)15a^3c^4z$.

52. $4ab^3c) - 9a^3b^3c^3$.

53. $35x^4y^5z^6) - 7x^5y^4z^3u^3w$.

54. $-28m^6n^6)21m^4n^3q^3$.

55. $-34a^3x^2)12c^4a^3x^3b$.

56. $-21e^4f^3g) - 18e^3f^3h^4g$.

57. $-17uvx^2) - 17ux^2$.

58. $40c^2) - 15cx$.

59. $-9a^3x^4)6abcx$.

60. $-29r^2s^4t^4x) - 29r^2s^3t^4x$.

61. $x^{-4}y^2z^{-7})x^{-2}y^2z^{-5}$.

62. $a^{-1}c^{-5})a^{-2}c^{-4}$.

63. $ac^{-1}x^4)a^{-3}c^{-1}x^{-4}$.

64. $2s^{-1}k^{-3}t^{-1})12s^6k^{-2}m^{-5}t$.

65. $a^6b^{-2}c^{-3}d^4)a^{-6}b^{-1}c^4$.

66. $3a^{-1}b^{-2}c^{-3}) - 15a^2c^{-4}$.

67. $-4c^{-1}d^{-5}z) - 20c^{-1}d^{-4}z^{-3}$.

68. $-8a^{-1}aa^2)24aa^{-1}a^{-2}$.

69. $x^{\frac{1}{3}})x^{\frac{3}{4}}$.

70. $x^{-2})x^{-\frac{1}{3}}$.

71. $m^{-\frac{1}{3}})m^{-2}$.

72. $a^{\frac{1}{5}})a^{\frac{7}{10}}$.

73. $c^{-1}x^{-2}) - c^2x^3$.

74. $3m^{\frac{1}{2}}n^{\frac{1}{3}})6m^{\frac{1}{3}}n^{\frac{1}{2}}$.

75. $11a^{-2}b^{-1}x^3)13a^{-\frac{3}{4}}b^{-\frac{1}{2}}x^3$.

76. $9x^{\frac{1}{10}}y^{\frac{1}{2}}z)36x^{\frac{4}{5}}y^{\frac{7}{12}}z^{\frac{3}{4}}$.

77. $a^{\frac{7}{8}}c^{\frac{1}{2}}e^{\frac{1}{6}}) - a^{-\frac{15}{8}}ce^{\frac{7}{6}}$.

78. $-4c^{\frac{1}{4}}x^{\frac{1}{6}}) - 12c^{\frac{8}{5}}x^{\frac{3}{4}}$.

79. $7(x+y) \div (x+y)$.

80. $5a(a+b) \div 5a$.

81. $4(u+v)^2 \div 2(u+v)$.

82. $18x(h+k) \div 3x(h+k)$.

83. $10a^2(c+x)^2 \div 5a(c+x)$.

84. $a(b+c)(e+f) \div a(e+f)$.

85. $12a(m+n)x \div 3x$.

86. $15(x+y)3a(z+w) \div 5a$.

87. $18(a+c)5xy(m+n) \div 9y(a+c)$.

88. $7xz^3(p-q) \div xz^3(p-q)$.

89. $13c^3x^3(m-n)^3 \div 13(m-n)^3c^3x^3$.

90. $16(h+u)^3(c-d)^2 \div 4(h+u)(c-d)$.

91. $18rs^2t^2(h+g)^4 \div 9(h+g)rst$.

92. $-5z^3(1+a^2)^7 \div 3z(1+a^2)^4$.

93. $28v^3(v+1)^3 \div -4v(v+1)^2$.

94. $-32c^5(c^2-2c+1)^4 \div 16c^4(c^3-2c+1)^3$.

95. $18x^{11}(a+b)^9(a-3)^5 \div 9x^7(a-3)^3(a+b)^4$.

96. $9(c-x)^{13}(u+v)^{17} \div 9^2(u+v)^{17}(-x+c)^{13}$.

97. $12(a-b)^3 4q^4(r-s^2) \div 16(a-b)(r-s)q^6$.

98. $5ax^2(x+y)^3 9a(z+u) \div 15(x+y)^5(z+u)a^3x^3$.

99. $6c^{-1}x(a-c)^{-2}3x^{-1}(p+q) \div 54(a-c)^2x(p+q)cd$.

Division of Polynomials by Monomials.

100. Divide $25+20$ by 5.

101. $27-18$ by 9.

102. $-48+32$ by 8.

103. $24+20-16$ by -4.

104. $-36-28-12$ by -4.

105. $9a+12b-3c$ by 3.

106. $16a^2-12c^2$ by -4.

107. $4c^3-5c^4$ by $-c^3$.

108. $-18a^5-27a^2$ by $-9a$.

109. $ad+bd-cd$ by d.

110. $12x^3+9x^2-6x^4$ by $-3x^2$.

111. $c^5-2c^4-3c^3$ by $-c^3$.

112. $-20x^5-30x^4$ by $-10x^3$.

113. $34a^3c^2 - 51a^2c^3$ by $-17a^2c^2$.

114. $-12a^4x + 8a^3x - 4a^2x$ by $-4a^2$.

115. $15p^3k^2 - 30p^2k + 10p^3ky$ by $5p^2$.

116. $8a^2b^5c^4 - 16a^5b^3c^4 + 4a^4b^7c$ by $-4a^2b^3$.

117. $60x^6y^4z^5 - 48x^4y^5z^6 + 36x^5y^6z^4$ by $-6x^2y^2$.

118. By $12xy^3z^2$.

119. By $-4x^4y^4z^4$.

120. $42am^3r^3st^2 - 63bc^3r^3st^2 + 105r^3s^3t^2x - 84r^3kst^2$ by $-21r^3st^2$.

121. $12a^4x^3h^2 - 18a^3bx^3h^2 + 8a^3ch^2x^3 - 10a^3dh^2x^3$ by $2a^3h^2x^3$.

122. $-24pq^2 + 36p^2q^2rs^3 - 72p^2q^2rt^3 + 84pq^2rx^2$ by $-12pq^2$.

Simplify

123. $3x^2 - (x^3 - 4x) \div x$.

124. $4c^3d(1 - 2c) - (10c^3d^2 + 8c^3d^3) \div -2c^3d$.

125. $(7a^3 - 14a^2) \div -7a - (-2a^2c + 3ac) \div ac$.

126. $(xy - x^2y + 2x^3y) \div xy - (4x^2 + 4x^3) \div 2x$.

Divide

127. $-xyzw + xyz^2w^2 - yz$ by $-xyzw$.

128. $15a^4b^5 - 7a^3b^3 + 9a^2b^4$ by $-3a^2b^3$.

129. $12c^3de^4 + 16c^4de^7f - 3d^2c^5e^4g$ by $4c^2de^3$.

130. $20m^6n^3pq + 21m^4hn - 35m^5npq^{10}$ by $5m^3npq$.

131. $4a^2cx - 2b^2x^4 - z^2$ by $2b^2x$.

132. $-15abc + 7a^2b - 5ab^2$ by $-3abc$.

133. $54x^3y^3z + 18x^4y^3k^5l - 7xy^2k^9$ by $9xyk$.

134. $36ac^3x^4 - 54ae^3 + 34z^3c$ by $-18acx^3$.

135. $24kl^6mn - 36akm - 48kl^2m$ by $12klm$.

136. $18abcde - 7a^2cdh + 5e^{12}$ by $-2abcde$.

137. $12acz^4 + 11c - 10a^3c^4z^2y$ by $5a^2c^4z$.

138. $36fx^3c - 5r^2s^2x^3 + 63f^2x^4c$ by $21f^2x^3c$.

139. $2a^2 - 3ac + \frac{3}{2}a^2c^3$ by $-\frac{1}{2}a$.

140. $\frac{1}{2}x^4y^2 - \frac{3}{2}x^3y^3$ by $-\frac{3}{2}x^3y^2$.

141. $-2m^4n^3 + \frac{7}{2}m^3n^4$ by $\frac{7}{3}m^3n$.

142. $-\frac{5}{2}a^2 + \frac{5}{2}aw + \frac{10}{3}a$ by $-\frac{5}{6}a$.

143. $x^3z^2 - \frac{6}{25}x^3z^3 + \frac{2}{5}x^3z^4$ by $\frac{6}{5}xz^2$.

144. $-\frac{3}{2}x^3 + \frac{7}{2}cx - 9dx$ by $-\frac{3}{2}x$.

145. $\frac{5}{2}a^2 - \frac{5}{3}ax + \frac{11}{3}a$ by $-\frac{5}{6}a$.

146. $\frac{1}{4}m^2q - \frac{1}{16}mnq - \frac{3}{8}mpq$ by $\frac{3}{8}mq$.

147. $3x^3(a-x) + 12x(a+x)$ by $3x$.

148. $7(a-2)^2 - 3x(a-2)$ by $(a-2)$.

149. $12(a+c)^2 - 4b(a+c) + 8d(a+c)$ by $-4(a+c)$.

150. $7x^4(x-y)^4 - 6x^3(x-y)^3 + 3x^2(x-y)^2$ by $x^2(x-y)^2$.

151. $15(a-x)^3 - 3a(a-x)^2$ by $5(a-x)$.

152. $2(a-c)^4 - m(a-c)^5 - 6(a-c)^6$ by $-2(a-c)^2$.

153. $28x^{\frac{7}{2}} - 32x^{\frac{3}{4}}$ by $4x^{\frac{1}{4}}$.

154. $x^{\frac{3}{4}} - x^{\frac{3}{8}}y^{\frac{3}{8}}$ by $-x^{\frac{1}{4}}$.

155. $m^{\frac{5}{6}} - m^{\frac{1}{6}}n^{\frac{2}{3}} + n^{\frac{2}{3}}$ by $m^{\frac{1}{6}}$.

156. $15x^{\frac{5}{6}} - 25x^{\frac{4}{3}}$ by $5x^{\frac{2}{3}}$.

157. $32x^{\frac{19}{12}} - 30x^{\frac{1}{2}}z^{\frac{3}{4}}$ by $8x^{\frac{1}{2}}$.

Division of Polynomials by Polynomials.

Divide 1248 by 3 ; 62611 by 127. What is the difference between these two examples?

Note that, in the first, the work is very much shortened by being performed mentally and only the result of the division written down, while, in the second, each step must be written down in detail, and repeated in exact order until the division is completed.

Notice carefully what the necessary steps are and the order in which they must be taken, namely, (1) dividing a portion of the dividend by the divisor (or by a portion of the divisor, when the example is very difficult); (2) writing the result obtained in the quotient; (3) multiplying the whole divisor by the quotient already obtained; (4) writing the result of this multiplication under the dividend; (5) subtracting this product from the dividend; (6) bringing down and annexing to the remainder the next figure of the dividend; and constant repetition of these processes, in this same order, continued so long as the remainder will contain the divisor. This method must always be used, in algebra, when the divisor is a polynomial, except in some special cases which will be given later. If these steps be thoroughly learned at the start, "long division" in algebra will present no serious difficulty.

Divide

158. $x^2 - 9$ by $x - 3$.

159. $x^2 - 25$ by $x + 5$.

160. $x^2 - 144$ by $x - 12$.

161. $a^2 - 121$ by $a + 11$.

162. $81 - a^4$ by $9 - a^2$.

163. $9x^2 - 49$ by $3x + 7$.

164. $25x^2 - 256$ by $-5x + 16$.

165. $-64x^4 + 36$ by $-8x^2 - 6$.

166. $1 - 100x^6$ by $-10x^3 + 1$.

167. $-49 + 81c^{14}$ by $9c^7 + 7$.

168. $x^2 - 5x + 6$ by $x - 2$.

169. $x^2 + 5x - 6$ by $x - .1$

170. $x^2 + x - 12$ by $x + 4$.

171. $c^2 - 2c - 15$ by $c - 5$.

172. $w^2 - w - 56$ by $w + 7$.

173. $z^2 - z - 30$ by $z - 6$.

174. $1 + 5x + 6x^2$ by $1 + 2x$.

175. $1 - 8m - 9m^2$ by $1 + m$.

176. $1 - 6m + 9m^2$ by $1 - 3m$.

177. $x^2 + 11x + 30$ by $x + 6$.

178. $x^2 + 11x + 28$ by $x + 4$.

179. $1 - 2e - 48e^2$ by $1 - 8e$.

180. $m^4 - 9m^2 + 8$ by $m^2 - 1$.

181. $c^4 - c^2 - 20$ by $c^2 + 4$.

182. $n^4 - 4n^2 - 21$ by $n^2 - 7$.

183. $1 - 19x^4 - 20x^8$ by $1 + x^4$.

184. $a^6 + a^3 - 72$ by $a^3 - 8$.

185. $p^6 - 10p^3 + 9$ by $p^3 - 9$.

186. $y^4 - 26y^2 + 120$ by $y^2 - 6$.

187. $1 - 35b^3 + 300b^6$ by $1 - 15b^3$.

188. $x^2 - y^2$ by $x - y$.

189. $a^2 - c^2$ by $a + c$.

190. $a^4 - x^4$ by $a^2 - x^2$.

191. $x^2 + 2xy + y^2$ by $x + y$.

192. $4m^2 + 4mn + n^2$ by $2m + n$.

193. $p^2 - 6pq + 9q^2$ by $p - 3q$.

194. $25c^2 + 30cd + 9d^2$ by $5c + 3d$.

195. $16e^2 - 16ef + 4f^2$ by $4e - 2f$.

196. $-r^2 + s^2$ by $-r + s$.

197. $a^4 + 2a^2x^2 + x^4$ by $a^2 + x^2$.

198. $12a^2 - 7ax - 12x^2$ by $3a - 4x$.

199. $15n^2 + 17mn - 4m^2$ by $3n + 4m$.

200. $12u^2 - 11uv - 36v^2$ by $4u - 9v$.

Solve the next fourteen examples both algebraically and arithmetically, and compare the results.

201. $9 - 4$ by $3 - 2$.

202. $25 - 16$ by $5 + 4$.

203. $121 - 49$ by $11 - 7$.

204. $144 - 64$ by $12 + 8$.

205. $49 + 42 + 9$ by $7 + 3$.

206. $64 - 80 + 25$ by $8 - 5$.

207. $144 - 96 + 16$ by $12 - 4$.

208. $64 - 8$ by $4 - 2$.

209. $125 + 27$ by $5 + 3$.

210. $3^2 - 2^2$ by $3 - 2$.

211. $5^2 - 4^2$ by $5 + 4$.

212. $4^3 - 2^3$ by $4 - 2$.

213. $7^3 - 5^3$ by $7 - 5$.

214. $7^3 - 4^3$ by $7 + 4$.

215. $x^3 + 1$ by $x + 1$.

216. $x^3 - 1$ by $x - 1$.

217. $a^3 - 8$ by $a - 2$.

218. $x^3 - y^3$ by $x - y$.

219. $c^3 + z^3$ by $c + z$.

220. $64a^3 - 8x^3$ by $4a - 2x$.

221. $27a^3x^3 - 64c^3$ by $3ax - 4c$.

222. $-8y^3 + 27x^6$ by $-2y + 3x^2$.

223. $-27a^6 - 125x^3$ by $3a^2 + 5x$.

224. $a^6 + b^6$ by $a^2 + b^2$.

225. $m^9 - n^6$ by $m^3 - n^2$.

226. $1728x^6 - 343a^9$ by $-7a^3 + 12x^2$.

227. $125x^6 - 64y^3$ by $5x^2 + 4y$.

228. $343a^3x^6 + 216a^6x^3$ by $7ax^2 + 6a^2x$.

229. $x^3 - 9x^2 + 27x - 27$ by $x - 3$.

230. $a^3 - 5a^2 - 46a - 40$ by $a + 4$.

231. $x^4 + xy^3 + x^3y + y^4$ by $x + y$.

232. $z^3 - z^2w + zw^2 - w^3$ by $z - w$.

233. $m^3 - mn^2 - m^2n + n^3$ by $m - n$.

234. $pq^3 - mpq - mq^2 + m^2$ by $pq - m$.

235. $6a^3 - 2a^2 - 6a - 2$ by $2a - 2$.

236. $c^3 - 86c - 140$ by $c - 10$.

237. $48y^3 - 76cy^2 - 64c^2y + 105c^3$ by $2y - 3c$.

238. $a^4 - 6a^3 + 5a^2 + 24a - 36$ by $a^2 - 4$.

239. $2x^4 - 8x^3y + 18xy^3$ by $2x^2 - 6xy$.

240. $39ax^2 + 30a^3 - 20x^3 - 43a^2x$ by $6a - 5x$.

241. $ax^3 + 3a^2x^2 - 2a^3x - 2a^4$ by $x - a$.

242. $4m^3 + 4m^2 + 29m + 21$ by $2m - 3$.

243. $6x^3 - 13x^2z + 4xz^2 + 3z^3$ by $2x - 3z$.

244. $6x^3y - 17x^2y^2 + 14xy^3 - 3y^4$ by $2x - 3y$.

245. $49a^3 - 72ax^2 + 27x^3$ by $7a - 3x$.

246. $x^3 + y^3$ by $x + y$.

247. $a^3 - x^3$ by $a + x$.

248. $1 - 2a^2$ by $1 + a$.

249. x^2 by $x + y$.

250. $1 + 4a$ by $1 - 6a$.

251. $3c^3 - 7c - 9$ by $c + 1$.

252. $8a + 5a^3 - 8a^2 + 7$ by $5a - 3$.

253. $m^3 - 6m^2 + 11m + 2$ by $m - 2$.

254. 1 by $1 - x$.

255. $a^2 - 7a + 11$ by $a - 2$.

256. $3b^2 + 5b - 9$ by $b - 4$.

257. $1 + x$ by $1 - x$.

258. $z^3 - 17z^2 - 15z - 13$ by $2z - 6$.

259. $y^4 - 19y^2 + 4y - 2$ by $5y - 5$.

260. $6a^5c^3 + 12a^3c^5 - 14a^4c^6 + a^6 - 1$ by $2a^3 - c$.

261. $x^4 - y^4$ by $x + y$.

262. $a^5 + c^5$ by $a + c$.

263. $c^7 + d^7$ by $c + d$.

264. $x^5 - 1$ by $x^3 + 1$.

265. $x^5 + 32w^{10}$ by $x + 2w^2$.

266. $x^{40} - y^4$ by $x^{10} + y$.

267. $12a^2c^2 + 5b^4 - 32a^2b^2 + 48a^4 - 3b^2c^2$ by $4a^2 - b^2$.

268. $6x^2y^2 - 4xy^3 - 4x^3y + y^4 + x^4$ by $x - y$.

269. $6a^4 - 96$ by $3a - 6$.

270. $32x^5 + 243$ by $2x + 3$.

271. $4a^4 - 18a^3 + 22a^2 - 7a + 5$ by $2a - 5$.

272. $2a^2 - 16a + 6$ by $a + 3$.

273. $a^6x^6 - 64$ by $-ax + 2$.

274. $-16x^3 + 46x^2 - 39x + 9$ by $-3 + 8x$.

275. $a^8 - 256$ by $a^2 + 4$.

276. $36x^7 - 36y^7$ by $9x - 9y$.

277. $16m^4 + 16m + 3$ by $4m^2 - 4m + 3$.

278. $81c^4 - 81c^2 - 36cd - 4d^2$ by $9c^2 + 9c + 2d$.

279. $32 - 80a + 80a^2 - 40a^3 + 10a^4 - a^5$ by $4 - 4a + a^2$.

280. $a^3c^3 - 9ac^3 - 8c^3$ by $a^2c^2 + 2ac^2 + c^2$.

281. $-3ax^3 + 4a^3x - 9a^2x^2 + 6a^4 + 2x^4$ by $-ax + 3a^2 - 2x^2$.

282. $x^8 - 2x^7 + 3x^6 - 3x^5 + 3x^4 - 3x^3 + 3x^2 - 2x + 1$ by $x^3 - x + 1$.

283. $3x^2y^2 + 48c^4d^2x^2 + 5c^3xy + cy^3 + 16c^5d^2 - 5c^3x - 16c^4d^2y - y^3 - 15c^2x^3$ by $c - y + 3x^2$.

284. $a^9 - a$ by $a^7 - a^5 + a^3 - a$.

285. $-c^8x^8 + 1$ by $-c^3x^3 + c^2c^2 - cx + 1$.

286. $243a^5e^5 - a^{10}e^{10}$ by $81a^4e^4 + 27a^5e^5 + 9a^6e^6 + 3a^7e^7 + a^8e^8$.

287. $ax + cx + ay + bx + bz + by + cz + cy + az$ by $x + y + z$.

288. $312c^4d^2 - 32c^3d^3 + 20c^3d^2 - 160c^2d^4 + 88c^2d^3 - 12c^2d^2$ by $12c^3d + 8cd^2 - 2cd$.

289. $x^3 + y^3 + z^3 - 3xyz$ by $x + y + z$.

290. $x^8 + x^6z^2 + x^4z^4 + x^2z^6 + z^8$ by $x^4 + x^3z + x^2z^2 + xz^3 + z^4$.

291. $2m^3 - 3m^2 - 6m - 1$ by $2m^2 - 5m - 1$.

292. $6p^3 - p^2 - 14p + 3$ by $3p^2 + 4p - 1$.

293. $6k^5 - 13k^4 + 4k^3 + 3k^2$ by $3k^3 - 2k^2 - k$.

294. $w^4 + 3w^3 + 2w + 15$ by $w - 3$.

295. $3a^6 + 2a^4 - 7$ by $a - 5$.

296. $16y^3 - 19 + 39y - 46y^2$ by $8y - 3$.

297. $11a^3 + 30a^4 - 82a^2 - 5a + 3$ by $-4 + 3a^2 + 2a$.

298. $12c^4 + 6c - 5c^3 + 20 - 33c^2$ by $-5 + c + 4c^2$.

299. $-35n^2 - 71n^3 + 9 + 30n + 28n^4$ by $-13n + 4n^2 + 6$.

300. $-9a^2 + 23ab - 18acx - 21b^2 + 32bcx - 12c^2x^2$ by $3a - 7b + 6cx$.

301. $(a - c)^2 + 1$ by $(a - c) + 1$.

302. $6(u + s)^2 - (u + s) - 15$ by $3(u + s) - 5$.

303. $(x-y)^3 - 3(x - y)^2z + 3(x - y)z^2 - z^3$ by $(x-y)-z$.

304. $[k^2+(m+n+p)k^2+(mn+mp+np)k+mnp]$ by $k+n$.

305. $[cd(x^2 + z^2) + xz(c^2 + d^2)]$ by $cx + dz$.

306. $[x^4 + (2f^2 - e^2)x^2 + f^4]$ by $x^2 + ex + f^2$.

307. $[(a^2x^2 + b^2y^2) - (a^2b^2 + x^2y^2)]$ by $[(a + y)(x + b)]$.

308. $m^5 - n^5 - 5mn(m^3 - n^3) + 10m^2n^2(m - n)$ by $(m+n)^2 - 4mn$.

309. $[xyz-y^2(x + z) + x^2(y+z)+z^2(x+y)]$ by $xy+xz+yz$.

310. $[r(r - 1)c^3 + (r^3 + 2r - 2)c^2 + (3r^2 - r^3)c - r^4]$ by $(rc^2 + 2r - c^2)$.

311. $\frac{1}{2}x^3 + x^2 + \frac{3}{8}x + \frac{3}{4}$ by $\frac{1}{2}x + 1$.

312. $\frac{1}{3}a^3 + \frac{17}{6}a^2 - \frac{5}{6}a + \frac{2}{3}$ by $\frac{1}{3}a + 3$.

313. $\frac{27}{8}y^3 + \frac{27}{125}$ by $\frac{3}{2}y + \frac{3}{5}$.

314. $\frac{3}{4}x^3 - \frac{7}{8}x^2z + \frac{7}{72}xz^2 - \frac{1}{8}z^3$ by $\frac{1}{2}x - \frac{4}{3}z$.

315. $\frac{a^3}{c} - 2c^2 - \frac{3a^2}{2} + \frac{3c^3}{a}$ by $2a - 3c$.

316. $x^4 - \frac{1}{x^4}$ by $x - \frac{1}{x}$.

317. $m^3 + 5m + \frac{5}{m} + \frac{1}{m^3}$ by $m + \frac{1}{m}$.

318. $a^{\frac{3}{2}} - x^{\frac{3}{2}}$ by $a^{\frac{1}{2}} - x^{\frac{1}{2}}$.

319. $p^4 - \frac{5}{4}p^3 + \frac{11}{8}p^2 - \frac{1}{2}p$ by $p^2 - \frac{1}{2}p$.

320. $\frac{2}{3}x^2 - \frac{5}{8}x - \frac{1}{6}y^2 + \frac{13}{6}y - 6$ by $\frac{2}{3}x + \frac{1}{3}y - 3$.

321. $\frac{1}{4}m^2 - \frac{7}{2}mn + 6n^2 + \frac{37}{12}m - 7n + 1$ by $\frac{1}{4}m + 3 - 3n$.

322. $\frac{1}{3}x^4 + \frac{1}{12}x^3 + \frac{31}{8}x^2 - \frac{17}{4}x + 6$ by $\frac{2}{3}x^2 - \frac{5}{6}x + 1$.

323. $\frac{9}{8}h^4 - \frac{3}{2}h^3 + \frac{1}{2}h^2 - \frac{2}{9}$ by $\frac{3}{2}h^2 - h - \frac{2}{3}$.

324. $\frac{9}{16}x^4 - \frac{3}{4}x^3 - \frac{7}{4}x^2 + \frac{4}{3}x + \frac{16}{9}$ by $\frac{3}{8}x^2 - \frac{1}{4}x - \frac{2}{3}$.

325. $x - y$ by $x^{\frac{1}{2}} + y^{\frac{1}{2}}$.

326. $a + x$ by $a^{\frac{1}{3}} + x^{\frac{1}{3}}$.

327. $27c^{-2} + 8d^{-3}$ by $3c^{-\frac{2}{3}} + 2d^{-1}$.

328. $p^{\frac{4}{3}} - q^{\frac{4}{3}}$ by $p^{\frac{1}{3}} + q^{\frac{1}{3}}$.

329. $m^3 + 5m^{\frac{3}{2}} + 6m$ by $m^{\frac{1}{2}} + 3$.

330. $4t^{\frac{5}{2}} - 3t^{\frac{5}{3}}u^{\frac{1}{2}} - u$ by $t^{\frac{5}{3}} - u^{\frac{1}{2}}$.

331. $12x^{\frac{3}{2}} + 35x + 25x^{\frac{1}{2}}$ by $4x^{\frac{1}{2}} + 5$.

332. $r - s$ by $r^{\frac{1}{2}} - s^{\frac{1}{2}}$.

333. $a + a^{\frac{1}{2}}c^{\frac{1}{2}} + c$ by $a^{\frac{1}{2}} - a^{\frac{1}{4}}c^{\frac{1}{4}} + c^{\frac{1}{2}}$.

334. $m^{\frac{1}{2}} - 3m^{\frac{1}{3}}n^{-\frac{1}{6}} + 3m^{\frac{1}{6}}n^{-\frac{1}{3}} - n^{-\frac{1}{2}}$ by $m^{\frac{1}{3}} - 2m^{\frac{1}{6}}n^{-\frac{1}{6}} + n^{-\frac{1}{3}}$.

335. $x^{\frac{5}{2}} - x^2 - 4x^{\frac{3}{2}} + 6x - 2x^{\frac{1}{2}}$ by $x^{\frac{3}{2}} - 4x^{\frac{1}{2}} + 2$.

336. $a^{\frac{4}{3}}x^{-\frac{2}{3}} - 49a^{\frac{2}{3}}x^{-\frac{1}{3}} + 56a^{\frac{1}{3}} - 16x^{\frac{1}{3}}$ by $a^{\frac{2}{3}} - 7a^{\frac{1}{3}}x^{\frac{1}{3}} + 4x$.

337. $x^3 - 4y + 6xz^{\frac{1}{3}} + 9z^{\frac{2}{3}}$ by $x + 2y^{\frac{1}{2}} + 3z^{\frac{1}{3}}$.

338. $c^{\frac{5}{2}} + c^{\frac{3}{2}}d - cd^{\frac{3}{2}} - d^{\frac{5}{2}}$ by $c + c^{\frac{1}{2}}d^{\frac{1}{2}} + d$.

339. $3x^{-1} - 23x^{-2} - 5x^{-3} + 80x^{-4} + 50x^{-5}$ by $x^{-1} - 6x^{-2} - 10x^{-3}$.

340. $24y^3 - 58y^{\frac{5}{3}} + 5y^{\frac{4}{3}} - 4y + 3y^{\frac{2}{3}} + 12y^{\frac{1}{3}} - 32$ by $2y^{\frac{2}{3}} - 3y^{\frac{1}{3}} - 4$.

341. $x^{\frac{3}{2}} - \frac{1}{2}x^{\frac{1}{2}}$ by $x^2 + \frac{1}{2}$.

SYNTHETIC DIVISION.

Divide

342. $a^4 - a^3 - 7a^2 + a + 6$ by $a^2 - a - 6$.

343. $b^4 + b^3 - 9b^2 - 16b - 4$ by $b^2 + 4b + 4$.

344. $1 + 2c^3 - c^5 + 2c^4$ by $1 - c + c^2$.

345. $4 + 9p^4 - 13p^3$ by $3p^2 - 2 + p$.

346. $16q^4 + 8q + 3$ by $4q^4 - 4q + 3$.

347. $y - 4y^3 - 9y^5 + 12y^4$ by $1 + 2y - 3y^2$.

348. $13a^3 + 71a - 70a^2 - 20 + 6a^4$ by $4 + 3a^2 - 7a$.

349. $n^7 - 3n^6 + n^5 - 4 + 12n - 4n^3$ by $n^5 - 4$.

350. $6x^5 + 20x^4 - 12x^3 - 48x^2 + 22x + 12$ by $6x^2 + 14x + 4$.

351. $v^6 - 3v^5 + 6v^4 - 7v^3 + 6v^2 - 3v + 1$ by $v^2 - v + 1$.

352. $a^5 + 4a^3 - 92a - 3a^4 + 26a^2 + 55$ by $a^2 - 3a + 11$.

353. $3p - 28 + 7p^3 - 25p^4 - 6p^5 + 81p^2$ by $-5p + 4 - 3p^2$.

354. $h^4 - 25h^2 + 40h - 16$ by $h^2 + 5h - 4$.

355. $32 - 80c + 80c^2 - 40c^3 + 10c^4 - c^5$ by $4 - 4c + c^2$.

356. $k^6 + 4k^5 - 3k^4 - 16k^3 + 11k^2 + 12k - 9$ by $k^3 - k^2 - 5k + 3$.

357. $a^5 - 2a^4 - 6a^3 + 4a^2 + 13a + 6$ by $a^3 + 3a^2 + 3a + 1$.

358. $4x^6 - 49x^4 + 76x^3 - 16$ by $2x^3 + 5x^2 - 6x - 4$.

359. $x^5 + 243$ by $9x^2 + 81 - 3x^3 - 27x + x^4$.

360. $2z^5 + 3z^4 - 8z^3 - 20z^2 + 30z + 60$ by $2z^3 - 5z^2 + 10$.

SPECIAL CASES IN MULTIPLICATION.

Case I.

The square of any polynomial is the sum of the squares of its several terms and twice the product of each term multiplied in succession by each of the terms that follow it.

Illustration: $(3+4+5+6)^2 = 3^2+4^2+5^2+6^2+2(3\times4)$
$+2(3\times5)+2(3\times6)+2(4\times5)+2(4\times6)+2(5\times6)$
$=9+16+25+36+24+30+36+40+48+60=324.$

$(a+3c-4d)^2 = a^2+(3c)^2+(-4d)^2+2(a\cdot3c)+2(a\cdot-4d)$
$+2(3c\cdot-4d) = a^2+9c^2+16d^2+6ac-8ad-24cd.$

Case II.

The product of the sum and difference of two quantities is the difference of their squares.

Illustration: $(5+3)(5-3) = 25-9 = 16.$

$$(2a+3x)(2a-3x) = 4a^2-9x^2.$$

Case III.

The product of any two binomials is (1) the product of the first terms of the binomials, and (2) the algebraic sum of the cross products of the binomial terms, and (3) the product of the second terms of the binomials.

Illustration: $(5+4)(3-2) = 15+2-8.$

5	5 + 4	+ 4
3	3 − 2	− 2
15	12 − 10	− 8

$$(2a + 3x)(3a - 4y) = 6a^2 + 9ax - 8ay - 12xy.$$

$2a$	$2a + 3x$	$+ 3x$
$3a$	$3a - 4y$	$- 4y$
$6a^2$	$9ax - 8ay$	$- 12xy.$

Case I.

Find by inspection the value of

1. $(3 + 2)^2$.	2. $(5 + 4)^2$.	3. $(1 + 6)^2$.
4. $(9 + 2)^2$.	5. $(0 + 8)^2$.	6. $(a + 2)^2$.
7. $(c + 11)^2$.	8. $(5 + b)^2$.	9. $(8 + x)^2$.
10. $(3 + m)^2$.	11. $(k + 9)^2$.	12. $(z + 12)^2$.
13. $(3 - 2)^2$.	14. $(7 - 4)^2$.	15. $(5 - 1)^2$.
16. $(9 - 0)^2$.	17. $(10 - 6)^2$.	18. $(7 - 3)^2$.
19. $(-3 + 7)^2$.	20. $(3 - 7)^2$.	21. $(8 + 2)^2$.
22. $(-8 - 2)^2$.	23. $(x - 1)^2$.	24. $(1 - x)^2$.
25. $(a - 5)^2$.	26. $(5 - a)^2$.	27. $(-5 + a)^2$.
28. $(c - 4)^2$.	29. $(y - 6)^2$.	30. $(-8 + m)^2$.
31. $(-10 - q)^2$.	32. $(a - c)^2$.	33. $(z + k)^2$.
34. $(m - n)^2$.	35. $(-e + f)^2$.	36. $(3c - y)^2$.
37. $(w + 2h)^2$.	38. $(d - 4y)^2$.	39. $(3a + 2x)^2$.
40. $(5c - 2d)^2$.	41. $(-2x + 3y)^2$.	42. $(-4z - 3u)^2$.
43. $(7c + 5x)^2$.	44. $(9y - 10k)^2$.	

Case II.

(Note: To save space, the product of the sum and difference of two quantities will be indicated by writing the quantities but once, with the sign $\pm$ or $\mp$.)

45. (3 ± 1).	46. (5 ± 4).	47. (7 ± 3).

48. (6 ± 0). 49. (11 ∓ 8). 50. $(x \pm 1)$.

51. $(a \mp 3)$. 52. $(v \mp 5)$. 53. $(2 \pm z)$.

54. $(3 \mp g)$. 55. $(x \pm h)$. 56. $(a \mp c)$.

57. $(3m \mp 4)$. 58. $(3x \pm 2w)$. 59. $(a \pm 4x)$.

60. $(2ab \pm c)$. 61. $(4ax \mp 3cy)$. 62. $(6a^2 \mp x)$.

63. $(a^2 \pm b^3)$. 64. $(3m^2 \mp 4n^4)$. 65. $(2a^2b \pm 3cd^2)$.

66. $(ax^3 - b^2y)^2$. 67. $(5x^4 + 6x^3)^2$. 68. $(4abc + 6de)^2$.

69. $(4cdn + 3a^2x^3y^3)^2$. 70. $(3x^2y^3z^4 \pm 5mn^7)$.

71. $(12x^3y^4k^5 - 0u^{17}v^{21})^2$. 72. $(9a^3m^3nx \pm 3y^5)$.

73. (1 ± 25). 74. $(5a^2c - 10ab^3)^2$.

75. $(6x^2y + 12xy^3)^2$. 76. $(3acx \pm 9b^2d^2y^2)$.

77. $(axy + xyz)^2$. 78. $(15 \pm 3bx^2y)$.

79. $(5a^3b^2c - 5ab^3c^2)^2$. 80. $(100 + 1)^2$.

81. $(m^8y^3 - n^{11}x^2)^2$. 82. $(1 - 12x^5y^{13}z^8)^2$.

83. $(0 + 25a^{17}x^{14})^2$. 84. $(7a^2b^2c^2d^2 \pm 8e^3f^3g^3h^3)$.

85. $(-3x^4y^2 + 5w^5z^3)^2$. 86. $(-10a^9c^{10}y^{11} - 8bd^2x^3)^2$.

87. $(-9a^3b^4c^5d + 12a^2b^3xy)(12xb^3ya^2 - 9c^5a^3db^4)$.

88. $(1 + \frac{1}{8})^2$. 89. $(3 + \frac{1}{5})^2$. 90. $(2 - \frac{1}{8})^2$.

91. $(4 \pm \frac{1}{3})$. 92. $(2a + \frac{1}{2})^2$. 93. $(x + \frac{1}{2}y)^2$.

94. $(a^2 \pm \frac{1}{2})$. 95. $(2a \pm \frac{1}{4})$. 96. $(2x^2 - \frac{1}{3})^2$.

97. $(\frac{2}{3} \mp 4ab)$. 98. $(\frac{1}{3}a^2b - \frac{2}{3}ab)^2$.

99. $(\frac{3}{4}c^3 - \frac{1}{2}a^3c)^2$. 100. $(\frac{1}{2}pq^2 + \frac{1}{3}p^2q^3)^2$.

101. $\left(a + \frac{1}{a}\right)^2$. 102. $\left(\frac{r}{2} - \frac{s}{3}\right)^2$. 103. $\left(\frac{ax}{3} + \frac{cz}{2}\right)^2$.

104. $\left(\frac{x}{a}-\frac{y}{c}\right)^2$. 105. $\left(\frac{x}{a}-\frac{y}{2e}\right)^2$. 106. $\left(\frac{x}{2a}-\frac{m}{b}\right)^2$.

107. $\left(ax+\frac{m}{ax}\right)^2$. 108. $\left(\frac{x^2}{2a}-\frac{y^2}{2b}\right)^2$.

109. $\left(\frac{x}{x+y}+\frac{y}{x+y}\right)^2$.

110. $(x^{\frac{1}{2}} \pm 3)$. 111. $(a^{-1}x - 2c^2)^2$.

112. $(5a^{-2} + 3a^3b^2)^2$. 113. $(4x \mp 5x^{-1})$.

114. $(7x^{-1} \pm 9y^{-2})$. 115. $(x^{-\frac{3}{2}} + xy)^2$.

116. $(4a^{\frac{1}{2}} + 2c^{-1})^2$. 117. $(m^{-1}n - 2n^2)^2$.

118. $(c^{\frac{1}{2}} \pm x^{\frac{1}{2}})$. 119. $(2x^{\frac{1}{2}} \mp 3y^{-2})$.

120. $(2^{\frac{1}{2}}x^{\frac{1}{2}} \pm 3^{\frac{1}{2}}y^{\frac{1}{2}})$. 121. $(\frac{1}{2}ac^{-2} + \frac{2}{3}x^{-1}c^{-1})^2$.

122. $(\frac{1}{2}ab^{-2} \mp \frac{2}{3}c^{-\frac{1}{2}}d^{\frac{1}{3}})$. 123. $(3^{\frac{1}{2}}4^{\frac{1}{2}}5^{\frac{1}{2}} \pm 2^{-1}3^{-2}4^{-3})$.

124. $(\sqrt{x} \pm 7)$. 125. $(2\sqrt{x} \mp 6)$. 126. $(3\sqrt{x} + 4)^2$.

127. $(x \pm \sqrt{3})$. 128. $(\sqrt{a} \pm \sqrt{13})$.

129. $(3\sqrt{a} \pm 2\sqrt{5})$. 130. $(a \pm \sqrt{-3})$.

131. $(3c^2 \mp 2\sqrt{-6})$.

132. $3^3(a^2 - x^2)^2$.

133. $(a + 2c)(a - 2c)(a^2 + 4c^2)$.

134. $(3a + 2)(3a - 2)(9a^2 + 4)$.

135. $(x^3 - 3a)(x^3 + 3a)(x^4 - 9a^2)$.

136. $(a^2 - x^2)(a^2 + x^2)(a^4 - x^4)$.

137. $3(4a^2 + 25c^2)(2a + 5c)(2a - 5c)$.

138. $(2 + \overline{a - c})(2 - \overline{a - c})$.

139. $(3 - \overline{a + x})^2$.

140. $(5 + \overline{m + n})^2$.

141. $\{(a + b) + (-c)\}^2$.

142. $(\overline{a + b} + c)(\overline{a + b} - c)$.

143. $(\overline{a + b} + \overline{c + d})^2$.

144. $\{(x + h) - (x - h)\}^2$.

145. $(a + x)^2(a - x)^2$.

146. $(a^2 + 6\{a + x\})(a^2 - 6\{a - x\})$.

147. $(1 + a)^2 - (1 + a)(1 - a)$.

148. $(3c + 4d)^2(3c - 4d)^2$.

149. $(a - 5)(a - 2)(a + 2)(a + 5)$.

150. $(x - 2)(x + 2)(x^2 + 4)(x^4 + 16)$.

151. $(c^2 - d)(c^3 + d^4)(c^4 + d^2)(c^2 + d)$.

152. $(x^2 + 2ac)(x^2 - 2ac)(x^8 + 16a^4c^4)(x^4 + 4a^2c^2)$.

153. $(m^3n^4 - 3mn^3)(m^3n^4 + 3mn^3)(m^6n^8 + 9m^2n^6)$
$(m^{12}n^{16} + 81m^4n^{12})$.

154. $(2 - x^n)^2$.

155. $(a^{-p} - c^{-q})^2$.

156. $(x^n \pm 5)$.

157. $(3a^{-1/n} - 2b^{-x/2})^2$.

158. $(1 + 2 + 3)^2$.

159. $(4 + 2 + 6)^2$.

160. $(5 + 3 - 2)^2$.

161. $(6 - 3 - 2)^2$.

162. $(-4 + 2 + 5)^2$.

163. $(-3 + 4 + 5)^2$.

164. $(2 - 4 - 6)^2$.

165. $(3 - 5 + 2)^2$.

166. $(-6 + 8 - 2)^2$.

167. $(-2 - 3 - 4)^2$.

168. $(m + n - 4)^2$.

169. $(a + 3 - c)^2$.

170. $(5 - x - y)^2$.

171. $(x^2 + 3x - 2)^2$.

172. $(a + 2x - 3c)^2$.

173. $(2a^2 - 7a + 8)^2$.

174. $(a^2 + c^2 - x^2)^2$.

175. $(x^3 + y^3 + z^3)^2$.

176. $(a^3 - b^3 - c^3)^2$.

177. $(x^2 - 2y^2 + 4z^2)^2$.

178. $(1 - e + e^2)^2$.

179. $(2x^3 - 3x^2 + 4x)^2$.

180. $(ax + cx + ac)^2$.

181. $(x^2 - xy + y^2)^2$.

182. $(a^4 - 4a^2c^2 + c^4)^2$.

183. $(abc - bcd - cde)^2$.

184. $(a + b + c)(a + b - c)$.

185. $(1 + x + x^2)(1 + x - x^2)$.

186. $(2x^2 - 3x + 4)(2x^2 + 3x + 4)$.

187. $(- k^2 + kl + r)(k^2 + kl + r)$.

188. $(2x + 3y + 4z)(- 2x + 3y - 4z)$.

189. $(1 - n + n^2)(1 + n + n^2)$.

190. $(2x - 3a + 4c)(2x + 3a - 4c)$.

191. $(m + n - p + q)^2$.

192. $(1 - x + x^2 - x^3)^2$.

193. $(c - a + x)^2 - (c + a - x)^2$.

194. $(x + y + z)^2 + (x - y + z)^2 + (x + y - z)^2 + (-x + y + z)^2$.

195. $(x + y + z)^2 - x(y + z - x) - y(x + z - y) - z(x + y - z)$.

196. $(a + b + c - d)(a + b - c + d)$.

197. $(b - c - d - e)(b + c + d - e)$.

198. $(a^3 + a^2 + 2a - 1)(- a^3 + a^2 - 2a - 1)$.

199. $(1 + 2a + 3c + 4x)(1 + 2a - 3c - 4x)$.

200. $(x^3 - 3x^2y + 3xy^2 - y^3)(x^3 + 3x^2y + 3xy^2 + y^3)$.

201. $(1 + \frac{1}{2} + \frac{1}{3})^2$.

202. $(\frac{1}{2}a + 3 + \frac{1}{3}c)^2$.

203. $(\frac{1}{2}a + \frac{1}{3}c - \frac{1}{4}x)^2$.

204. $(\frac{3}{4}a^2 - a + \frac{1}{2}x)^2$.

205. $(\frac{1}{2}a - \frac{1}{3}b - \frac{2}{3}c)^2$.

206. $(\frac{1}{4}x^3 + \frac{1}{2}x^2 - x)^2$.

207. $(\frac{1}{4}a^2 + \frac{1}{3}c^2 + \frac{1}{2}e^2)(\frac{1}{4}a^2 + \frac{1}{3}c^2 - \frac{1}{2}e^2)$.

208. $(\frac{1}{2}x - \frac{2}{3}y + \frac{3}{4}z)(\frac{1}{2}x + \frac{2}{3}y + \frac{3}{4}z)$.

209. $(\frac{2}{3}a^2 - \frac{1}{2}c^3 - \frac{1}{3}x^3)(\frac{2}{3}a^2 + \frac{1}{2}c^3 + \frac{1}{3}x^3)$.

210. $(-\frac{1}{2}ac - \frac{2}{3}ax + \frac{3}{4}ay)(\frac{1}{2}ac + \frac{2}{3}ax + \frac{3}{4}ay)$.

211. $(3a^{\frac{1}{2}} - 2 - a^{-\frac{1}{2}})^2$.

212. $(c+c^{\frac{1}{2}}-4)(c+c^{\frac{1}{2}}+4)$.

213. $(2p^{\frac{1}{2}} + 2 + 3p^{-\frac{1}{2}})(2p^{\frac{1}{2}} + 2 - 3p^{-\frac{1}{2}})$.

214. $(4 - m^{\frac{1}{2}} + m^{-1})(4 + m^{\frac{1}{2}} + m^{-1})$.

215. $(x^a + 5 + 3x^{-a})(x^a - 5 - 3x^{-a})$.

216. $\{(a + x)^{\frac{1}{2}} - (a - x)^{-\frac{1}{2}}\}^2$.

217. $(\sqrt[3]{x^2} + 2 + 2x^{-\frac{2}{3}})^2$.

218. $(x^{\frac{3}{2}} + 2\sqrt{xy} + 3y^{\frac{3}{2}})^2$.

Case III.

219. $(x + 3)(x + 2)$.

220. $(x + 3)(x + 1)$.

221. $(x + 2)(x + 1)$.

222. $(x + 4)(x + 2)$.

223. $(x + 3)(x + 4)$.

224. $(x + 5)(x + 2)$.

225. $(x + 1)(x + 4)$.

226. $(a + 8)(a + 4)$.

227. $(x - 4)(x - 3)$.

228. $(x - 3)(x - 1)$.

229. $(x - 7)(x - 2)$.

230. $(x - 1)(x - 8)$.

231. $(x - 9)(x - 6)$.

232. $(x - 5)(x - 10)$.

233. $(m - 1)(m - 12)$.

234. $(c - 25)(c - 2)$.

235. $(x + 2)(x - 1)$.

236. $(x - 2)(x + 1)$.

237. $(x + 7)(x + 5)$.

238. $(x + 7)(x - 5)$.

239. $(x - 7)(x + 5)$.

240. $(a - 6)(a + 8)$.

241. $(k - 15)(k - 2)$.

242. $(1 - x)(1 - 4x)$.

243. $(1 + 2a)(1 + 5a)$.

244. $(1 + 3c)(1 - 4c)$.

245. $(1 - 9e)(1 + 8e)$.

246. $(a^2 + 2)(a^2 + 5)$.

247. $(x^2 - 8)(x^2 - 1)$.
248. $(v^2 + 7)(v^2 - 4)$.
249. $(x^3 - 1)(x^3 - 2)$.
250. $(a^3 + 4)(a^3 + 3)$.
251. $(y^3 - 5)(y^3 + 3)$.
252. $(z^3 - 4)(z^3 + 5)$.
253. $(2a + 1)(2a + 3)$.
254. $(3x - 2)(3x - 4)$.
255. $(4m + 5)(4m - 2)$.
256. $(6z - 6)(6z + 10)$.
257. $(7b + 4)(7b + 2)$.
258. $(1 - 2c^3)(1 - 4c^3)$.
259. $(3a^2 + 3)(3a^2 - 8)$.
260. $(4m - 8)(4m + 1)$.
261. $(5k^2 - 4)(5k^2 + 6)$.
262. $(2 + 3x)(2 + 4x)$.
263. $(3 - y)(3 - 4y)$.
264. $(4 + 5z)(4 - 2z)$.
265. $(5 - 2u)(5 + 8u)$.
266. $(6 + v)(6 + 11v)$.
267. $(3 - 2a^2)(3 + 4a^2)$.
268. $(4 + 7x^2)(4 - x^2)$.
269. $(6 - x)(4 - x)$.
270. $(5 + y)(1 + y)$.
271. $(2 + m)(10 + m)$.
272. $(8 - c)(6 - c)$.
273. $(3 + d)(7 + d)$.
274. $(9 - k)(1 - k)$.
275. $(7 + n)(3 + n)$.
276. $(8 - z)(3 - z)$.
277. $(1 - 2x)(1 + 12x)$.
278. $(2 - 2x)(2 + 3x)$.
279. $(3 + 5c)(3 - c)$.
280. $(5 + 6y)(5 - 2y)$.
281. $(9 - 2a)(9 + 3a)$.
282. $(7 - 2k)(7 - 4k)$.
283. $(6 + 10z)(6 - 9z)$.
284. $(8 - b)(8 + 7b)$.
285. $(10 + x)(10 - 3x)$.

286. $(x^{\frac{1}{2}} + 4)(x^{\frac{1}{2}} + 1)$.
287. $(a^{\frac{1}{2}} - 5)(a^{\frac{1}{2}} - 3)$.
288. $(c^{\frac{1}{3}} + 4)(c^{\frac{1}{3}} - 1)$.
289. $(y^{\frac{2}{3}} - 4)(y^{\frac{2}{3}} + 7)$.
290. $(x^{-2} + 6)(x^{-2} + 5)$.
291. $(z^{-1} - 8)(z^{-1} - 3)$.
292. $(m^{-3} + 9)(m^{-3} - 4)$.
293. $(k^{-2} - 4)(k^{-2} + 10)$.

294. $(x^{\frac{1}{2}}+1)(x^{\frac{1}{2}}+2)$.
295. $(w^{\frac{1}{2}}-7)(w^{\frac{1}{2}}-6)$.
296. $(n^{\frac{1}{4}}+9)(n^{\frac{1}{4}}-3)$.
297. $(e^{\frac{2}{3}}-4)(e^{\frac{2}{3}}+8)$.
298. $(2a^{-1}-1)(2a^{-1}+3)$.
299. $(4x^{\frac{1}{3}}+1)(4x^{\frac{1}{3}}-7)$.
300. $(6+4x^{-\frac{1}{2}})(6-4x^{-\frac{1}{2}})$.
301. $(1-a^{-\frac{1}{2}})(1+2a^{-\frac{1}{2}})$.
302. $(3a^{-\frac{2}{5}}-4)(3a^{-\frac{2}{5}}+10)$.

303. $(x+a)(x-3a)$.
304. $(x+2y)(x+5y)$.
305. $(c-4w)(c-6w)$.
306. $(y-2z)(y+7z)$.
307. $(m+n^2)(m+5n^2)$.
308. $(e^2-6f)(e^2+2f)$.
309. $(ax-by)(ax-3by)$.
310. $(ab+bc)(ab-4bc)$.
311. $(cd-2cx)(cd+3cx)$.
312. $(x^2-2x)(x^2-5x)$.
313. $(2a-3c)(2a+4c)$.
314. $(3m-4n)(3m-6n)$.
315. $(3x+y)(3x+4y)$.
316. $(2c+5ay)(2c-6ay)$.
317. $(4ac-5ax)(4ac+2ax)$.
318. $(2x^2+3y^2)(2x^2+8y^2)$.
319. $(4x^3-\frac{1}{2})(4x^3+\frac{3}{4})$.
320. $(x-a)(x-b)$.
321. $(x+c)(x+d)$.
322. $(x+a)(x-c)$.
323. $(x-m)(x+n)$.
324. $(a+2x)(a+3y)$.
325. $(k-2l)(k-5l)$.
326. $(v-3a)(v-4x)$.
327. $(7-2a)(5+2a)$.
328. $(3+5c)(4+5c)$.
329. $(6+ax)(7-ax)$.
330. $(-4+cd)(8+cd)$.
331. $(-3+4ax)(-5+4ax)$.
332. $(-2-6x^2)(-9+6x^2)$.
333. $(-7^n+xy)\,(7^n-3xy)$.
334. $(-x^{-3}+2cd)\,(x^{-3}+cd)$.
335. $(2x+a)(3x-y)$.
336. $(5c-n)(c-2a)$.

337. $(-x+3a)(2x-4c)$. **338.** $(4a+3w)(-2a-k)$.

339. $(-3m+2p)(-2n-2q)$.

340. $(2ay-3x)(-4ay+c)$. **341.** $(1-5ac)(3x-2ac)$,

342. $(2a+ay)(-3a+3ax)$. **343.** $(3-7cd)(2ab+cd)$.

344. $(-2ax-2ay)(-5ax-4ab)$.

345. $(2x-1)(x+4)$. **346.** $(3x+2)(x+4)$.

347. $(2c-3)(c-8)$. **348.** $(x+5)(3x+2)$.

349. $(3x-7)(2x+5)$. **350.** $(3x+7)(2x-5)$.

351. $(5x+3)(2x-6)$. **352.** $(2a+9)(a+3)$.

353. $(a-3c)(2a-2c)$. **354.** $(x+2y)(2x+y)$.

355. $(3a-x)(2a-4x)$. **356.** $(n-1)(4n+p)$.

357. $(1+x)(a-2x)$. **358.** $(4e+3f)(e+5f)$.

359. $(2y-5a)(3y-6a)$. **360.** $(ax-3c)(2ax+4c)$.

361. $(3ay+ax)(2ay-4ax)$. **362.** $(2x-3y)(4x-7y)$.

363. $(3ab-4cd)(2ab+cd)$. **364.** $(-3x+4y)(2x-5y)$.

365. $(-4a-7x)(-3a-2x)$.

SPECIAL CASES IN DIVISION.

Determine what binomial or binomials, if any, will exactly divide the following expressions, and give the quotient or quotients in each case:

1. $a^3 - 1$.
2. $x^3 - 8$.
3. $c^3 - 27$.
4. $m^3 - 64$.
5. $1 - 8x^3$.
6. $1 - 27x^3y^3$.
7. $n^3 - 125$.
8. $1 - 8y^3$.
9. $8x^3 - 1$.
10. $1 - 64a^3x^3$.
11. $27 - m^3n^3$.
12. $27c^3d^3 - 8$.
13. $x^3 - 512$.
14. $k^3 - 216$.
15. $w^3 - 343$.
16. $216 - e^3$.
17. $1 - 343y^3$.
18. $8a^3 - 27$.
19. $x^3y^3 - 1$.
20. $(x^6 - 1) \div (x^3 - 1)$.
21. $(c^{12} - 27) \div (c^4 - 3)$.
22. $x^3 - y^3$.
23. $x^3y^3 - z^3$.
24. $(a^6 - x^6) \div (a^2 - x^2)$.
25. $8a^3 - b^3$.
26. $(x^9 - y^9) \div (x^3 - y^3)$.
27. $8a^3 - 27b^3$.
28. $64a^3c^3 - 27x^3$.
29. $729x^3 - 512z^3$.
30. $(c^3 - 125x^3) \div (c^3 - 5x^3)$.
31. $a^6x^3 - y^9$.
32. $8x^3 - 125y^6$.
33. $64m^3n^3 - 343$.
34. $729a^6 - 125x^3$.
35. $512m^9n^3 - 27x^6$.
36. $27a^4 - 16c^3$.
37. $8m^{15}n^3 - p^{12}$.
38. $x^4 - 1$.
39. $1 - 16x^4$.
40. $a^4 - m^4$.
41. $16 - x^4$.
42. $81 - a^4$.
43. $x^4 - c^4z^4$.
44. $625u^4 - v^4$.
45. $x^5 - 1$.
46. $(a^6 - x^6) \div (a - x)$.
47. $(a^3 - c^3) \div (a^2 - c^2)$.
48. $16 - n^4$.

49. $64a^6 - x^6$.
50. $1 - x^7$.
51. $x^5 - 243y^5$.
52. $81c^4 - 256x^4$.
53. $243m^5 - 32n^5$.
54. $128p^7 - q^{14}$.
55. $a^5 - 32c^5$.
56. $243k^5 - 1$.
57. $x^5 - 32y^{10}$.
58. $x^{15} - y^{10}$.
59. $81a^8 - 16c^4$.
60. $a^{10}b^{15}c^{20} - y^{25}$.
61. $a^{-5} - c^{-5}$.
62. $x^{\frac{3}{5}} - y^{\frac{3}{5}}$.
63. $a^{-\frac{7}{5}} - x^{-7}$.
64. $\frac{a^3}{x^3} - c^9$.
65. $1 - \frac{8}{27}x^3$.
66. $\frac{1}{8}a^3 - v^3$.
67. $\frac{1}{243}a^5 - x^5$.
68. $\frac{1}{125}m^3 - \frac{1}{64}n^3$.
69. $(a - x) \div (a^{\frac{1}{3}} - x^{\frac{1}{3}})$.
70. $(x^3 - y^3) \div (x^{\frac{3}{2}} - y^{\frac{3}{2}})$.
71. $(x^3 - y^3) \div (x^{\frac{3}{5}} - y^{\frac{3}{5}})$.
72. $(a - 243x^{\frac{5}{3}}) \div (a^{\frac{1}{5}} - 3y^{\frac{1}{3}})$.
73. $1 - 8a^{-3}$.
74. $x^{-4} - 1$.
75. $x^{\frac{3}{2}} - 27$.
76. $(x^3 - 81a^3) \div (x^{\frac{1}{2}} - 3a^{\frac{1}{2}})$.
77. $(a^{4x} - x^6) \div (a^x - x^{\frac{3}{2}})$.
78. $x^{\frac{5}{3}} - 1$.
79. $x^{\frac{7}{3}}y^{-\frac{14}{5}} - z^{\frac{14}{5}}w^{-\frac{7}{3}}$.
80. $\frac{1}{8}m^3n^3 - 125x^{-3}y^{-3}z^{-3}$.
81. $[(a + c)^3 - x^3]$.
82. $[(x - y)^4 - z^4]$.
83. $[m^4 - (p - q)^4]$.
84. $[a^4c^4 - (b + d)^4]$.
85. $[(a + b)^5 - c^5]$.
86. $[k^7 - (r + s)^7]$.
87. $[(a + x)^3 - c^3]$.
88. $[8m^6n^6 - (mn + 1)^3]$.
89. $[(x^2 - ac)^3 - 8p^3q^3]$.
90. $(ax + by)^3 - (ax - by)^3$.
91. $1 + x^3$.
92. $m^3 + 8$.
93. $1 + 8x^3$.
94. $1 + 64a^3$.
95. $8a^3x^3 + 1$.
96. $x^3 + 343$.
97. $a^3b^3 + 512$.
98. $512x^9y^3 + 27$.
99. $216 + 1000x^3$.

100. $x^3 + y^3$. **101.** $8c^3 + d^3$. **102.** $a^5 + x^5$.

103. $27a^3 + c^3$. **104.** $r^5 + 32s^5$.

105. $64a^3 + 1000y^3$. **106.** $729m^3 + 216n^3$.

107. $1 + a^9$. **108.** $a^5 + 243x^5$.

109. $1024a^5 + 243b^5$. **110.** $(x^9 + y^9) \div (x^6 + y^3)$.

111. $a^{10}b^5 + c^{15}$. **112.** $64a^3c^3 + 27x^3$.

113. $343x^3 + 1000z^3$.

114. $a^3 + \frac{1}{8}x^6$. **115.** $1 + x^{-3}$. **116.** $27a^3 + \frac{1}{64}c^3$.

117. $64x^{-3} + \frac{1}{27}y^3$. **118.** $a^{10} + \frac{1}{32}x^5$. **119.** $x^{\frac{1}{2}} + 1$.

120. $a^{\frac{3}{5}} + c^{\frac{3}{7}}$. **121.** $x^{-5} + m^{\frac{5}{2}}$. **122.** $x^{\frac{3}{2}} + y^{\frac{3}{2}}$.

123. $ax^{\frac{5}{2}} + c^5y$.

124. $a^4 - 1$. **125.** $1 - 81x^4$. **126.** $y^4 - 256$.

127. $625 - a^4$. **128.** $a^4 - x^4$. **129.** $p^4 - q^{12}$.

130. $a^6 - x^3$. **131.** $81 - x^{12}$. **132.** $x^6 - 1$.

133. $x^4 - 81a^4$. **134.** $16x^4 - 1$. **135.** $a^6 - 729c^6$.

136. $81p^4 - 16q^4$. **137.** $256a^4 - 10000$.

138. $625m^4 - 1$. **139.** $x^6 - 64$. **140.** $81m^4 - 256n^4$.

141. $a^8c^{12} - 256x^{16}$. **142.** $x^{12} - 625z^8$. **143.** $1296x^4 - a^8$.

144. $256x^{16} - y^8$.

145. $\frac{a^4}{x^4} - c^4$. **146.** $x^{-6} - y^{-4}$. **147.** $1 - 81a^{-8}$.

148. $x^{\frac{4}{5}} - y^{\frac{4}{5}}$. **149.** $a^{4m} - x^6$.

150. $(a - b) \div (a^{\frac{1}{2}} - b^{\frac{1}{2}})$. **151.** $(x - x^{-1}) \div (x^{\frac{1}{2}} - x^{-\frac{1}{2}})$.

152. $x - 81z^{\frac{4}{3}}$. 153. $256a^{-\frac{4}{5}} - x^{-\frac{4}{5}}$.

154. $\frac{1}{16}a^{\frac{4}{3}} - \frac{1}{81}b^8$.

155. $a^6 + 1$. 156. $c^{10} + 1$. 157. $x^{12} + 1$.

158. $729x^6 + y^3$. 159. $64 + a^6$. 160. $a^{10} + y^{10}$.

161. $a^6 + x^6$. 162. $x^{12} + y^{12}$. 163. $x^{12} + 64y^6$.

164. $1 + 729a^6$. 165. $1000000a^6 + 64x^{12}$.

166. $a^6 + 64$. 167. $\frac{1}{64}a^6 + c^{-12}$.

168. What are the quotients when the divisor is $a + x$ and the dividends are, respectively, $a^2 - x^2$, $a^3 + x^3$, $a^5 + x^5$, $a^6 - x^6$, $a^9 + x^9$?

169. When the divisor is $m - n$ and the dividends are, respectively, $m^2 - n^2$, $m^3 - n^3$, $m^4 - n^4$, $m^6 - n^6$, $m^7 - n^7$?

170. When the divisor is $p + 1$ and the dividends are, respectively, $p^2 - 1$, $p^3 + 1$, $p^5 + 1$, $p^7 + 1$, $p^8 - 1$?

171. When the divisor is $q - 1$ and the dividends are respectively, $q^2 - 1$, $q^3 - 1$, $q^5 - 1$, $q^7 - 1$, $q^9 - 1$?

172. Which of the following expressions are divisible by $x + y$? $x^{250} - y^{250}$; $x^{251} + y^{251}$; $x^{452} + y^{452}$; $x^{553} - y^{553}$. Which are divisible by $x - y$?

173. Which of the following quantities are divisible by $a + 1$? $a^{127} - 1$; $a^{126} - 1$; $a^{125} + 1$; $a^{124} + 1$. Which by $a - 1$?

174. What are the last three terms in the quotients of the divisible quantities in the previous example?

175. What does $x^2 - xy + y^2$ suggest?

176. What does $x^3 + 3x^2y + 3xy^2 + y^3$ suggest?

177. What does $c^2 - 2cd + d^2$ suggest?

178. What does $m^2 + mn + n^2$ suggest?

179. What does $h^2 - k^2$ suggest?

180. What does $(p^2 + q^2)(p + q)(p - q)$ suggest?

181. What does $a^2 + a + 1$ suggest?

In the following examples write the second factor required to produce, with the one given, the sum or difference of two cubes:

182. $xy + 1$.	**183.** $a - 5x$.	**184.** $2ab + 3cd$.
185. $4x^2 - 5y$.	**186.** $a^2 + 4c$.	**187.** $xy + 8$.
188. $mn - 7y$.	**189.** $h - 4k^2$.	**190.** $xyz + 2$.
191. $x^{-2} + a$.	**192.** $a + y^{\frac{1}{2}}$.	**193.** $c^n - d$.

194. $\frac{1}{3}m^2 - n$.

195. Write the cubes in each of the above examples.

FACTORING.

Case I. — When the Factors are Monomials.

The factors of $8 = 2 \cdot 2 \cdot 2$, of $15 = 5 \cdot 3$, of $36 = 3 \cdot 3 \cdot 2 \cdot 2$, of $13 = 13 \cdot 1$, of $a^3 = a \cdot a \cdot a$, of $ab = a \cdot b$, of $5x^3 = 5 \cdot x \cdot x$, of $12a^3b = 2 \cdot 2 \cdot 3 \cdot a \cdot a \cdot b$, of $6x^3y^2 = 2 \cdot 3 \cdot x \cdot x \cdot x \cdot y \cdot y$.

What are the prime factors of

1. 12. 2. 30. 3. 18. 4. 45. 5. 50.

6. 98. 7. 169. 8. 242. 9. 105. 10. 154.

What are the equal factors of

11. 4. 12. 9. 13. 16. 14. 49. 15. 121.

16. 36. 17. 27. 18. 125. 19. 216.

20. 64. [Three sets.]

21. What name is given to one of the equal factors of a quantity?

22. Two equal factors of $16x^6y^4$.

23. Square root of $49x^8$.

24. Three equal factors of $125a^3x^6$.

25. Cube root of $1000m^6n^{12}$.

26. Four equal factors of $81p^{12}q^8$.

27. Fourth root of $625a^8c^4$.

28. Five equal factors of $32z^5$.

29. Fifth root of $243a^{10}$.

30. Is 1 a perfect square? A perfect cube? A perfect nth power? Why?

31. Is x^6 a perfect square? A perfect cube? A perfect nth power? Why?

32. Is -1 a perfect square? A perfect cube? A perfect nth power? Why?

33. Is $-x^2$ a perfect square? Why? Can $-x^2$ be separated into two factors one of which is a perfect square? If so, what are they?

34. Cube root of $-125k^3l^6$. **35.** Of $1728a^9c^{12}$.

36. Fourth root of $256a^4m^8$. **37.** Of $-81a^4$.

38. Cube root of $64a^6$. Square root?

39. Cube root of $-64a^6$. Square root?

40. Find as many roots as possible of $256a^{16}$.

What are the prime factors of

41. $18x^3y^2$. **42.** $27mn^2p^3$. **43.** $21x^4y^3z^2$.

44. $49m^2n^3p^4q^5$. **45.** $64a^2c^4x^3$. **46.** $-64c^3d^5$.

47. $-72m^2n^3p$. **48.** $-210a^3kc^2l$. **49.** $-(64c^3x^4)$.

50. $-(-39a^4xc^3z^2)$.

51. Two equal factors of $64x^{12}y^6$. Three equal factors of one of these?

52. Separate $7x^2y$ into two factors, in three different ways.

53. Separate a^3x^2 into two factors in five different ways.

54. Separate $45a^3c$ into three factors, two of which are equal.

55. Separate $80p^3q$ into two factors, one of which is a perfect square.

56. What is the greatest perfect square contained as a factor in $96a^5x^3$?

57. The greatest perfect cube contained as a factor in $-500m^7b^5$?

58. The greatest perfect fourth power contained as a factor in $405k^5l^3$?

59. The greatest perfect fifth power contained as a factor in $486a^5c^7x^{11}$?

60. The greatest perfect sixth power contained as a factor in $384x^{10}y^{11}z^{12}$?

61. Two equal factors of a^{2n}? **62.** Of x?

63. Five equal factors of a?

Factors of

64. x^{-3}. **65.** y^{n+2}. **66.** z^{n+1}.

67. $u^{\frac{3}{4}}$. **68.** $6y^{\frac{2}{5}}$. **69.** $a^{-\frac{3}{7}}$.

70. p^{-1}. **71.** $16a^{\frac{3}{5}}x^{\frac{2}{3}}$. **72.** -7^{n-1}.

73. a^{-3n}. **74.** a^x. **75.** $-5^{n+1}a^{-3}x^{2m}$.

76. $6x^{-1}y^{-\frac{1}{2}}z^{-n}$. **77.** $51x^{-\frac{2}{3}}y^n$. **78.** $13^{-a}x^{n+1}y^{\frac{2}{5}}$.

79. $-(-\frac{1}{4}x^{n-3}y^{\frac{1}{3}}z^{2p})$. **80.** $-6a^{-2}b^{-n-1}c^{-\frac{2}{3}}$.

Two factors of

81. $12xy^2$, if one is $3x^{\frac{1}{2}}$. **82.** $8a^{\frac{2}{5}}c^5$, if one is $4ac$.

83. $15p^2q$, if one is $3(pq)^{\frac{1}{2}}$. **84.** $5a^mb^{\frac{3}{5}}$, if one is $5ab$.

85. $7a^{-\frac{1}{2}}c^{n-1}$, if one is $7^{\frac{1}{2}}ac^{-2}$.

Two equal factors of

86. $4x^2y^3$. **87.** $5ax^n$. **88.** $6x^{\frac{1}{3}}y^{-1}$.

89. $9a^{3n}c^{\frac{1}{2}}x^{-2}$. **90.** $7x^{3m}y^{\frac{1}{3}}z^{-5n}$. **91.** $\frac{1}{4}a^{-n}b^{\frac{1}{2}}c$.

92. $5^{n}x^{a+2}y^{-\frac{1}{2}}$.

What are the factors of x?

Ans.: $x^{\frac{1}{2}} \cdot x^{\frac{1}{2}}$; $\sqrt{x} \cdot \sqrt{x}$; $x^{\frac{2}{3}} \cdot x^{\frac{1}{3}}$; $x^{\frac{3}{4}} \cdot x^{\frac{1}{4}}$; $x^{\frac{1}{2}} \cdot x^{-\frac{1}{2}}$; $x^{n} \cdot x^{1-n}$; $x^{a-1} \cdot x^{2-a}$; $x^{\frac{1}{3}} \cdot x^{\frac{1}{3}} \cdot x^{\frac{1}{3}}$; $x^{\frac{1}{2}} \cdot x^{\frac{1}{4}} \cdot x^{\frac{1}{8}} \cdot x^{\frac{1}{16}}$; etc., etc. That is, any powers of x the sum of whose exponents equals unity.

CASE II. — WHEN ONE OF THE FACTORS IS A MONOMIAL.

What are the factors of

93. $2x - 4$. **94.** $5 + 10a$. **95.** $15ax + 20by$.

96. $7x + 14y + 21z$. **97.** $6 - 12k + 18l$.

98. $16x + 4a - 8ax$. **99.** $12x^{3} + 15x^{2} - 18y^{5}$.

100. $x^{3} + x$. **101.** $m^{2}n + mn^{2}$. **102.** $ab - bc$.

103. $4k^{3} - 3k^{2}$. **104.** $x^{2} + 2xy$. **105.** $2xy - y^{2}$.

106. $c^{3}d + d$. **107.** $a^{2}c^{2} - a^{3}c^{3}$. **108.** $a^{3}b - ab^{3}$.

109. $5q^{2} - q$. **110.** $15x^{2} + 35$. **111.** $12a^{3} - 30a^{4}x$.

112. $8a^{2}c + 4ac^{2}$. **113.** $21a^{3}b^{2}c - 63ab^{2}c^{3}$.

114. $14ac - 7d$. **115.** $-15a^{2}x^{4} + 10a^{4}x^{2}$.

116. $56x^{3}y^{3} - 33a^{4}z^{3}$. **117.** $15a^{5}m^{2}yz^{3} - 12b^{3}m^{3}n^{3}y^{3}$.

118. $216a^{3}b^{4}c^{4}d^{5}x^{3}y - 96a^{2}c^{5}d^{5}x^{3}$.

119. $185a^{3}y^{3}z^{3} - 37a^{3}bcd^{3}y^{3}z$. **120.** $12a^{8}c^{3}x^{4}yz^{3} - 5a^{7}c^{3}x^{4}yz^{3}$.

121. $25a^{4}c^{3}x^{3}y^{4}z^{5} + 25a^{3}c^{3}x^{4}y^{4}z^{5}$.

122. $216m^{5}n^{3}p^{3}q^{8} + 36m^{2}n^{3}p^{3}qs^{9}$.

123. $56a^{5}c^{3}d^{3}m^{7}n^{5}x^{3}z + 224bd^{5}mxz^{4}$.

124. $2xy + 2y^{2} + 2yz$. **125.** $25a^{2} + 10a - 5$.

126. $ax - bx + cx$.

127. $3a + 12 - 9c$.

128. $25 - 20p^2 - 15p^2q^2$.

129. $n - 3n^2 - 5n^3$.

130. $7abc + 8bcd + 9cde$.

131. $a^2x^2 + a^2y - a^2$.

132. $cd + cx + cy$.

133. $3x^2y - 3xy^2 - y^3$.

134. $x^3 + 2x^2y - xy^2$.

135. $a^6 + a^4x - a^2x^2$.

136. $x^4 - 5x^3y + 20x^2z^2$.

137. $8a^3 + 6a^4 + 4a^5$.

138. $5h^3 - 30h^2 + 10h$.

139. $x^2yz - xy^2z + xyz^2$.

140. $a^4c^2 - a^3c^3 + a^2c^4$.

141. $8pq^3 - 12p^2q^2 + 4p^3q$.

142. $3x^2y^3 - 6x^2y^5 + 3xy^4$.

143. $3m^5n - 21m^4n^2 + 27m^3n^4$.

144. $3abc - 6bcd - 3bcx$.

145. $42x^2y - 14xy + 56x^3y^2$.

146. $12m^3n^2 + 24m^2n^3 - 60m^3n^3x$.

147. $3a^4 - 3a^3b + 6a^2b^2$.

148. $a^3b^2y - c^4d^3z + e^6f^7u$.

149. $70p^4q^5 + 126p^2q^4 - 112p^5q^6$.

150. $14a^4b^4x^2y - 6ax^5y^5 + 10ax^2y$.

151. $3a^4b^3c^2 - 6a^2b^4c^3 + 12a^3b^3c^4$.

152. $54m^3n^6 - 108m^6n^8 + 243m^8n^9$.

153. $54x^3y^6 - 26a^4c^3 + 27x^3z^4$.

154. $45a^7k^{10} - 90a^5k^7 + 360a^4k^8$.

155. $32x^3y^6 + 96x^6y^6 + 128x^8y^9$.

156. $26a^4c^3x - 39a^4c^3x^2 + 52a^3c^3x^2$.

157. $8ab^3c + 5ac^3d - 5aa^2e + 8ae^2f$.

158. $8c^4d + 3c^3d - 18c^3d^3 + 33b^3c^5d$.

159. $4k^3 - 16k^3l + 24k^3l^2 + 12k^3l^3$.

160. $ac^2 + 3a^2c^3q + 5a^3c^3q^2 - a^4c^4q^3$.

161. $3a^3x^3 + 6a^4x^2 - 9a^2x^4 + 12a^3x^3$.

162. $4r^4s - 12r^4s^2 - 16r^4s^3 + 8r^4s^4$.

163. $2m^3n + 6m^2n^2 + 6mn^3 + 2n^4$.

164. $5pq^3 - 15pq^2x + 15pqx^2 - 5px^3$.

165. $a - 3a^2 + 6a^3 + 7a^4 - 8a^5 + 3a^6 - a^7$.

166. $13ab^3 + 78ab^2 + 156ab - 104a$.

167. $22a^2c^3m^2p^3x - 33a^2cm^2p^3x^2 - 44ac^3m^2p^2x^2$.

Factor examples 168–177 in such a way that the binomial factor shall contain no fraction.

168. $2ac - \frac{1}{2}c^2$.

169. $3c^3d^3e^4 + \frac{1}{3}c^3d^2e$.

170. $10m^2n^2p^2 - \frac{1}{5}mn^2p^2$.

171. $\frac{1}{4}x^2y^2 - 3x^2y^4$.

172. $3x^2y^2 + \frac{2}{4}a^2x^2$.

173. $\frac{4}{5}a^2c - 2ab^2$.

174. $\frac{1}{8}pq - \frac{3}{4}qr$.

175. $\frac{2}{5}ax - 3ay$.

176. $4ac + \frac{2}{7}ax$.

177. $\frac{1}{2}klmn + \frac{2}{3}klst$.

Case III. — Polynomials Whose Terms, When Grouped, Have a Common Factor.

178. $6(c + x) - 12$.

179. $18(a^2 + b^2) - 9$.

180. $(a + b)x + (a + b)y$.

181. $(m + n)k + (m + n)l$.

182. $a(r - s) + b(r - s)$.

183. $(e + f)m - (e + f)n$.

184. $a(g + h) - c(g + h)$.

185. $6(u - v) - a(u - v)$.

186. $a(x + y)b + c(x + y)d$.

187. $c^2(c + y) + y^2(c + y)$.

188. $g(k + l) - h(k + l)$.

189. $3x^4(m - 2n) + 5(m - 2n)$.

190. $3x(x-1)-3(x-1)$. **191.** $2(a-2b)^2+6(a-2b)$.

192. $m(a-1)-a+1$. **193.** $x(a-c)-(c-a)$.

194. $3a(c-d)+d-c$.

195. $3cd^2(d-2c)^2+12c^3d(d-2c)$.

196. x^3+x^2+x+1. **197.** y^3-y^2+y-1.

198. $1-q+q^2-q^3$. **199.** $1-n+3n^2-3n^3$.

200. $1+x-x^2-x^3$. **201.** $ap+bp+aq+bq$.

202. $x+y+ax+ay$. **203.** $ay-by+az-bz$.

204. $c-cd+4-4d$. **205.** $5mn-hn-5mp-hp$.

206. $x^3-xy+x^2y^2-y^3$. **207.** $ce^2-cf^2+de^2-df^2$.

208. $amn+bcmn-aq-bcq$. **209.** $c^3a^2+d^2a^2-ed^2-ec^2$.

210. $am^2p+aqp^2-aqm^5-ap^3$.

211. $x^3y^3+x^3yzu^2+xy^3z^2u+z^3u^3$.

212. $2+3x-8x^2-12x^3$. **213.** $5f^3+10f^2+f+2$.

214. $10ac-2a-15bc+3b$. **215.** $3xy+3zw-xw-9zy$.

216. $6ax+10bx-18ay-30by$.

217. $20mn-35pn-8mq+14pq$.

218. $24xy-32zy-6xu+8zu$.

219. $45x^3+20x^2y^2-63xy+28y^3$.

220. $33mnpq-21p^3q+22mnx-14px$.

221. $ey^2-fy^3+ey-gy^3-fy-gy$.

222. $ax-ay+bx-by-cy+cx$.

223. $2xy+3zy-5yu+8xw+12zw-20uw$.

CASE IV. — WHEN THE FACTORS ARE POLYNOMIALS.

A.

Type-forms : $a^2 + 2ab + b^2 = (a + b)^2$,

$a^2 - 2ab + b^2 = (a - b)^2$.

$a^2 - b^2 = (a + b)(a - b)$.

What are the factors of

224. $x^2 + 2x + 1$. 225. $a^2 - 4a + 4$. 226. $m^2 + 8m + 16$.

227. $d^2 - 10d + 25$. 228. $p^2 - 36$. 229. $1 + 2y + y^2$.

230. $1 - 6x + 9x^2$. 231. $49 - a^4$. 232. $64 - 16k + k^2$.

233. $16 - 81$. 234. $g^2 - 2gh + h^2$. 235. $9m^2 - 36n^2$.

236. $7^2 - 5^2$. 237. $11^2 - 49$. 238. $a^2 + x^2 - 2ax$.

239. $4x^2 + 4x + 1$. 240. $121 - 9w^2$. 241. $49 + 70 + 25$.

242. $25 - 1^2$. 243. $x^2 + 49 - 14x$.

244. $a^4 + 2a^2b^2 + b^4$. 245. $c^4 - 2c^2 + c^2$.

246. $x^4 + 2x^2y^2 + y^6$. 247. $a^{10}x^2 - 9y^4$.

248. $25k^4 - 4l^2m^2$. 249. $a^{10} - 64a^5 + 1024$.

250. $x^2 + 2xy - y^2$. 251. $64b^2 - 32bx + 4x^2$.

252. $64d^2 + 16x^2 - 64dx$. 253. $16x^2y^2 - 8xy^2z + y^2z^2$.

254. $49a^2 - 112ab + 64b^2$. 255. $36r^2 - 84rs + 49s^2$.

256. $36c^2 - 54cd + 49d^2$. 257. $25x^2 + 15xz + 9z^2$.

258. $25a^2 + 9c^2 + 30ac$.

259. $9a^4b^4c^4 - 36a^2b^4c^4d^2 + 36b^4c^4d^4$.

260. $25a^2b^4 + 70a^2b^2cx + 49a^2c^2x^2$.

261. $25x^{16} - 121b^{12}$. 262. $9a^{48} - 4z^{12}q^{16}$.

263. $x^{10}y^{10} - x^{16}y^{16}$. 264. $25a^{18} - 49b^{30}c^{30}$.

265. $-80x+25+64x^2$. 266. $121x^4+286x^2y+169y^2$.

267. $21a^2b^2cx+49a^2c^2x^2+9a^2b^4$.

268. $625x^4-256c^6$. 269. $1-100x^6y^2z^{10}$.

270. $(575)^2-(425)^2$. 271. $(750)^2-(250)^2$.

272. $(101)^2-(99)^2$. 273. $7^2a^4+5^2z^6-70a^2z^3$.

274. $289x^2y^2z^2+9y^2z^2d^2-102xy^2z^2d$.

275. $361a^2b^2c^2-76lmnabc+4l^2m^2n^2$.

276. $x^{\frac{1}{2}}-9$. 277. $x^4+x^2+\frac{1}{4}$. 278. $9a^{\frac{2}{3}}+6a^{\frac{1}{3}}+1$.

279. $4a^{-2}-4a^{-1}+1$. 280. $.0001x^4-1$.

281. $d^2-2+\frac{1}{d^2}$. 282. $49x^{\frac{2}{3}}-81y^{-2}$.

283. $c^2+\frac{1}{c^2}+2$. 284. $\frac{1}{a^2}+\frac{2}{a}+1$.

285. $\frac{a^2}{c^2}-2+\frac{c^2}{a^2}$. 286. $\frac{m^2}{n^2}-4+\frac{4n^2}{m^2}$.

287. $4x^{\frac{2}{3}}-49x^{\frac{4}{5}}$. 288. $a-b$.

289. $16x^4y-9z^2k^2$. 290. $x+2\sqrt{xy}+y$.

291. $a-b-2a^{\frac{1}{2}}b^{\frac{1}{2}}$. 292. $x^{\frac{2}{3}}+y^{\frac{4}{5}}-2x^{\frac{1}{3}}y^{\frac{2}{5}}$.

293. $\frac{a^2}{b^2}+2+\frac{b^2}{a^2}$. 294. $\frac{x^4}{y^4}-2+\frac{y^4}{x^4}$.

295. $a^2+2a\sqrt{x}+x$. 296. $x-2b\sqrt{x}+b^2$.

297. $x-2\sqrt{x\cdot y}+y$. 298. $a^2b-2ax\sqrt{b}+x^2$.

299. $a+2\sqrt{a}+1$. 300. $x^{\frac{2}{3}}-y^{\frac{2}{3}}$.

301. $4a^{-6}-9x^{-4}$. 302. $r-s$.

303. $x^2 + 2x^{\frac{3}{2}} + x$.

304. $\frac{1}{4}x^6 + \frac{1}{3}x^4 - \frac{2}{3}x^5$.

305. $\frac{9}{25}a^4 + \frac{1}{4}b^6 + \frac{3}{5}a^2b^3$.

306. $\frac{1}{16}a^4 + 16x^2 + 2a^2x^2$.

307. $16x^{-1} + 8x^{-\frac{1}{2}} + x^{-2}$.

308. $\frac{1}{9}a^6c^4 - \frac{2}{3}a^5c^2 + \frac{1}{9}a^4$.

309. $\frac{1}{4}m^4x - \frac{1}{2}m^2n^2x^{\frac{1}{2}} + \frac{1}{4}n^4$.

310. $\frac{a^2}{m^2} - \frac{2ac}{mn} + \frac{c^2}{n^2}$.

311. $\frac{x^2}{a^2} + 4\frac{xy}{ac} + 4\frac{y^2}{c^2}$.

312. $9a^{\frac{2}{3}} - 6 + a^{-\frac{2}{3}}$.

313. $16x^{\frac{2}{3}} + 24 + 9x^{-\frac{2}{3}}$.

314. $\frac{1}{9}a^{\frac{2}{3}} - \frac{2}{3} + a^{-\frac{2}{3}}$.

315. $b^{\frac{1}{2}} - 4 + 4b^{-\frac{1}{2}}$.

316. $(a + b)^2 - c^2$.

317. $c^2 - (a + b)^2$.

318. $g^2 - (e - f)^2$.

319. $(m + n)^2 - 4$.

320. $4 - (m + n)^2$.

321. $(r + 2s)^2 - t^2$.

322. $t^2 - (r + 2s)^2$.

323. $(2p + 3q)^2 - 4x^2$.

324. $4x^2 - (2p + 3q)^2$.

325. $4k^4 - (c + 5d)^2$.

326. $(a^2 + 3)^2 - 36c^2$.

327. $36c^2 - (a^2 + 3)^2$.

328. $9p^2 - (3g - 5h)^2$.

329. $(7m - n^2)^2 - q^2$.

330. $q^2 - (7m - n^2)^2$.

331. $1 - (a + b)^2$.

332. $(a + x)^2 + 2(a + x) + 1$.

333. $(x - 4)^2 - 4(x - 4) + 4$.

334. $(2x - 4)^2 - 4(2x - 4) + 4$.

335. $(x^2 - 10x)^2 - 50(x^2 - 10x) + 625$.

336. $(x^2 - y)^2 - 10(x^2 - y)y + 25y^2$.

337. $16(c + d)^2 + 8(c + d) + 1$.

338. $4(x - y)^2 - 12(x - y) + 9$.

339. $(m + n)^2 + 4p(m + n) + 4p^2$.

340. $4a^2c^2 + 4(e+f)ac + (e+f)^2$.

341. $(a+b)^2 - (c+d)^2$. 342. $(e+f)^2 - (g-h)^2$.

343. $(k-l)^2 + (m+n)^2$. 344. $(p-q)^2 - (x-y)^2$.

345. $(a+3)^2 - (b-4)^2$. 346. $(a-2x)^2 - (x-4)^2$.

347. $(2x+y)^2 - (3z+w)^2$. 348. $(2-4x)^2 - (3a+4c)^2$.

349. $4(x+y)^2 - (x-y)^2$. 350. $9(c+d)^2 - 4(c-d)^2$.

351. $(8x-5)^2 - 3x+7)^2$. 352. $(a+x)^4 - (a-x)^4$.

353. $(3x+2y)^2 - (2x-3y)^2$. 354. $x^2+2xy+y^2-(a+b)^2$.

355. $(c+d)^2 + a^2 + 2ab + b^2$.

356. $m^2 - 2mn + n^2 - (p-q)^2$.

357. $(a^2c^2 - c^2)^2 - (a^2c^2 + c^2)^2$.

Resolve into factors and simplify.

358. $(a+b)^2 - a^2$. 359. $e^2 - (f-g)^2$.

360. $(x^2+y^2)^2 - 4x^2y^2$. 361. $(c+d)^2 - (c-d)^2$.

362. $(3x+1)^2 - (2x-1)^2$. 363. $(a+3c)^2 - 4c^2$.

364. $9a^2 - (3a-5b)^2$. 365. $16x^2 - (3x+1)^2$.

366. $(5a+2c)^2 - (3a-c)^2$. 367. $(7a+3)^2 - (5a-4)^2$.

368. $(a-c-3)^2 - (a-3)^2$. 369. $(12a+x)^2 - (11a-x)^2$.

370. $(x-2y+3z)^2 - (x-z)^2$.

371. $x^2 + 2x + 1 - y^2$. 372. $c^2 - 2c + 1 - d^2$.

373. $1 - a^2 - 2ab - b^2$. 374. $x^2 - 2xy + y^2 - z^2$.

375. $m^2 + 2mn + n^2 - 9$. 376. $p^2 - 2pq + q^2 - 36$.

377. $25 - a^2 + 2ax - x^2$. 378. $1 - 2a + a^2 - 16x^2y^2$.

379. $a^2 + 4ac + 4c^2 - 4a^2c^2$. 380. $m^2 - q^2 + n^2 - 2mn$.

381. $2gh - k^2 + g^2 + h^2$.

382. $a^2 - 2h - h^2 - 1$.

383. $4 - a^2 - x^2 - 2ax$.

384. $k^2 + 8l - l^2 - 16$.

385. $a^2 - b^2 + 2bc - c^2$.

386. $4m^2 - p^2 - q^2 + 2pq$.

387. $9e^2 - 9g^2 + 12fg - 4f^2$.

388. $-14ac - 49c^2 - a^2 + 16a^2c^2$.

389. $4ab - 4a^2 + 1 - b^2$.

390. $2cd - c^2 - d^2 + x^2$.

391. $16n^2 - 8mn - 49k^2 + m^2$.

392. $x^2y^2 - 2xyz + z^2 - 25x^2y^2$.

393. $a^2 + 2a + 1 - x^2 - 2xy - y^2$.

394. $c^2 + 2cd + d^2 - p^2 + 2pq - q^2$.

395. $e^2 + 4ef + 4f^2 - m^2 + 2mn + n^2$.

396. $a^2 - b^2 + 1 - 2a - 4bc - 4c^2$.

397. $a^2 - y^2 + x^2 - b^2 + 2ax + 2by$.

398. $2ah - g^2 + a^2 - 2gk - k^2 + h^2$.

399. $m^2 - n^2 + 2n - 1 - 2mp - p^2$.

400. $4x^2 + 4xz - 4y^2 - z^2 - 4x + 1$.

401. $4c^2 - 12cf - d^2 - h^2 - 2dh + 9f^2$.

402. $4x^2 - 9z^2 - 4 + 4xy + y^2 + 12z$.

403. $-c^2 + d^2 + 16x^2 - 36 - 12c - 8dx$.

404. $x^2 + 6yz - 9y^2z^2 - 10xy - 1 + 25y^2$.

405. $p^4 - 25q^6 + 8p^2q^2 - 9 + 30q^3 + 16q^4$.

B.

Factoring is rendered much easier by becoming familiar with the following type-forms :

$a^2 + b^2$ is prime,

$a^2 - b^2 = (a + b)(a - b)$,

$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$,

$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$,

$a^4 + b^4$ is prime,

$a^4 - b^4 = (a^2 + b^2)(a + b)(a - b)$.

$a^5 + b^5 = (a + b)(a^4 - a^3b + a^2b^2 - ab^3 + b^4)$,

$a^5 - b^5 = (a - b)(a^4 + a^3b + a^2b^2 + ab^3 + b^4)$,

$a^6 + b^6 = (a^2 + b^2)(a^4 - a^2b^2 + b^4)$,

$a^6 - b^6 = (a + b)(a^2 - ab + b^2)(a - b)(a^2 + ab + b^2)$,

$a^7 + b^7 = (a+b)(a^6 - a^5b + a^4b^2 - a^3b^3 + a^2b^4 - ab^5 + b^6)$,

$a^7 - b^7 = (a-b)(a^6 + a^5b + a^4b^2 + a^3b^3 + a^2b^4 + ab^5 + b^6)$,

$a^8 + b^8$ is prime,

(NOTE: The sum of the even powers can not be factored when the index is some power of 2, such as 2, 4, 8, 16, etc.)

$a^8 - b^8 = (a^4 + b^4)(a^2 + b^2)(a + b)(a - b)$,

$a^2 + 2ab + b^2 = (a + b)^2$,

$a^2 - 2ab + b^2 = (a - b)^2$,

$a^4 + 2a^2b^2 + b^4 = (a^2 + b^2)^2$,

$a^4 - 2a^2b^2 + b^4 = (a^2 - b^2)^2 = (a + b)^2(a - b)^2$,

$a^6 + 2a^3b^3 + b^6 = (a^3 + b^3)^2 = (a + b)^2(a^2 - ab + b^2)^2$,

$a^6 - 2a^3b^3 + b^6 = (a^3 - b^3)^2 = (a - b)^2(a^2 + ab + b^2)^2$.

406. $x^3 + 1$. 407. $y^3 - 1$. 408. $1 + c^3$.

409. $1 - x^3$. 410. $a^3 + 8$. 411. $8 - z^3$.

412. $a^3 - 27$. 413. $p^3 + 64$. 414. $64 - q^3$.

415. $125 + w^3$. 416. $216 - d^3$. 417. $h^3 - 343$.

418. $k^3 + 512$. 419. $t^3 - 512$. 420. $x^3 - 1000$.

421. $729 - u^3$. 422. $x^3 + y^3$. 423. $a^3 - x^3$.

424. $q^3 - n^3$ 425. $a^6b^3 - 1$. 426. $s^6 - t^6$.

427. $k^6 + l^6$. 428. $8a^3 - 1$. 429. $8e^3 + f^3$.

430. $1 + 27y^3$. 431. $1 - 64x^3$. 432. $8a^3 + 27x^3$.

433. $27c^3 - 64d^3$. 434. $m^3n^3 - 125t^3$. 435. $c^{12} + 512$.

436. $343a^3 + 64b^6$. 437. $64 - h^9$. 438. $8a^3b^3 - 216c^3$.

439. $512a^3 - 729x^3$. 440. $343a^3b^3 = 512x^3y^3$.

441. $64x^{12}y^3 - 27g^6h^9$. 442. $216a^6x^6 + 343b^9y^9$.

443. $512c^3d^{15} - 729s^{21}t^6$. 444. $1000a^3b^6c^9 - 729x^{12}y^{15}z^{18}$.

445. $x^5 - 1$. 446. $a^5 + 1$. 447. $c^7 + 1$.

448. $1 + a^7$. 449. $a^5 - 32$. 450. $x^5 + 243$.

451. $a^7 - x^7$. 452. $a^5c^5 + x^5$. 453. $32 + k^5$.

454. $243z^5 - 1$. 455. $32x^5 + 243y^5$.

456. $a^6 - 1$. 457. $x^6 - 64$. 458. $n^{10} + p^{10}$.

459. $x^{12} + y^{12}$. 460. $64a^6 + x^6$. 461. $729 + c^6$.

462. $x^8 + y^8$. 463. $16x^8 - a^4c^{12}$. 464. $1 - .0001a^4$.

C.

Type-forms.

$$R. \quad \begin{cases} x^2 + 10x + 21 = (x+7)(x+3), \\ x^2 - 10x + 21 = (x-7)(x-3), \\ x^2 + 4x - 21 = (x+7)(x-3), \\ x^2 - 4x - 21 = (x-7)(x+3). \end{cases}$$

$$S. \quad \begin{cases} 25x^2 + 15x + 2 = (5x+2)(5x+1), \\ 25x^2 - 15x + 2 = (5x-2)(5x-1), \\ 25x^2 + 5x - 2 = (5x+2)(5x-1), \\ 25x^2 - 5x - 2 = (5x-2)(5x+1). \end{cases}$$

$$T. \quad \{ 54x^2 - 15x - 56 = (9x+8)(6x-7).$$

NOTE: Such trinomials are usually factored without much difficulty by inspection, the right factors of the first and last terms being found after one or two trials. But sometimes this is not so easy, especially if large numbers be used. Try to factor $16x^2 - 15x - 196$ by inspection, for instance. When examples like type-forms "S" or "T" present any difficulty, they can always be changed to the form "R" and thus solved with comparative ease. For example: $16x^2 + 49x - 60$. First multiply through by the coefficient of the first term, expressing the result as follows: $(16x)^2 + 49 \cdot 16x - 960$. Next substitute a for $16x$ and we have $a^2 + 49a - 960$, which is readily resolved into the factors $(a+64)(a-15)$. Now these factors have been obtained only after subjecting the original quantity to two processes, (1) multiplying it by 16, and (2) substituting a for $16x$. To find the factors of the original quantity we must reverse these steps. Restoring $16x$ in place of a we have $(16x+64)(16x-15)$, and dividing by 16 we have the two factors of

$$16x^2 + 49x - 60 = (x+4)(16x-15).$$

465. $x^2 + 3x + 2$.

466. $x^2 - 4x + 3$.

467. $a^2 + 5x + 6$.

468. $c^2 - 5c + 4$.

469. $x^2 + x - 6$.

470. $x^2 + 7x + 12$.

471. $x^2 + x - 12$.

472. $x^2 - 7x + 12$.

473. $n^2 + n - 30$.

474. $p^4 - p^2 - 20$.

475. $a^2 - 9a - 36$.

476. $c^2 - 5c - 36$.

477. $q^2 - 16q - 36$.

478. $m^2 - 35m - 36$.

479. $x^2 + 15x + 36$.

480. $c^2 - 13c + 36$.

481. $f^2 - 37f + 36$.

482. $1 + 7a + 12a^2$.

483. $1 + a - 12a^2$.

484. $1 - 5a - 6a^2$.

485. $y^2 - 7y - 18$.

486. $z^2 + 11z - 12$.

487. $d^6 - 7d^3 + 12$.

488. $e^8 + 3e^4 - 4$.

489. $h^{10} - 9h^5 - 10$.

490. $n^2 + 7n - 60$.

491. $t^2 - 11t - 60$.

492. $u^2 - 2u - 60$.

493. $c^2 - 28c - 60$.

494. $g^2 + 59g - 60$.

495. $1 - 9a + 8a^2$.

496. $1 + 7c + 12c^2$.

497. $a^2 + ax - 2x^2$.

498. $m^2 + 4am + 3a^2$.

499. $x^2 + 13cx + 36c^2$.

500. $z^2 + 29qz + 100q^2$.

501. $a^2c^2 - 21ac + 20$.

502. $e^8f^4 + 7e^4f^2 + 12$.

503. $x^2y^2z^2 - 19xyz + 48$.

504. $n^2p^2q^2 - 13npq + 22$.

505. $a^2x^2 + 18axy + 65y^2$.

506. $r^2s^2 + 23rsz + 90z^2$.

507. $x^2 - (a + c)x + ac$.

508. $x^4y^4 + 20x^2y^2z + 51z^2$.

509. $y^2z^2 - 28abyz + 75a^2b^2$.

510. $x^4 + 37x^2y^2 - 120y^4$.

511. $1 - 8x^3 - 105x^6$.

512. $204 - 5a - a^2$.

513. $c^2 - 21c + 104$.

514. $d^2 - 30d + 225$.

515. $a^2 - 49ab + 600b^2$.

516. $m^4n^4 + 37m^2n^2 + 300$.

517. $20 + 9a + a^2$.

518. $72 + 27q^3 + q^6$.

519. $44 - 15f^2 + f^4$.

520. $96 + 22x^2 + x^4$.

521. $120 - 26k + k^2$.

522. $192 - 32n^3 - n^6$.

523. $120 - 7cd - c^2d^2$.

524. $a^2 + (b + c)d + d^2$.

525. $y^2 + (a - b)y - ab$.

526. $x^4 + (a^2 + c^2)x^2 + a^2c^2$.

527. $z^2 - (a + b)z + ab$.

528. $x^2 + (3a - 2c)x - 6ac$.

529. $(a - b)^2 + 7(a - b) + 10$.

530. $(a + c)^2 + 2(a + c) - 15$.

531. $(x+y)^2+23(x+y)+60$.

532. $(p-q)^2-24(p-q)-112$.

533. $2x^2+5x+2$. 534. $3c^2+4c+1$. 535. $3m^2+5m+2$.

536. $4x^2-5x+1$. 537. $4a^2+a-3$. 538. $2y^2+3y-2$.

539. $2a^2-a-1$. 540. $2n^2+7n+6$. 541. $3p^2-10p+3$.

542. $4s^2+5s-6$. 543. $2-3c-2c^2$. 544. $3a^2-a-10$.

545. $8+18x-5x^2$. 546. $3+11d-4d^2$.

547. $10-5m-15m^2$. 548. $20-9z-20z^2$.

549. $40+6a-27a^2$. 550. $3x^2-17xy+10y^2$.

551. $12x^2-23xy+10y^2$. 552. $12x^2-17xy+6y^2$.

553. $10w^2+7wz-33z^2$. 554. $6x^2+35xz-6z^2$.

555. $21c^2+26cx-15x^2$. 556. $6p^2-5pq-6q^2$.

557. $-10x^4+7x^2y^2+12y^4$. 558. $18a^2x^2-71axp-45p^2$.

559. $ac^2+(a-b)c-b$. 560. $mnx^2-(m^2-n^2)x-mn$.

561. $10(x+y)^2+7q(x+y)-6q^2$.

562. $7(k-l)^2-37p(k-l)+10p^2$.

563. $6(a^2+c^2)^2-9(a^2+c^2)x^2-15x^4$.

564. $2(p^2-q^2)^2-4k(p^2-q^2)-6k^2$.

D. — Special Devices.

Type-forms.

$4a^4+b^4=4a^4+4a^2b^2+b^4-4a^2b^2=(2a^2+b^2)^2-(2ab)^2$.

$x^4+x^2y^2+y^4=x^4+2x^2y^2+y^4-x^2y^2=(x^2+y^2)^2-(xy)^2$.

565. $1+4a^4$. 566. $64m^4+1$. 567. x^4+64.

568. $81+4c^4$. 569. m^4+4n^4. 570. a^4+4x^4.

571. $x^4 + 64y^4$. 572. $64p^4 + 81$. 573. $4q^4 + 625$.

574. $1024 + k^8$. 575. $x^4 + 324$. 576. $4a^4 + 256$.

577. $4c^4 + 1296x^4$. 578. $2500x^4 + 1$. 579. $x^2 + y^2$.

580. $a^2 + n^4$. 581. $9x^4 + 4z^2$. 582. $a^4c^8 + 64e^8f^4$.

583. $x^4 + x^2 + 1$. 584. $x^4 - 6x^2 + 1$.

585. $a^4 + b^4 - 7a^2b^2$. 586. $c^4 - 6c^2d^2 + d^4$.

587. $z^4 + w^4 - 18z^2w^2$. 588. $k^4 + 9k^2 + 81$.

589. $a^4 + 16a^2 + 256$. 590. $p^4 + q^4 - 11p^2q^2$.

591. $1 + 3s^2 + 4s^4$. 592. $1 + 9a^4c^4 + 2a^2c^2$.

593. $a^4x^4 + 13a^2x^4 + 49x^4$. 594. $81m^4 + 9m^2y^2 + y^4$.

595. $16h^4 - 28h^2k^2 + k^4$. 596. $4c^4 + 9y^4 - 24c^2y^2$.

597. $36a^4 + 144a^2 + 576$.

598. $25x^4y^4 + 5x^2y^2z^2w^2 + 9z^4w^4$.

599. $4c^4d^8 + 8c^2d^4e^4f^2 + 121e^8f^4$.

E. — The Factor Theorem.

§ I. The result obtained by subtracting any number or quantity from itself is zero. For example: $4 - 4 = 0$; $817 - 817 = 0$; $x - x = 0$; $acy - acy = 0$, etc., etc.

§ II. The result obtained by multiplying any number or quantity by zero is zero. For example: $5 \times 0 = 0$; $1728 \times 0 = 0$; $F \times 0 = 0$; $(c + d) \times 0 = 0$, etc., etc.

§ III. Now if we let F represent one of the factors of a quantity, it is obvious that, when the quantity is zero, the other factor must be 0, that is, that $F \times 0 = 0$.

But in § I we have seen that $a - a = 0$. Substituting this value of 0 in the first term of the equation $F \times 0 = 0$ we have $F \times (a - a) = 0$, or $(a - a)F = 0$.

We have to remember, then, that, *in order that a product may be zero, it is necessary and sufficient that one of the factors be zero.*

§ IV. This theorem affords a convenient method of solving equations and of factoring quantities not, otherwise, easily factorable.

Thus, the equation $x^2 - 3x + 2 = 0$ may be written $(x - 2)(x - 1) = 0$.

As one of these factors must equal 0, we put $x - 2 = 0$, whence $x = 2$; and $x - 1 = 0$, whence $x = 1$. The equation $x^2 + (a - b)x - ab = 0$ may be written $(x + a)(x - b) = 0$, whence $x + a = 0$ and $x = -a$; and $x - b = 0$ and $x = b$.

§ V. It will be found by multiplying together several polynomial factors, arranged according to a common letter or letters, that the *first* term of the product is *the product of the first terms of the factors*, and that the *last* term of the product is *the product of the last terms of the factors.*

It follows that a polynomial which contains one or more binomial factors may be factored by resolving its first and last terms into factors, and then determining which of the binomials that can be formed by uniting these factors (two and two), with the sign + or − are factors of the given polynomials.

A simple method of testing which of the factors is contained, without performing the division, depends upon the converse of "The Factor Theorem," *i. e., if one factor of an expression is equal to zero, then the whole expression must be.*

Suppose we wish to find the factors of $x^3 - 4x^2 - 9x + 36$. The possible factors of 36 are $+1, -1, +2, -2, +3, -3, +4, -4, +6, -6, +9, -9, +18, -18, +36, -36$. Thus there are sixteen possible binomial factors, at least. To test all these by division would be too tedious.

But to decide whether $x - 1$ is a factor, it is only necessary to suppose that $x = 1$: in that case $x - 1 = 0$. Now as $x^3 - 4x^2 - 9x + 36 = 0$, if we substitute 1 for x, the result should equal 0. But $1 - 4 - 9 + 36 = 24$, and therefore $x - 1$ is not a factor.

In the same way $x + 1$ is not, because, if $x = -1$, $x^3 - 4x^2 - 9x + 36$ becomes $-1 + 4 + 9 + 36 = 48$.

$x = +2$ gives $8 - 16 - 18 + 36 = 10$,

$x = -2$ gives $-8 - 16 + 18 + 36 = 30$,

$x = 3$ gives $27 - 36 - 27 + 36 = 0$.

Therefore $x - 3$ is a factor.

$x = -3$ gives $-27 - 36 + 27 + 36 = 0$.

Therefore $x + 3$ is a factor.

$x = 4$ gives $64 - 64 - 36 + 36 = 0$.

Therefore $x - 4$ is a factor.

As each of these factors is prime and their product is the given quantity, $x^3 - 4x^2 - 9x + 36$, there are no other possible factors.

(NOTE: As a good test of the value of "the factor theorem" let the student, later, try to solve in any other way the following simple example in quadratics: $x^2 + y = 7 : x + y^2 = 11$.)

§ VI. The use of "synthetic division" sometimes shortens the process, especially when there are four or more binomial factors.

Resolve into factors

$$x^4 + 2x^3 - 16x^2 - 2x + 15.$$

Try first $x - 1$.

$$\begin{array}{r|l} 1 & 1 + 2 - 16 - 2 + 15 \\ +1 & \quad 1 + 3 - 13 - 15 \\ \hline & 1 + 3 - 13 - 15 = x^3 + 3x^2 - 13x - 15 = \text{first quotient.} \end{array}$$

Try second $x + 3$.

$$\begin{array}{c|l} 1 & 1 + 3 - 13 - 15 \\ +3 & \quad 3 + 18 + 15 \\ \hline & 1 + 6 + 5 = x^2 + 6x + 5 = \text{second quotient.} \end{array}$$

By inspection $x^2 + 6x + 5 = (x + 1)(x + 5)$.

Therefore the factors of $x^4 + 2x^3 - 16x^2 - 2x + 15$ are $(x - 1)(x - 3)(x + 5)(x + 1)$.

Determine, without performing the division, whether either or both of the binomials given with each expression are factors or not.

600. $x^2 - 3x + 2$. $x - 1$, $x - 2$.

601. $2x^2 + x - 1$. $x - 1$, $x + 1$.

602. $x^2 - 5x + 2$. $x - 1$, $x - 2$.

603. $x^2 - 7x + 6$. $x + 3$, $x - 2$.

604. $x^2 + 6x + 6$. $x + 3$, $x + 2$.

605. $x^3 - 3x^2 + 4$. $x + 2$, $x - 2$.

606. $x^3 - 19x + 30$. $x - 2$, $x - 3$.

607. $x^3 - 43x + 42$. $x + 6$, $x - 7$.

608. $2x^3 - 5x + 39$. $x + 3$, $x - 3$.

609. $6x^3 + 7x + 13$. $x + 1$, $x + 13$.

610. $x^3 + x^2 - 4x - 4$. $x - 2$, $x + 2$.

611. $x^3 + 3x + x + 3$. $x + 1$, $x + 3$.

612. $x^3 - 8x^2 + 17x - 10$. $x - 2$, $x - 5$.

613. $32x^{10} - 33x^5 + 1$. $x + 1$, $x - 1$.

614. $x^6 + 4x^4 + x^2 - 6$. $x^3 - 1$, $x^3 + 2$.

615. $x^6 - 6x^4 - 19x^3 + 84$. $x^3 - 3$, $x^3 + 4$.

616. $2x^4 + 4ax^3 - 5a^2x^2 - 3a^3x + 2a^4$. $x + a$, $x - a$.

617. $75x^4 + 140x^3 - 223x^2 + 92x - 12$. $x - \frac{1}{5}$, $x - \frac{2}{5}$.

618. $x^n + a^n$. $x + a$, $x - a$.

619. $x^n - a^n$. $x + a$, $x - a$.

Resolve into factors

620. $x^3 - 2x^2 - x + 2$. 621. $x^3 + 6x^2 + 11x + 6$.

622. $x^3 - x^2 - 8x + 12$. 623. $x^3 - 2x^2 - 5x + 6$.

624. $x^3 - x^2 - 17x - 15$. 625. $x^3 + 3x^2 - 25x + 21$.

626. $x^3 + x^2 - 17x + 15$. 627. $x^3 - 19x - 30$.

628. $x^3 - 43x + 42$. 629. $x^3 + 3x^2 - 10x - 24$.

630. $x^3 - 13x - 12$. 631. $x^3 - 7x^2 - 8x - 12$.

632. $2x^3 - 9x^2 + 13x - 6$. 633. $2x^3 - 4x^2 - 2x + 4$.

634. $3x^3 + 6x^2 - 3x - 6$. 635. $x^3 - 18x - 35$.

636. $3x^4 - 7x^2 - 20$.

637. $5(x^2 + 3) - (x - 4)(x + 5) - 68$.

638. $7(x^2 - 1) - (x + 3)(x - 4) = x^2 + 41$.

639. $17 + x = (x + 3)^2 - 28$.

640. $3x^2 - 17ax - 28a^2$. 641. $6x^2 - 11x + 3$.

642. $3x^3 - 4x^2 - 6x + 4$. 643. $x^3 + 21x + 342$.

644. $x^4 - 3x^3 - 3x^2 + 12x - 4$.

645. $x^5 - 2x^4 - 6x^3 + 8x^2 + 5x - 6$.

646. $3x^2 + (5x + 2)^2 = 12x^3 - 8$.

647. $x^4 - 5a^2x^2 + 4a^4$. 648. $x^4 - x^3 - 4x - 16$.

649. $2x^3 + 31x^2 + 62x - 39$. 650. $3x^3 - 46x^2 + 9x + 2$.

HIGHEST COMMON FACTOR.

A. Monomials.

Find the highest common factor of

1. 52, 91.
2. 1835, 1995.
3. 2945, 3441.
4. 84, 126, 210.
5. 98, 140, 217.
6. 270, 198, 153.
7. apx^2, a^2px.
8. abd, acd, bcd.
9. $2x^4y$, $5x^3y^2$.
10. $9a^2x$, $4ax^2$.
11. $40p^3q^5$, $125p^5q^4$,
12. $\frac{3}{2}x^3y^2z^4$, $2y^3$, z^5.
13. $17xy^2z$, $34x^3yz$, $51xyz^2$.
14. a^2bpzw, b^3gzw. a^3bzs^2.
15. $mn^2p^3q^4$, $m^4np^2q^3$, $m^3n^2pq^2$.
16. $17a^5x^2y^3$, $51a^4xy^5$, $68a^6x^4y^3$.
17. $126e^3f^4$, $21e^2f^6g^3$, $147e^4f^5g$.
18. $117x^7y^3z^6$, $104x^3y^4z^5$, $156x^5y^4z^3$.
19. $24a^2b^2c^3$, $16a^4c^4$, $72a^3b^3c^3x$.
20. $30g^3h^4k^5$, $24g^3h^2k^3$, $72g^3h^3k^5$, $36g^4h^4k^4$.
21. $35m^2n^2p^3$, $30m^3n^2x^3y^3z$, $40m^3n^3x^3$, $21m^3n^3p^3x$, $45n^2x^3y^3$.
22. $105c^3d^3e^4f^2g^5$, $35c^3d^4e^5fg^4$, $28c^4d^3e^3f^5g^3$, $49c^5d^7e^3f^3g^2$.
23. $(a-b)^2(a+c)^3$, $(a-b)^3(a+c)^2$.
24. $39(p-q)^3(a+p)^2(b+p)$, $51(p-q)^2(p+b)(a+p)$.
25. $3a^2c^{\frac{2}{3}}$, $2acde$, $4a^{\frac{1}{3}}cd$.

B. Factorable by Inspection.

26. a^4-1, a^3-1.
27. x^5+x^3, x^4-1.
28. $8-x^3$, x^2-4.
29. $6(a+1)^2$, $8(a^3-1)$.
30. $13(a^6+c^6)$, $39(a^4-c^{-4})$.

31. $7(p^2-1)$, $28(p+1)^3$. 32. a^2+ap+p^2, $5(a^3-p^3)$.

33. k^2+3k-4, k^2-2k+1, k^2-1.

34. $s^2+9st+14t^2$, s^2-4t^2. 35. b^2+3b+2, b^2+6b+8.

36. $x^3y^3+xy^5$, $x^5-x^2y^4$.

37. $m^3n^3+2m^3n^2$, $m^3n^4-4m^4n^3$.

38. a^2-1, a^2+3a+1. 39. p^3q+3q^3, $p^6-9p^2q^4$.

40. $c^6d^2-4c^4d^4$, $c^6d^2-16c^3d^6$.

41. $a^2-15a+36$, $a^2-9a-36$.

42. $w^3-7wz+6z^2$, w^3-wz^2.

43. $27(x^2-y^2)$, $51(x+y)^2$.

44. $(a^2c-ac^2)^2$, $ac(a^2-c^2)^2$. 45. a^4-c^4, $a^3-a^2c-ac^2+c^3$.

46. q^3-1, q^3-1, q^3+q-2.

47. $1-y^2$, $1+y^3$, y^2+5y+4.

48. a^2-x^2, $(a+x)^2$, $a^2+3ax+2x^2$.

49. $a^2-4a-21$, $a^2-12a+35$, $a^2+5a-84$.

50. m^2-n^2, m^3-n^3, $m^2-7mn+6n^2$.

51. $s^2-6st+9t^2$, $s^2+st-12t^2$, $s^2-st-6t^2$.

52. a^3+27, a^3+6a+9.

53. a^2+ac, a^5+c^5, $3a^2+3c^2$.

54. b^4+9b^2+20, b^4+7b^2+10, $4b^2-100$.

55. $12x^4y^3z-48w^4z$, $15v(x^2y+2w^2)^2$.

56. $16p^4-81q^4$, $4p-6q$, $4p^2-8pq+3q^2$.

57. a^2-1, a^3-1, $(a-1)^2$. 58. x^3+y^3, $2x^2+2xy+2y^2$.

59. $z^2-8z+16$, $5z^3-20z^2+20z$.

60. $6(e^4 - f^4)$, $4(e^6 - f^6)$.

61. $h^3 - h$, $2h^2 - 4h + 2$, $h^3 + h^2 - 2h$.

62. $k^2 - 6k + 9$, $k^2 + 2k - 15$, $k^3 - 27$.

63. $a^2 - 9$, $a^3 + 27$, $a^4 - 81$, $6a^2x + 18ax$.

64. $x^3 - x^2y - xy^2 + y^3$, $x^2 + 2xy + y^2$, $3x^2 - 3y^2$.

65. $2cd + 2ce - 3df - 3fe$, $4c^2 - 12cf + 9f^2$.

66. $c^3d^4 - c^3e^3$, $d^5 - d^3e$, $d^3e^3 - de^5$.

67. $4x^2 - 60x + 144$, $x^3 - 9x - 36$, $2x^2 - 28x + 48$.

68. $7m^3n^2 + 28m^3np + 28m^3p^2$, $n^4 - 16p^4$.

69. $a^3 + 3a^2b + 3ab^2 + b^3$, $5ax + 5by$, $3a^2 - 3b^2$.

70. $m^2n - m^2p - mnq + mpq$, $2m^3 - 2mnq$, $4m^4 + 4mq^3$.

71. $x^3 + 6x^2 + 12x + 8$, $x^2 - 4$, $xy + 2y - xz - 2z$.

72. $x^2 - 9x + 20$, $x^2 - x - 12$, $x^2 - 2x - 8$.

73. $4p^2 - 12pq + 9q^2$, $8p^3 - 27q^3$, $4p^2 - 6pq - 6pr + 9qr$.

74. $8x^3 - 36x^2y + 54xy^2 - 27y^3$, $4x^2 - 8xy + 3y^2$.

75. $a^3 - 3a^2 - 7a + 21$, $a^3 - 3a^2 - 3a + 9$, $4a^2 - 16a + 12$.

76. $-6a^2b^2 - 6a^2bc$, $2ab^2 + 2ac^2 + 4abc$, $3ac^3 + 3ab^3$

77. $4a^2 - 12ac + 9c^2$, $4a^2 - 9c^2$, $10ax - 2a - 15cx + 3c$.

78. $c^5 - c^3 - c + 1$, $ac^4 + 2bc^4 + ac^3 - a + 2bc^3 - 2b$.

C. One or More Factorable by Inspection.

79. $a^2 - 5a - 24$, $a^3 + 4a^2 - 26a + 15$.

80. $c^2 - c - 42$, $c^3 + 4c^2 - 47c - 210$.

81. $n^3 + 9n^2 + 27n - 98$, $n^2 + 12n - 28$.

82. $9b^3 + 53b^2 - 9b - 18$, $b^2 + 11b + 30$.

83. $a^4 + 2a^3 + 2a^2 + 2a + 1$, $a^4 + 67a^2 + 66$.

84. $35v^3 - 7v + 5wv - w$, $25v^3 - 1$.

85. $z^3 + z^2 + z + 1$, $z^3 + 9z^2 + 26z + 18$.

86. $a^3 + 4a^2 - 5a$, $a^3 - 6a + 5$.

87. $c^3 - 3c + 2$, $c^4 - 6c^3 + 8c - 3$.

88. $p^3 - 3p^2 + 4$, $p^3 - 2p^2 - 4p + 8$.

89. $15a^3 - 19a^2 + 4$, $9a^3 - 9a^2 - 4a + 4$.

90. $q^4 - 2q^3 + q^2 - 2q$, $q^5 + q^3 - 8q^2 - 8$.

91. $m^2 + mp + mn + np$, $2m^2 - mn - 3n^2$.

92. $x^3 - 3x^2 - x + 12$, $x^2z + xz - 12z$.

93. $a^2 - 2ax - 3x^2$, $a^4 + 10a^3x + 35a^2x^2 + 50ax^3 + 24x^4$.

94. $k^3 - 5k^2 - k + 5$, $2k^2 - 15k + 25$.

95. $t^3 + 2t^2 - 3t$, $2t^3 + 5t^2 - 3t$.

96. $3m^3 + mn^2 - 3m^2n - n^3$, $4m^2 - 5mn + n^2$.

97. $cd^4 + cd^3 + cd^2 + cd$, $2c^3d^2 + c^3d - c^2$.

98. $ax^3 + ax^2 + 2ax + 2$, $x^4 - x^2 + x - 1$.

99. $y^3 - y^4 - y^2 + y$, $y^5 - 2y^4 - y^3 - 2y^2 + y$.

100. $3x^2 + x - 2$, $4x^3 + 2x^2 - x + 1$.

101. $s^3 + 5s^2 - s - 5$, $s^2 - 2s + 1$, $s^3 + 7s^2 - s - 7$.

102. $e^2 - 3ef + 2f^2$, $e^2h - f^2h$, $e^3 - e^2f - ef + f^2$.

103. $a^3 - a^2 + a - 1$, $3a^2 - 2a - 1$, $a^3 - a^2 - a - 1$.

104. $h^3 - 7h + 6$, $h^3 - h^2 - 10h + 10$, $h^3 + h^2 - 5h + 3$.

105. $12c^2y^2 + 4y^4 + 4c^3y + 12cy^3$, $2c^3 + 2y^3$, $10c^5 + 10c^3y^2 + 20c^4y$.

106. $a^3 - 7a^2 + 16a - 12$, $2a^3 - 9a^2 + 7a + 6$, $a^3 - 3a^2 - 4a + 12$.

107. $2l^3 - 13l + 15$, $10l^2 - 23l + 12$, $2l^3 + 5l - 12$.

108. $x^3 + 3xy - 4x - x^2y - 3y^2 - 4y$, $x^4 + 4x^2y - 4x^2 + 3y^2 - 4y$, $x^3 - 2x^2 - 4x + 8 + 3xy - 6y$.

D. Not Easily Factorable.

(Note: Synthetic division renders the work much easier.)

109. $y^3 + 7y - 48$, $y^3 - 16y + 21$.

110. $m^4 - 3m^2 + 2$, $m^4 + 3m^2 - 10$.

111. $l^3 - 23l + 28$, $l^3 - 10l - 24$.

112. $p^4 + p^2 + 25$, $p^3 - 4p + 15$.

113. $z^3 - 26z - 60$, $z^3 + 15z - 306$.

114. $y^3 - 2y^2 + 6y - 5$, $x^3 + 4y - 5$.

115. $c^3 + 2c^2 - 3$, $c^3 + c^2 - 2$.

116. $q^3 + 2q^2 - 3$, $q^3 + 2q^2 - 5q + 2$.

117. $3x^2 - 2x - 1$, $4x^3 - 2x^2 - 3x + 1$.

118. $a^3 - 7a^2b + 11ab^2 - 5b^3$, $a^3 + 3a^2b - 9ab^2 + 5b^3$.

119. $2c^3 + 7c^2 - 9$, $c^3 + c^2 - 5c + 3$.

120. $a^3 + 2a^2 + 2a + 1$, $a^3 - 2a - 1$.

121. $35 + 19b^2 + 2b^4$, $21 - 7b + 3b^2 - b^3$.

122. $x^3 - y^{-3}$, $x^5 - y^{-5}$.

123. $2x^3 - 12x^2 + 21x - 10$, $x^4 - 8x^3 + 21x^2 - 20x + 4$.

124. $3h^2 + h - 10$, $6h^3 - 7h^2 - 22h + 32$.

125. $2d^3 - d^2 - d - 3$, $3d^3 + d^2 + d - 2$.

126. $20a^4 + a^2 - 1$, $75a^4 + 15a^3 - 3a - 5$.

127. $9k^3 - 9k - 10$, $3k^3 + k^2 - 4k - 10$.

128. $2q^3 + 9q^2 - 25$, $4q^3 - 21q + 10$.

129. $n^3 + 2n - 4t^2 + 8t - 3$, $n^3 - 4t^2 + 12t - 9$.

130. $3a^4 + 2a^3 - 2a - 3$, $2a^3 - 3a^2 - 2a + 3$.

131. $2a^4 - 9a^3 + 12a^2 - 3a - 2$, $2a^4 + a^3 - 9a^2 + a + 1$.

132. $m^4 + 2m^2 + 9$, $7m^3 - 11m^2 + 15m + 9$.

133. $3x^3 - 18x^2 + 36x - 24$, $x^3 - 3x^2 + 4$.

134. $6x^3y - 7x^2y^2 + 5xy^3 - 2y^4$, $2x^4 + 5x^3y - 2x^2y^2 + 3xy^3$.

135. $8x^3 + 4x^2y - 4xy^2$, $4xy^2 - 2y^3 + 6x^2y$.

136. $15x^3 + 11x^2y - 8xy^2 - 4y^3$, $12x^3 - 5x^2y - 11xy^2 + 6y^3$.

137. $3a^4 - 7a^3c - 5a^2c^2 - ac^3 - 6c^4$, $2a^3 - 3a^2c - 8ac^2 - 3c^3$.

138. $27x^5y^3 - 9x^3y^5 - 18x^4y^2$, $36x^6 + 9x^3 - 27x^4 - 18x^5$.

139. $9a^4 - 64a^2 + 3a + 8$, $a^4 + a^3 - 8a^2 - 5a + 3$,
$8a^4 + 3a^3 - 64a^2 + 9$.

140. $x^3 - 2x^2 + 1$, $x^3 - 4x + 3$, $2x^3 + x^2 - 7x + 4$.

141. $2x^3 - 11x^2 + 19x - 10$, $2x^3 - 9x^2 + 13x - 6$,
$4x^3 - 20x^2 + 31x - 15$.

142. $a^3 - 2a^2 - 3a + 20$, $a^3 + 5a^2 - 9a + 35$, $a^3 - 4a^2 + 9a - 10$.

143. $x^3 - 19x^2 + 119x - 245$, $3x^2 - 38x + 119$.

144. $3x^6 - 3x^4 - 6x^3 + 9x^2 - 3x$, $2x^5 - 4x^4 + 8x^3 - 12x^2 + 6x$.

145. $16a^2x^2 + 6a^2x^4 - 28a^2x^3$, $14ax - 8a + ax^3 - 7ax^2$.

146. $3y^3 - 14y^2 + 16y$, $5y^3 - 10y^2 + 7y - 14$, $y^3 - 7y^2 + 16y - 12$.

147. $3a^3 + 8a^2 - 8a + 11$, $9a^4 + 51a^3 + 45a^2 - 119a - 154$.

148. $2g^4 + 9g^3 + 14g + 3$, $3g^4 + 15g^3 + 5g^2 + 10g + 2$.

149. $3a^3 - 4a^2 + 9a - 8$, $2a^3 + 5a^2 + a - 8$.

150. $6x^3 - 17x^2 + 2x + 15$, $2x^3 + 3x^2 - x - 12$.

151. $10m^6 - 7m^4 - 2m^3 + 8$, $5m^7 - 5m^6 - m^5 + m^4 + 11m^3 - 11m^2 + 12m - 12$.

152. $3y^4 + 5y^3 - 7y^2 + 2y + 2$, $2y^4 + 3y^3 - 2y^2 + 12y + 5$.

153. $8c^4 - 16c^3 + c + 2$, $16c^4 + 4c^3 + 1$.

154. $4x^4 - 6x^3 + x^2 + 4x - 3$, $2x^4 - x^3 - 4x^2 + 3x$.

155. $a^3 + 4a^2x - 8ax^2 + 24x^3$, $4a^6 - 4a^5x + 32a^3x^3 - 32a^2x^4$.

156. $24x^5 + 16x^4y - 36x^3y^2 - 12x^2y^3 + 8xy^4$, $4x^5y - 4x^3y^3 + 4x^2y^4 - xy^5$.

157. $a^4 - 12a^2 + 8a + 3$, $a^4 + a^3 - 53a + 51$.

158. $21a^3 - 32a^2 - 54a - 7$, $21a^3 - 4a^2 - 15a - 2$.

159. $x^7 - 3x^6 + x^5 - 4x^2 + 12x - 4$, $2x^4 - 6x^3 + 3x^2 - 3x + 1$.

160. $x^5 + x^4 - 2x^3 + x^2 + x + 4$, $x^6 - 2x^5 - 4x^3 - 2x + 1$.

161. $3y^4 + 2y^2 - 17y - 22$, $7y^4 - 3y^3 - 11y - 66$.

162. $6x^5 + 20x^4 - 12x^3 - 48x^2 + 22x + 12$, $x^6 + 4x^5 - 3x^4 - 16x^3 + 11x^2 + 12x - 9$.

163. $x^4 - (a + b)^2x^2 + 2ab(a + b)x - a^2b^2$, $x^4 - (a^2 + b^2)x^2 + a^2b^2$.

164. $2c^2 - 13c + 18$, $2c^3 + (2b - 9)c^2 - (9b + 6)c + 27$.

165. $x^4 - qx^3 + (p - 1)x^2 + qx - p$, $x^4 - px^3 + (q - 1)x + px - q$.

LOWEST COMMON MULTIPLE.

1. 4, 3, 2.
2. 6, 2, 3.
3. 5, 10, 4.
4. 12, 4, 8.
5. 8, 10, 12.
6. 5, 15, 9, 18.
7. 18, 24, 36.
8. 21, 39, 57.
9. 64, 400, 512, 1024.
10. 323, 391, 629.
11. ab, bc, cd.
12. x^2, xy, y^2.
13. a, m^2n, x^2y.
14. qr, rs, st, qt.
15. $4ab$, $8ac$, $6bc$.
16. $8x^2$, $12x^3$, $18x^4$.
17. cy^3, a^2, xz^3.
18. $2mn^2$, $3np$, $6m^2p$.
19. ab, bc, cd, ac, ad, bd.
20. $20xy^3$, $90z^4w^3$, $27y^3z^3$.
21. $12a^2bc^4$, $18ab^5c$, $20a^4c^3$.
22. $7a^3c^2$, $-21a^2c^3$, $-343ac$.
23. $3a^3bc^3$, $5ab^3c^2$, $15a^2b^2c$, $20a^3b^3c^3$.
24. $14a^3b^2c^4$, $20a^7bc^5$, $25a^3b^3c^3$, $28abcd$.
25. $6mn^2pq^5$, $9m^5n^4p^3$, $15mn^2p^3$, $20m^5n^4p^5q^2$.
26. x^3y, x^2z, x^3w, w^2v, x^2z, y^2w, xy^3, yz^3, zw^3, xz^2, xw^3, xv^2.
27. $10x^4y^{a+1}z^3$, $15x^5y^{a-1}z^2$.
28. $12x^ay^{-3}z^{\frac{1}{2}}$, $36x^{a+1}y^3z^{\frac{1}{2}}$, $60x^{a-1}y^{-1}z$.
29. $a^2 + ab$, a.
30. 6, $3x - 3y$.
31. $2a(x - y)$, $a^2(x - y)$.
32. $ax + bx$, $ay + by$.
33. $ax + x^2$, $abc + bcx$.
34. $a^2 - 2ab + b^2$, $a^2 - b^2$.
35. $a^2 - 1$, $a^2 - a$.
36. $x + 1$, $x - 1$, $x^2 - 1$.
37. $2c - 1$, $4c^2 - 1$.
38. $x - 3$, x, $4x^2 - 12$.
39. $3a$, $x - a$, $(x - a)^2$.
40. $a(a + b)$, $b(a + b)$.

41. $3(x+2)$, $5(x+2)$. 42. $2(x+1)$, $4(x+1)^2$.

43. $4(x+3)$, $(x-1)(x+3)$.

44. $3(p-q)(p+q)$, $2(p+q)^2$.

45. $a(a+1)$, $a(a+1)(a-2)$, $a(a-2)$.

46. ef, $gh+gk$, hk. 47. $c+x$, $2c$, $2c(c-x)$.

48. $(m+n)(p+q)$, $(m+n)(p-q)$.

49. x, $x-y$, x^2-y^2. 50. p^2q, p^2-pq, $px-qx$.

51. $4x^2y$, $4x^2-2xy$, $2x-y$.

52. $2a+2x$, $4a-4x$, a^2-x^2.

53. $(2x-4)(3x-6)$, $(x-3)(4x-8)$, $(2x-6)(5x-10)$.

54. $x-1$, $x+1$, x^2+1, x^4-1.

55. $2c^2$, c^2-d^2, $4c^2-4cd$, $2c+2d$.

56. $x+y$, $(x+y)^2$, x^2-y^2, $(x-y)^2$.

57. $4b^3$, $8a^2$, $a+b$, $2a^2$, $4a^2b+2ab^2$.

58. $m^2+2mn+n^2$, m^3-n^3, $m^2-2mn+n^2$.

59. p^2-1, p^3+1, p^3-1. 60. $a-1$, a^2+a+1, a^3-1.

61. $3a-2x$, $9a^2-4x^2$, $9a^2-12ax+4x^2$.

62. $(y-y^2)^2$, y^2-1, $2(1+y)y$.

63. $pq^2(m-n)$, $p^2q(n-m)$, $-p^2q^2(m-n)$.

64. $8x^2+16x$, x^3+4x^2+4x, x^2.

65. $3(x-y)^2$, $6(x+y)(x-y)$, $12(x+y)^2$.

66. $4(a^2+ax)$, $8(ax-x^2)$, $12(a^2-x^2)$.

67. $3a(a^2-a+1)$, $2a^2(a+1)$, $4a^3+4$.

68. a^2+ax+x^2, $a-x$, a^3-x^3.

69. $x(x^3+8)$, $2x^2(x^2+4x+4)$, x^4+2x^3.

70. c^4+3c^3, $2c^4-6c^3+18c^2$, c^5+27c^2.

71. $8p^6q-27q^4$, $4p^4q^2+6p^3q^3+9q^4$, $2p^3-3pq$.

72. x^2+3xy, $xy-3y^2$, x^3-9y^2, x^2+9y^2.

73. a^2-a-20, $a^2-9a+20$.

74. $a+1$, a^2-2a-3.

75. x^3-1, x^2+3x+2.

76. $a^2+4a-21$, a^2-3a, a^2+7a.

77. x^2-2x-3, x^3-4x+3.

78. p^2+7p+6, $p^2+11p+30$.

79. c^3+c-6, c^3-9, c^3-6c+8.

80. $z^4-18z^3+81z^2$, z^3-13z^2+36z.

81. x^2+5x+4, x^3+2x-8, $x^3+7x-12$.

82. x^2+x-12, x^3-x-20, x^2-4x+3.

83. $4a^2-25$, $2a^3-5a^2-4a+10$.

84. $ax+bx+ay+by$, $3(a^2-b^2)$, $(x+y)^2$.

85. $9x^2+12x+4$, $27x^3+8$, $6ax^3+4ax^2$.

86. $ax-ay-bx+by$, $x^2-2xy+y^2$, $3a^2b-3ab^2$.

87. $a^2-b^2-c^2+2bc$, $a^2-b^2+c^2+2ac$.

88. $x^2-(a+b)x+ab$, $x^2-(b+c)x+bc$, $x^2-(a+c)x+ac$.

89. x^2-3x+2, x^3-4x+4, $x^3-7x^2+14x-8$.

90. x^2-4a^2, $x^3+2ax^2+4a^2x+8a^3$, $x^3-2ax^2+4a^2x-8a^3$.

ONE OR MORE FACTORABLE BY INSPECTION.

91. $25x^2-1$, $5x^2-9x-2$.

92. a^3-8a+3, a^6+3a^5+a+3.

93. $x^3 + x^2 - 4x - 4$, $x^3 + 6x^2 + 11x + 6$.

94. $y^3 + 4y^2 - 8y + 24$, $y^4 - y^3 + 8y - 8$.

95. $x^4 - 2x^3 + x^2 - 8x + 8$, $4x^3 - 12x^2 + 9x - 1$.

96. $a^4 - a^3 + 2a^2 + a - 3$, $a^4 + a^3 + 2a^2 + 5a + 3$.

97. $a^3 + ab^2 - ac^2 - 2a^2b$, $2a^2 - ab - b^2 + 2ac + bc$.

98. $4x^3 - x^2y - 3xy^2$, $3x^3 - 3x^2y + xy^2 - y^3$.

99. $x^4 - 2x^3 - 3x^2 + 8x - 4$, $x^4 - 5x^3 + 20x - 16$.

100. $x^3 + 2x^2 + 2x + 1$, $2x^3 + x^2 + x - 1$.

101. $x^3 + 2x^2 - x - 2$, $x^3 + 3x + 2$.

102. $x^4 - 4x^2 + 12x - 9$, $x^3 + 5x^2 + x - 3$.

103. $2x^3 + x^2 - 5x - 3$, $x^4 + 2x^3 + 2x^2 + x$.

104. $4p^3 + 5p^2 - 8p - 1$, $8p^3 + 10p^2 - 5p - 1$.

105. $a^3 + 3a^2 - 10a - 24$, $a^3 - a^2 - 14a + 24$, $a^4 - 3a^3 - 4a^2 + 12a$.

106. $8x^{-3} - 8x^{-2}y^{-1} + 4x^{-1}y^{-2} - y^{-3}$, $4x^{-3} + 2x^{-2}y^{-1} - x^{-1}y^{-2} + y^{-3}$.

107. $x^3 + x^2 - 2$, $x^3 + 2x^2 - 3$.

108. $12a^2 - 4a - 21$, $6a^2 - 17a + 12$.

109. $2a^2 + a + 3$, $a^2 - 6a - 5$.

110. $2n^3 - 5n^2 - 2n + 2$, $n^3 + n^2 - 8n - 6$.

111. $p^3 + 10p^2 + 29p + 20$, $p^3 + 6p^2 + 11p + 6$.

112. $3c^3 + 2c^2 + 51c + 70$, $3c^3 + 14c^2 - 19c - 70$.

113. $6q^3 - 7q^2 + 5q - 2$, $4q^4 - 5q^2 + 4q - 3$.

114. $2y^3 - 19y + 3$, $2y^3 - 6y^2 + 3y - 9$.

115. $2k^3 + 5k^2 - k - 6$, $2k^3 + k^2 - k + 3$.

116. $2a^4 - a^3 + a - 12$, $2a^4 - 3a^3 + 4a^2 - 5a - 4$.

117. $3z^4 - 14z^3 - 6z^2 + 5z$, $2z^4 - 11z^3 + 3z^2 + 10z$.

118. $x^3 - 2x^2y + xy^2 - 2y^3$, $x^3 - 2x^2 - 2x^2y + 4xy + x - 2y$.

119. $a^3 - a^2 - 18$, $a^3 - 8a - 3$, $a^2 + 2a - 12$.

120. $a^3 + 2a^2 + 9$, $a^4 + a + 6$.

121. $4x^3 - 9x - 4$, $2x^3 - 5x^2 + x + 2$.

122. $x^4 - 23x^2 - 10x$, $7x^3 - 34x^2 - 4x - 5$.

FRACTIONS.

Reduce the following fractions to their lowest terms :

(NOTE: Remember that a divisor of the numerator and denominator of a fraction is also a divisor of their sum and their difference.)

1. $\frac{9}{12}$.
2. $\frac{35}{42}$.
3. $\frac{45}{125}$.
4. $\frac{93}{126}$.
5. $\frac{78}{91}$.
6. $\frac{115}{138}$.
7. $\frac{319}{551}$.
8. $\frac{246}{369}$.
9. $\frac{703}{1073}$.
10. $\frac{1261}{1649}$.
11. $\frac{6 \cdot 5 \cdot 4 \cdot 3}{1 \cdot 2 \cdot 3 \cdot 4}$.
12. $\frac{7 \cdot 8 \cdot 9 \cdot 10}{5 \cdot 4 \cdot 3 \cdot 2}$.
13. $\frac{26 \cdot 25 \cdot 24}{13 \cdot 12 \cdot 10}$.
14. $\frac{5 \cdot 4 \cdot 3 \cdot 2}{5 \cdot 2 \cdot 3 \cdot 4}$.
15. $\frac{5^2 \cdot 4^2 \cdot 3^2}{3 \cdot 4 \cdot 5}$.
16. $\frac{ab}{ac}$.
17. $\frac{a^2x}{ax^2}$.
18. $\frac{10a^2x}{15ax^3}$.
19. $\frac{3mxz}{4amz}$.
20. $\frac{7x}{21acx}$.
21. $\frac{12cdx}{6bcxy}$.
22. $\frac{5efgh}{7fxy}$.
23. $\frac{a^2bc}{5a^2b^2}$.
24. $\frac{5abx^2}{10bx}$.
25. $\frac{4mnx}{6m^2n^2x^2}$.
26. $\frac{3mxy}{7amz}$.
27. $\frac{4mnx}{8axz}$.
28. $\frac{7cdp^2}{21c^2dp}$.
29. $\frac{17b^2cxy}{51xybc^2}$.
30. $\frac{mnp^3x^2}{p^2rsx^3}$.
31. $\frac{15m^3nx^4z}{25m^2n}$.
32. $\frac{4b^3c^3mx^4y}{9mxy^2b^2c^2}$.
33. $\frac{84x^2y^2z^2}{108xya^4c^3}$.
34. $\frac{12a^2bc^3dm^4n}{18a^5b^2c^3dx^3y^2}$.
35. $\frac{110ef^2g^3}{550e^7fg^4}$.

36. $\dfrac{16a^2b^3c^4d^5e^6}{24a^6b^5c^4d^3e^2}$. **37.** $\dfrac{121abcde}{22fghij}$. **38.** $\dfrac{106a^5mn^3x^4}{159m^6n^8x^2y}$.

39. $\dfrac{126a^6b^2c^9}{98ab^{10}c^2}$. **40.** $\dfrac{-91m^2pq^3x^4}{119mp^2q^2x^3}$. **41.** $\dfrac{65x^2y^3z^4w^5}{-26x^4y^3z^2w^4}$.

42. $\dfrac{-110c^3x^2y^7a^5}{-22a^3b^5c^2x^2}$. **43.** $\dfrac{-819a^5bc^6xy^3}{468a^6bc^5x^2y}$.

44. $\dfrac{444g^3h^2klx^3y}{-202g^2h^3k^2xy^2}$. **45.** $\dfrac{-533a^3c^5e^7h^4x^6w^3}{-799b^4d^6f^2k^8y^5z^7}$.

46. $cd : de$. **47.** $xyz : xz$. **48.** $2x : 8x^2$.

49. $3a^3c^2 : 4a^2c^3$. **50.** $12a^2b^3c^2 : 4abcd$.

51. $\dfrac{4(x+y)}{2(x+y)}$. **52.** $\dfrac{xy}{x^2+xy}$. **53.** $\dfrac{2ax}{a^2+ax}$.

54. $\dfrac{5(c+d)^2}{15(c+d)}$. **55.** $\dfrac{m^3-m^2n}{3mn}$. **56.** $\dfrac{4a+4b}{5a+5b}$.

57. $\dfrac{cdy+dy^2}{cey+ey^2}$. **58.** $\dfrac{a^2c+a^2}{ac^2+ac}$. **59.** $\dfrac{ab+ac^2}{c^2a-ab^2}$.

60. $\dfrac{a^2-1}{a-1}$. **61.** $\dfrac{c+1}{c^2-1}$. **62.** $\dfrac{x^2-1}{x^4-1}$.

63. $\dfrac{x-y}{x^2-y^2}$. **64.** $\dfrac{c^2-d}{c^2+cd}$. **65.** $\dfrac{7a-7x}{a^2-x^2}$.

66. $\dfrac{a^2c-ac^2}{ac^2-a^2c}$. **67.** $\dfrac{a^2c+a^2}{ac^2-a^2}$. **68.** $\dfrac{a^2-c^2}{a^3-c^3}$.

69. $\dfrac{(a+x)^2}{a^3-x^3}$. **70.** $\dfrac{(x+a)^3}{(a+x)^2}$. **71.** $\dfrac{x^2-y^2}{x^3+y^3}$.

72. $\dfrac{(x-z)^2}{x+z}$. **73.** $\dfrac{2x-1}{4x^2-1}$. **74.** $\dfrac{3a+1}{9a^2-1}$.

75. $\dfrac{x^2-y^2}{x^2+2xy+y^2}$. **76.** $\dfrac{x^3+2x^2}{x^2+4x+4}$. **77.** $\dfrac{a+bc}{ax+bcx}$.

78. $\dfrac{x^2-y^2}{x^3-y^3}$.

79. $\dfrac{k^3-1}{4k(k+1)}$.

80. $\dfrac{b^4-c^4}{b^3+bc}$.

81. $\dfrac{8a^3+1}{64a^6-1}$.

82. $\dfrac{p^3+3p^2}{p^2-9}$.

83. $\dfrac{3a^4+3a^2b^2}{5a^4+5a^2b^2}$.

84. $\dfrac{a^2-4x^2}{a^2+4ax+4x^2}$.

85. $\dfrac{12cd^2-6cd}{8d^2e-2e}$.

86. $\dfrac{4cy+2y^2}{8cy^3-2y^3}$.

87. $\dfrac{3rs+9s^2}{4r^2-36s^2}$.

88. $\dfrac{4a^2b+6a^2c}{8b^2-18c^2}$.

89. $\dfrac{mn-mnx}{2ax-2ax^2}$.

90. $\dfrac{6ab+8cd}{27a^2b^2x-48c^2d^2x}$.

91. $\dfrac{7ab^3x^3-7ab^3y^2}{14a^3bcx^3-14a^3bcy^3}$.

92. $\dfrac{10x^2+20xy+10y^2}{5x^3+5x^2y}$.

93. $\dfrac{4e^2-8ef+4f^2}{48(e-f)^2}$.

94. $\dfrac{3c^2d+3cd^2}{3c^2+6cd+3d^2}$.

95. $\dfrac{12m^3x^4+2m^2x^5}{18mn^2x+3n^2x^2}$.

96. $\dfrac{h^3-hk^2}{h^3+2h^2k+hk^2}$.

97. $\dfrac{12ax^5y^4z^2-28ax^3y^3z^2}{32x^6yz^3+8x^6y^4z^4}$.

98. $\dfrac{10x^3y+15xy^3-20x^2yz^2}{30xy^3+25xyz^3+10x^2yz^2}$.

99. $\dfrac{a^4-1}{a^6+1}$.

100. $\dfrac{b^6-1}{b^8-1}$.

101. $\dfrac{c^3+1}{c^9+1}$.

102. $\dfrac{m^4-n^4}{m^6-n^6}$.

103. $\dfrac{c^6+d^6}{c^8-d^8}$.

104. $\dfrac{a^6+x^6}{a^4-a^2x^2+x^4}$.

105. $\dfrac{a^6+2a^3x^3+x^6}{a^6-b^6}$.

106. $\dfrac{a(x-y)b^2c}{3a^2(y-x)bc}$.

107. $\dfrac{7\cdot 9+3}{7\cdot 9-2\cdot 3}$.

108. $\dfrac{(a-1)(a-2)}{(1-a)(2-a)}$.

109. $\dfrac{5\cdot 8-3\cdot 4}{2\cdot 4+12}$.

110. $\dfrac{(1-x)^2(1+x)^2}{(1-x)(x+1)}$.

111. $\dfrac{(x-3)(4+2x)}{2x^2-12x+18}$.

112. $\dfrac{a^2x+abx}{a^3+3a^2b+3ab^2+b^3}$.

113. $\dfrac{a^6-b^6}{(a+b)(a-b)}$.

114. $\dfrac{a+b}{a^{\frac{1}{2}}-b^{\frac{1}{2}}}$.

115. $\dfrac{a-b}{a+2a^{\frac{1}{2}}b^{\frac{1}{2}}+b}$.

116. $\dfrac{x^{\frac{4}{3}}-y^{\frac{4}{3}}}{x+y}$.

117. $\dfrac{x^2-x^{-2}}{x^2+2+x^{-2}}$.

118. $\dfrac{x^{\frac{1}{2}}+y^{-\frac{1}{2}}}{x-y^{-1}}$.

119. $\dfrac{x^{-2}+1}{x^{-2}+2x^{-1}+1}$.

120. $\dfrac{a^{5m}+32}{a^{4m}-16}$.

121. $\dfrac{a^{-2}-2a^{-1}c^{-1}+c^{-2}}{a^{-3}-c^{-3}}$.

122. $\dfrac{a-9c}{a-6a^{\frac{1}{2}}c^{\frac{1}{2}}+9c}$.

123. $\dfrac{x-6x^{\frac{1}{2}}+9}{x^{\frac{1}{2}}-27}$.

124. $\dfrac{d^{\frac{5}{3}}-1}{d^{\frac{2}{3}}-2d^{\frac{1}{3}}+1}$.

125. $\dfrac{a^{\frac{2}{3}}+x^{\frac{2}{3}}}{a^{\frac{4}{3}}-x^{\frac{4}{3}}}$.

126. $\dfrac{a^{\frac{4}{3}}-2+a^{-\frac{4}{3}}}{a^{\frac{4}{3}}-a^{-\frac{4}{3}}}$.

127. $\dfrac{c^{\frac{2}{3}}+32x^{\frac{2}{3}}}{c^{\frac{4}{3}}-4x^{\frac{1}{2}}}$.

128. $\dfrac{p^{\frac{1}{2}}-q}{p^{\frac{1}{3}}-2p^{\frac{1}{6}}q^{\frac{1}{3}}+q^{\frac{2}{3}}}$.

129. $\dfrac{a^{\frac{3}{2}}-27}{a-6a^{\frac{1}{2}}+3^2}$.

130. $\dfrac{x^2+2x+1}{x^2+3x+2}$.

131. $\dfrac{a^2-2a+1}{a^2-4a+3}$.

132. $\dfrac{m^2+2m+1}{m^2-3m-4}$.

133. $\dfrac{z^2-2z+1}{z^2+4z-5}$.

134. $\dfrac{x^2+2x+1}{x^2-6x-7}$.

135. $\dfrac{x^2+x-6}{x^2+5x+6}$.

136. $\dfrac{x^2+5x+6}{x^2+6x+9}$.

137. $\dfrac{a^2+3a+2}{a^2+6a+5}$.

138. $\dfrac{c^2-7c+12}{c^2-8c+15}$.

139. $\dfrac{k^2-k-20}{k^2+k-30}$.

140. $\dfrac{b^2-3b+2}{b^2-1}$.

141. $\dfrac{d^2-9}{d^2-6d+9}$.

142. $\dfrac{z^3-27}{z^2+2z-15}$.

143. $\dfrac{c^2-14c-51}{c^2-2c-15}$.

144. $\dfrac{a^2+7a+10}{a^2+4a-5}$.

145. $\dfrac{d^2-10d+25}{d^2-12d+35}$.

146. $\dfrac{n^2+5n-84}{n^3-14n^2+49n}$.

147. $\dfrac{p^2+3p-28}{p^3-64}$.

148. $\dfrac{y^2+y-72}{4xy^2-48xy+128x}$.

149. $\dfrac{21(x^2-y^2)}{70(x-y)^2}$.

150. $\dfrac{4a^2-16}{2a^2-2a-12}$.

151. $\dfrac{a^2-a-132}{a^3-1728}$.

152. $\dfrac{a^2-9ax+14x^2}{a^2-ax-2x^2}$.

153. $\dfrac{c^2+cd-2d^2}{c^3-d^3}$.

154. $\dfrac{27x+x^4}{18x-12x^2+2x^3}$.

155. $\dfrac{a^2x^2-16a^2}{ax^2+9ax+20a}$.

156. $\dfrac{p^3-8p^2+12p}{p^2-12p+36}$.

157. $\dfrac{x^3-x^2-56x}{x^4+x^3-42x^2}$.

158. $\dfrac{c^2-14ac+45a^2}{c^2-2ac-15a^2}$.

159. $\dfrac{a^3+x^3}{2a^3x-2a^2x^2+2ax^3}$.

160. $\dfrac{64x^3+27y^3}{16x^2+24xy+9y^2}$.

161. $\dfrac{49cd^2+112c^2d+64c^3}{49d^3-64c^2d}$.

162. $\dfrac{9-x^2}{x^2-5x+6}$.

163. $\dfrac{a^2-16}{16+a-2a^2}$.

164. $\dfrac{x^2 - x - 2}{4 - x^2}$.

165. $\dfrac{49 - x^2}{x^2 - 2x - 35}$.

166. $\dfrac{1 - x^2}{x^2 - 13x + 12}$.

167. $\dfrac{a^2 - 7ac + 6c^2}{c^2 - a^2}$.

168. $\dfrac{8x^2 - 1}{1 - 11x + 18x^2}$.

169. $\dfrac{14a^2 - 4a^3}{4a^2 - 28a + 49}$.

170. $\dfrac{2x^2 - 3x + 1}{x^2 + x - 2}$.

171. $\dfrac{x^2 - 10x + 21}{28 + 3x - x^2}$.

172. $\dfrac{x^{-2} - 9}{x^{-2} - 4x^{-1} + 3}$.

173. $\dfrac{z^{-4} - 2z^{-2} - 15}{z^{-4} + 9z^{-2} + 18}$.

174. $\dfrac{x + x^{\frac{1}{2}} - 6}{x - x^{\frac{1}{2}} - 12}$.

175. $\dfrac{a^{\frac{2}{3}} + 2a^{\frac{1}{3}} - 24}{a^{\frac{2}{3}} - 7a^{\frac{1}{3}} + 12}$.

176. $\dfrac{c^{\frac{4}{3}} - 2c^{\frac{2}{3}} - 8}{c^{\frac{4}{3}} + 8c^{\frac{2}{3}} + 12}$.

177. $\dfrac{q - 3q^{-\frac{1}{2}} + 2q^{-2}}{q + q^{-\frac{1}{2}} - 2q^{-2}}$.

178. $\dfrac{m^{\frac{2}{3}} - 5m^{\frac{1}{3}}n^{\frac{1}{3}} + 6n}{m^{\frac{2}{3}} + 2m^{\frac{1}{3}}n^{\frac{1}{3}} - 8n}$.

179. $\dfrac{x^2 + x - 6}{2x^2 + x - 15}$.

180. $\dfrac{x^2 - x - 12}{3x^2 - 3x - 36}$.

181. $\dfrac{b^2 + 11b + 30}{4b^2 + 25b + 6}$.

182. $\dfrac{c^2 + c - 20}{2c^2 + 7c - 15}$.

183. $\dfrac{a^2 - 3a - 18}{5a^2 + a - 42}$.

184. $\dfrac{p^2 - 7p + 10}{2p^2 - p - 6}$.

185. $\dfrac{2m^2 + 3m + 1}{m^2 - 1}$.

186. $\dfrac{3y^2 - 2y - 40}{y^2 - 3y - 4}$.

187. $\dfrac{d^2 - 4d - 32}{2d^2 - 8d - 64}$.

188. $\dfrac{x^2 - 3x - 10}{6x^2 - 0x - 24}$.

189. $\dfrac{2x^2+x-15}{2x^2+19x+35}$.

190. $\dfrac{2z^2+6z-6}{2z^2-11z+6}$.

191. $\dfrac{3c^2-5c-2}{3c^2-8c-3}$.

192. $\dfrac{2d^2-4d-16}{2d^2-0d-8}$.

193. $\dfrac{6q^2-7q-3}{2q^2+q-6}$.

194. $\dfrac{4w^2-8w+3}{4w^2+4w-3}$.

195. $\dfrac{6s^2+28s-10}{12s^2-28s+8}$.

196. $\dfrac{2a^2+19a+35}{3a^2+15a-42}$.

197. $\dfrac{6x^2-3x-9}{9x^2+21x+12}$.

198. $\dfrac{12c^2-cd-6d^2}{8c^2-54cd+36d^2}$.

199. $\dfrac{6x^2-xy-5y^2}{6x^2-13xy-15y^2}$.

200. $\dfrac{4a^2-5ac-6c^2}{8a^2+2ac-3c^2}$.

201. $\dfrac{x^3-5x^2+7x-3}{x^2-3x+2}$.

202. $\dfrac{a^2+9a+20}{a^3+7a^2+14a+8}$.

203. $\dfrac{c^3+10c^2+21c-18}{c^2-c-42}$.

204. $\dfrac{d^2+11d+30}{9d^3+53d^2-9d-18}$.

205. $\dfrac{x^3-x-12}{x^3-3x^2-3x-4}$.

206. $\dfrac{a^3-5a-14}{a^3+a^2-3a-2}$.

207. $\dfrac{c^3+c^2-c-1}{c^3+11c-12}$.

208. $\dfrac{y^2+8y+12}{y^3-3y+2}$.

209. $\dfrac{a^3+3a^2-4a}{7a^3-18a^2+6a+5}$.

210. $\dfrac{ac^2-15ac+54a}{c^3-18c^2+101c-180}$.

211. $\dfrac{x-xy+z-zy}{1-3y+3y^2-y^3}$.

212. $\dfrac{3a^2+2a-1}{a^3+a^2-a-1}$.

213. $\dfrac{c^3-3c^2+4c-2}{c^3-c^2-2c+2}$.

214. $\dfrac{n^3-3n^2+4}{n^3-2m^2-4n+8}$.

215. $\dfrac{xy+2y-yz-2z}{2xy-2y-2xz+2z}$.

216. $\dfrac{x^3-ax^2+b^2x-ab^2}{x^3-ax^2-b^2x+ab^2}$.

217. $\dfrac{a^5 - a^4c - ac^4 + c^5}{a^4 - a^3c - a^2c^2 + ac^3}$.

218. $\dfrac{x^2 - y^2 - 2yz - z^2}{x^2 + 2xy + y^2 - z^2}$.

219. $\dfrac{a^4 - a^3 - a + 1}{a^4 - 2a^3 - a^2 - 2a + 1}$.

220. $\dfrac{p^3 - p^2q - pq^2 + q^3}{p^3 + p^2q - pq^2 - q^3}$.

221. $\dfrac{k^3 - 6k^2 + 11k - 6}{k^3 - 2k^2 - k + 2}$.

222. $\dfrac{c^2 - c^2d + 3cd^2 - 3d^3}{4c^2d - cd^2 - 3d^3}$.

223. $\dfrac{m^4 - m^3 + 2m^2 + m + 3}{m^4 + 2m^3 - m - 2}$.

224. $\dfrac{h^3 + 3h^2 + 4h + 12}{h^3 + 4h^2 + 4h + 3}$.

225. $\dfrac{x^2 - (y + z + w)^2}{(x - y)^2 - (z + w)^2}$.

226. $\dfrac{(a^3 - b^3)(a + b)}{(a^3 + b^3)(a - b)}$.

227. $\dfrac{(p^3 + q^3)(p^2 + pq + q^2)}{(p^3 - q^3)(p^2 - pq + q^2)}$.

228. $\dfrac{x^2yz - y^2z + 2y^2z^2 - yz^3}{4x^2y^2 - (x^2 + y^2 - z^2)^2}$.

229. $\dfrac{a^3 + 2a^2 - 13a + 10}{a^3 + a^2 - 10a + 8}$.

230. $\dfrac{q^3 + 6q^2 + 11q + 6}{q^3 + 5q^2 - 6q}$.

231. $\dfrac{6y^3 + y^2 - 5y - 2}{6y^3 + 5y^2 - 3y - 2}$.

232. $\dfrac{b^3 - 4b^2 + 9b - 10}{b^3 + 2b^2 - 3b + 20}$.

233. $\dfrac{d^3 - 5d^2 + 11d - 15}{d^3 - d^2 + 3d + 5}$.

234. $\dfrac{c^3 - c^2x - cx^2 - 2x^3}{c^3 + c^2x + 3cx^2 + 2x^3}$.

235. $\dfrac{e^4 - e^3 - e + 1}{e^4 - 2e^3 - e^2 - 2e + 1}$.

236. $\dfrac{x^3 + 7x^2 - 6x + 2}{x^3 - 8x^2 + 2x + 11}$.

237. $\dfrac{n^3 + 3n^2 - 4n + 12}{n^3 + 4n^2 + 4n + 3}$.

238. $\dfrac{a^3 - 3a^2 + 4a - 2}{a^3 - a^2 - 2a + 2}$.

239. $\dfrac{u^3 - 3u^2 + 3u - 2}{u^3 - 4u^2 + 6u - 4}$.

240. $\dfrac{r^3 + 2r^2s - 2rs^2 - s^3}{r^3 - 3r^2s - 2rs^2 + 4s^3}$.

241. $\dfrac{4c^3 + c^2 + c - 3}{3c^3 + c^2 + c - 2}$.

242. $\dfrac{2a^3 + a^2c + ac^2 - c^3}{3a^3 + a^2c + ac^2 - 2c^3}$.

Reduce the following fractions to integral or mixed quantities:

243. $\frac{19}{5}$.

244. $\frac{12345}{19}$.

245. $\frac{10764}{23}$.

246. $\frac{16591}{47}$.

247. $\frac{a^2c^2x}{ac^2x}$.

248. $\frac{bc^2 - cd}{c}$.

249. $\frac{ac + x}{a}$.

250. $\frac{x^2 + xy}{x}$.

251. $\frac{24cy - 8cy}{cy}$.

252. $\frac{c^2 - de}{c}$.

253. $\frac{cx - cy - d}{c}$.

254. $\frac{xy - x^2 + z}{x}$.

255. $\frac{a^2 + ab + b^2}{a}$.

256. $\frac{pq^2 + pq - pq^2}{pq}$.

257. $\frac{a^2c + ac - a^2c}{ac}$.

258. $\frac{d^3 - bd^2 + cd}{d^2}$.

259. $\frac{cde - ef^2}{ce}$

260. $\frac{5ay + ab - x}{y}$.

261. $\frac{k^2 - k + 1}{k^2}$.

262. $\frac{6xy - 2c}{3x}$.

263. $\frac{18m^2nz - 12mz^2 - 5m}{6mz}$.

264. $\frac{xy + y^2}{y^2}$.

265. $\frac{a^2 - ax}{a^2}$.

266. $\frac{p^3 + p^2 + p}{p^2}$.

267. $\frac{5c^2z + 7az + az^2}{az}$.

268. $\frac{4h^3 + 6h^2k + 8hk^2}{2h^2}$.

269. $\frac{20cst - 3ct - 2t}{5cs}$.

270. $\frac{ax^3 - ax^4 + y}{ax^2}$.

271. $\frac{15x^2 - 4x + 7}{5x}$.

272. $\frac{lmn - lmn^2 + lmn^3}{lmn}$.

273. $\dfrac{9a^2cd - 18a^2c^2d + 12acd^2}{3acd}$.

274. $\dfrac{10a^3 - 5a + 2}{5a}$.

275. $\dfrac{5cd + ef + c}{c}$.

276. $\dfrac{25m^3 - 3n + 2p}{5m}$.

277. $\dfrac{8z^3 - 16z + 7y^2z^2}{8z}$.

278. $\dfrac{3c^2 - 9cd + w - 6c}{3c}$.

279. $\dfrac{A + B}{AB}$.

280. $\dfrac{4x^3 + 24x + 2}{4x}$.

281. $\dfrac{9q^2 + 18q - 4}{3q}$.

282. $\dfrac{32c^3 - 5x - 4}{4c}$.

283. $\dfrac{xy + yz + xz}{xyz}$.

284. $\dfrac{ax^2 - 2axy - a^2z^2}{a^2x^2}$.

285. $\dfrac{3^2 + 4^2 + 5^2}{2 \cdot 3 \cdot 4 \cdot 5}$.

286. $\dfrac{1 + x + x^2 + x^3}{x^3}$.

287. $\dfrac{A^3 - B^3}{A^{2 \cdot 3}B^{2 \cdot 3}}$.

288. $\dfrac{(x+y)^2}{x}$.

289. $\dfrac{(x-y)^3}{y}$.

290. $\dfrac{-7a^3 - 3a^2x + 2x^3}{-x}$.

291. $\dfrac{a(1-x) - b(1+x)}{ab}$.

292. $\dfrac{a^2(x-y) - b^2(x+y)}{ab}$.

293. $\dfrac{(a-c) + (p-q)}{(a-c)(p-q)}$.

294. $\dfrac{(x-1) - (y-1)}{(x-1)(y-1)}$.

295. $\dfrac{x}{x^{-1}}$.

296. $\dfrac{x^{-1}}{x}$.

297. $\dfrac{c^0}{c^{-1}}$.

298. $\dfrac{a^0}{a^2}$.

299. $\dfrac{4^{-3}}{4^{-2}}$.

300. $\dfrac{x^2}{x^{\frac{1}{2}}}$.

301. $\frac{p^{-\frac{1}{6}}}{p^{\frac{1}{3}}}$.

302. $\frac{q^{-2}}{q^{-5}}$.

303. $\frac{b^{-\frac{3}{4}}}{b^{-\frac{1}{4}}}$.

304. $\frac{x^{\frac{3}{5}}}{x^{-\frac{2}{5}}}$.

305. $\frac{x^2z^2}{y^2w}$.

306. $\frac{5cx^2}{3x^3y^2}$.

307. $\frac{3am^3}{3a^3m}$.

308. $\frac{m^3n}{x^{-1}y}$.

309. $\frac{b^3d^3}{x^{-3}d^{-2}}$.

310. $\frac{4a^{-3}c^3n^{-2}}{2a^{-2}c^{-2}}$.

Note carefully examples 311–323, and be able to solve such examples mentally.

311. $\frac{x+1}{x-1}$.

312. $\frac{x-1}{x+1}$.

313. $\frac{x+7}{x+3}$.

314. $\frac{x+7}{x-3}$.

315. $\frac{x-7}{x+3}$.

316. $\frac{x-7}{x-3}$.

317. $\frac{x+3}{x+7}$.

318. $\frac{x+3}{x-7}$.

319. $\frac{x-3}{x+7}$.

320. $\frac{x-3}{x-7}$.

321. $\frac{2x+5}{x+2}$.

322. $\frac{2x+5}{x-2}$.

323. $\frac{2x-5}{x-2}$.

324. $\frac{ax+2x^2}{a+x}$.

325. $\frac{x^2+1}{x+1}$.

326. $\frac{a^2-1}{a+1}$.

327. $\frac{2ab+2b^2}{a+b}$.

328. $\frac{ax^2y-ay}{x-1}$.

329. $\frac{a^2-ac-x}{a-c}$.

330. $\frac{c^2+cd+e}{c+d}$.

331. $\frac{m^2+n^2}{m+n}$.

332. $\frac{a^2+ax+x^2}{a+x}$.

333. $\dfrac{p^3q - pq^3}{p - q}$.

334. $\dfrac{x^3 - y^3}{x + y}$.

335. $\dfrac{mx + ny + 2nx}{m + n}$.

336. $\dfrac{4x^4y - 5}{2x^2y - 2}$.

337. $\dfrac{a + b + a^2 + 2ab + b^2}{a + b}$.

338. $\dfrac{e^2 + 2ef + f^2}{e - f}$.

339. $\dfrac{x^2 + 3x + 2}{x + 3}$.

340. $\dfrac{2a^2 - 6a - 1}{a - 3}$.

341. $\dfrac{5c^3 - 5d^3 + 2}{c - d}$.

342. $\dfrac{12a^4 + 19a^2 - 7a}{4a^2 + 1}$.

343. $\dfrac{8p^2 + 1}{2p + 3}$.

344. $\dfrac{q^3 - 3q - 5}{q - 1}$.

345. $\dfrac{a^2 + 3ax - 4}{a + x}$.

346. $\dfrac{x^9 - x^5}{x^3 - 1}$.

347. $\dfrac{m^{40} - n^4}{m^{10} - n}$.

348. $\dfrac{36c^7 - 36d^7}{9c - 9d}$.

349. $\dfrac{x^2 + 12x + 18}{x + 3}$.

350. $\dfrac{18 + 12x + x^2}{3 + x}$.

(NOTE: Compare carefully examples 349 and 350 and then their answers. Is the difference in their answers real or only apparent? Substitute 6 as the value of x in the answers and then compare results.)

351. $\dfrac{18x^3 - 3x^2 + 36}{3x^2 - 4x + 5}$.

352. $\dfrac{x^3 + ax^2 - 3a^2x - 3a^3}{x - 2a}$.

353. $\dfrac{a^3 - 2a^2}{a^2 - a + 1}$.

354. $\dfrac{a^4 + c^4}{a + c}$.

355. $\dfrac{x^5 + y^5}{x - y}$.

356. $\dfrac{a^3 - b^3}{a^2 - ab + b^2}$.

357. $\dfrac{1 - 2a^2}{1 + a}$.

358. $\dfrac{1 + 6a}{1 - 2a}$.

359. $\dfrac{1 + 7x}{1 - 6x}$

360. $\dfrac{1 - a}{1 + a^{-1}}$.

Reduce the following mixed expressions to fractions :

361. $5\frac{2}{3}$. 362. $15\frac{8}{17}$. 363. $8+\frac{2}{5}$. 364. $7-\frac{3}{4}$.

365. $\frac{94}{7}-8$. 366. $a+\frac{3a}{4}$. 367. $2x^2-\frac{4x^2}{3}$.

368. $5c-\frac{2c-3}{7}$. 369. $x+2y+\frac{3x-4y}{5}$.

370. $2m+4n-\frac{3m-8n}{3}$. 371. $\frac{5a-12x}{4}+6a+3x$.

372. $2a+3x-\frac{6a+9x}{3}$. 373. $4z-\frac{3y+8z}{3}$.

374. $a+b-\frac{3a-3b}{4}+1$. 375. $a+\frac{bc}{d}$.

376. $m+n+\frac{q^2}{m}$. 377. $h-k-\frac{z^2}{x}$. 378. $\frac{a-c}{c}+1$.

379. $2p+q-\frac{3q^2-2pq}{p}$. 380. $1+\frac{m-n-1}{n}$.

381. $5kl-\frac{kl^2-l^3}{l}$ 382. $1+3e-\frac{4g-h}{4g}$.

383. $4rs-\frac{t-x}{4r^2s^2}$. 384. $b+1+\frac{b+1}{b}$.

385. $2+3a-\frac{a-5}{4a}$. 386. $c-1+\frac{c-1}{c}$.

387. $a-1-\frac{a-1}{a}$. 388. $x-4+\frac{x+2}{3x}$.

389. $3b-\frac{1+2b^2}{b}$. 390. $3x-\frac{5xy-3}{2y}$.

391. $1-\frac{x^2+y^2-z^2}{2xy}$ 392. $c-\frac{x^2-y^2-c^2}{c}$.

393. $m - \dfrac{m(p-q)}{p}$.

394. $\dfrac{x^2-y^2}{2y} + (x-y)$.

395. $5pq + \dfrac{4pq-q^3}{q}$.

396. $(x-1)^2 - \dfrac{(x-1)^2}{x}$.

397. $2x - 3y + 4z + \dfrac{2x^2yz + 9xy^2z - 12xyz^2}{3xyz}$.

398. $c + \dfrac{1}{a^c}$.

399. $x + a + \dfrac{1}{a^x}$.

400. $\dfrac{x^{-2}}{y} + x^{-1} + y^{-1}$.

401. $a^{\frac{3}{2}} - a^{\frac{1}{2}} - \dfrac{1}{a^{\frac{1}{2}}}$.

402. $3x^{\frac{3}{4}} - \dfrac{1+2x^{\frac{3}{2}}}{x^{\frac{3}{4}}}$.

403. $1 - \dfrac{x}{x+y}$.

404. $\dfrac{c}{c-d} - 1$.

405. $1 + \dfrac{e}{f-h}$.

406. $3 + \dfrac{2}{a^2-1}$.

407. $\dfrac{m^2}{m+n} + n$.

408. $a - x + \dfrac{x^2}{a+x}$.

409. $b + d - \dfrac{d^2}{b-d}$.

410. $2x - 7 - \dfrac{4x^2-50}{2x+7}$.

411. $p + q - \dfrac{p^2-q^2-3}{p-q}$.

412. $a - x - \dfrac{a^2+x^2}{a+x}$.

413. $x + y - \dfrac{x^2+y^2}{x+y}$.

414. $p + q - \dfrac{p^2+q^2}{p-q}$.

415. $3m - 2n - \dfrac{3m^2-8n^2}{3m+2n}$.

416. $\dfrac{8a^2+7b^2}{3a-2b} - 2a - 3b$.

417. $\dfrac{x^2y-2z}{x+z} - xy + 2$.

418. $x + 5 - \dfrac{2x - 15}{x - 3}$.

419. $c^2 - cd + d^2 + \dfrac{2d^3}{c + d}$.

420. $x^3 + xy + y^2 - \dfrac{x^3 - y^3}{x - y}$.

421. $m + p - \dfrac{m^2 + p^2 + p}{m - p}$.

422. $q + 2 + \dfrac{q^2 + 4q + 4}{q - 2}$.

423. $c - \dfrac{2abc - 2b^2c}{a^2 - b^2}$.

424. $\dfrac{x^3z}{x^2 - z^2} - (x^3z^3 + x^1z)$.

425. $3c - 9 - \dfrac{3c^2 - 30}{c + 3}$.

426. $5k - 2t - \dfrac{3k^2 - 4t^2}{5k - 6t}$.

427. $\dfrac{2x^2}{x + y} - (x - y)$.

428. $a^2 + a + 1 + \dfrac{2}{a - 1}$.

429. $b^2 - 2bc + 4c^2 - \dfrac{6c^3}{b + 2c}$.

430. $x - a + y + \dfrac{a^2 - ay + y^2}{a + x}$.

431. $A - \dfrac{3}{4 + 5a^2}$.

432. $4Aa - \dfrac{A}{a + A}$.

433. $B - b - \dfrac{B^2 + b^2 - B - b}{B + b} - 1$.

434. $z + 2xy - 3xz - \dfrac{y^2z - 5xy^2z + x^3}{y^2 - yz}$.

435. $2m^2 - \dfrac{4m(m - 2)}{2m - 1} + 3m$.

436. $2s + 4st - 8rs + \dfrac{12rs^2 - 8rs^2t}{2s + 2rs}$.

437. $a^3 + 3ax + x^3 + \dfrac{9a^2x^2}{a^3 - 3ax + x^3}$.

438. $p - \dfrac{p^3 + q^3}{p^2 - pq + q^2} + q.$ 439. $\dfrac{(c^2-1)^2}{c^2+c+1} - (c^2-c+1).$

440. $2m + 5n + 9q + \dfrac{25n^2 - 81q^2}{2m - 5n + 9q}.$

Reduction to a Given Denominator.

(Note: In examples 441–475 the hyphen is to be read as follows; "is to be reduced to a fraction whose denominator shall be.")

441. $\frac{3}{4}$ - 12. 442. $\frac{7}{8}$ - 32. 443. 25 - 7.

444. $\frac{5}{9}$ - 27. 445. 9 - 13. 446. $\dfrac{5x}{8}$ - 32.

447. a - 3. 448. c - h. 449. $\dfrac{a}{c}$ - x.

450. $\dfrac{ax}{b}$ - aby. 451. 7 - $5x$. 452. $4y$ - 9.

453. $\dfrac{3cd}{2e}$ - $4c^2e^2f^2$. 454. $\dfrac{x}{y}$ - $ay + by$.

455. $\dfrac{a}{c}$ - $c^2 + cd$. 456. $x^2 - y + z$ - a^2x^2.

457. $p^2r^2 - q^2s^2$ - p^2q^2. 458. $\dfrac{a-x}{a+x}$ - $a^2 - x^2$.

459. $c + y$ - $c + y$. 460. $x^2 - y^2$ - $x - y$.

461. $a^2 + b^2$ - $a + b$. 462. $c^2d^2e^4$ - $x - y$.

463. $m^2 - n$ - $n^2 - m$. 464. $\dfrac{a+c}{a+2c}$ - $a^2 - 4c^2$.

465. $\dfrac{a+1}{a-1}$ - $a^4 - 1$. 466. $\dfrac{x}{x-y}$ - $y^2 - x^2$.

467. $\dfrac{k}{e+2z}$ - $4e^2z^2 - e^4$. 468. $\dfrac{5}{c-x}$ - $c^3 - x^3$.

469. $\frac{1}{x-2}$ $-x^2-7x+10$. 470. $\frac{x-2}{x-3}$ $-x^2-7x+12$.

471. $\frac{3x+2}{2x-5}$ $-6x^2-19x+10$.

472. $\frac{a^2+ax+x^2}{a+x}$ $-a^3+x^3$.

473. $\frac{1}{a-b-c}$ $-a^2-2ab+b^2-c^2$.

474. $\frac{c^2+cx+x^2}{c^2-cx+x^2}$ $-c^4+c^2x^2+x^4$.

475. $\frac{m-n-p}{m+n+p}$ $-m^2+2mp-p^2+n^2$.

Reduce to equivalent fractions having the lowest common denominator the following:

476. $\frac{2}{3}, \frac{3}{4}$.

477. $\frac{3}{5}, \frac{7}{8}$.

478. $\frac{3}{4}, \frac{5}{6}, \frac{7}{12}$.

479. $\frac{2}{7}, \frac{3}{8}, \frac{5}{16}$.

480. $\frac{2}{11}, \frac{3}{13}, \frac{7}{17}$.

481. $\frac{a}{3}, \frac{2a}{5}, \frac{7a}{10}$.

482. $\frac{x}{3}, \frac{y}{6}, \frac{z}{8}$.

483. $\frac{2a}{3}, \frac{3c}{5}, \frac{4x}{7}$.

484. $x, \frac{2}{3}, \frac{a}{4}, 5$.

485. $\frac{a}{2}, \frac{b}{3}, \frac{c}{4}, \frac{x}{y}$.

486. $\frac{2m}{n}, \frac{a}{c}$.

487. $\frac{x}{a}, \frac{3}{4}, \frac{z}{w}$.

488. $\frac{c}{2u}, \frac{d}{v^2}, \frac{e}{z}$.

489. $\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$.

490. $\frac{1}{2x}, \frac{1}{3y}, \frac{1}{4z}$.

491. $\frac{2c}{3x}, \frac{2 \cdot 3c^2}{3 \cdot 4c^3}$.

492. $\frac{3}{-x}, \frac{a}{-xy}, \frac{5c}{y}$.

493. $\frac{-x^3}{2 \cdot 3}, x, \frac{x^5}{2 \cdot 3 \cdot 4 \cdot 5}$.

494. $\frac{a}{3}, \frac{c}{a}, \frac{d}{a^2}$.

495. $\frac{a}{m}, \frac{b}{m^2}, \frac{c}{m^3}$.

496. $\frac{A^2}{a}, \frac{B^2}{b}, \frac{C^2}{c}$.

497. $\frac{2}{ax}, \frac{3}{by}, \frac{4}{a^2}$.

498. $f, \frac{f}{-g}, \frac{-f^2}{gh}$.

499. $\frac{x}{ab}, \frac{y}{ac}, \frac{z}{bc}$.

500. $\frac{cx}{ay^2}, \frac{cy}{ax^2}, \frac{xy}{a}$.

501. $\frac{x^2}{2ab}, \frac{y^3}{3xc}, \frac{z^3}{4bc}$.

502. $\frac{2ab}{3ac}, \frac{3}{4}, \frac{x}{a^2c}, \frac{1}{8}$.

503. $\frac{3x}{2y}, \frac{5b}{2c}, \frac{3y}{7x}, \frac{m}{n^2}$.

504. $\frac{2}{ab}, \frac{3}{2bc}, \frac{4}{3cd}, \frac{5}{4de}, \frac{6}{5ef}$.

505. $\frac{db}{a^2cx^2y}, \frac{ad}{c^2bx^2y^3}, 2, \frac{cx}{ax^2y^2}$.

506. $\frac{am}{a^2mx^3y^5}, \frac{-3}{-5}, \frac{cm}{ac^2y^4}, \frac{dx}{a^2x^4m^3}$.

507. $\frac{3x-7}{6}, \frac{4x-9}{18}$.

508. $\frac{a-3}{3}, \frac{a-2}{2}, \frac{a-1}{1}$.

509. $\frac{a+x}{3}, \frac{5a-1}{a}$.

510. $\frac{2a}{c}, x, \frac{c+1}{d}$.

511. $\frac{a-b}{ab}, \frac{a-c}{ac}, \frac{b-c}{bc}$.

512. $\frac{4a-5c}{5ac}, \frac{3a-2c}{12a^2c}$.

513. $\frac{a+2}{x}, \frac{a-3}{4}, \frac{x^2+4}{z}$.

514. $\frac{a-1}{a}, \frac{b-2}{b}, \frac{c-3}{c}$.

515. $\frac{x-y}{3y^2}, \frac{4}{3xy}, \frac{x+y}{4ax}$.

516. $\frac{a}{3}, \frac{x}{5}, \frac{1-x}{1+x}$.

517. $\frac{2a}{4}, 3, \frac{c+d}{c-d}$.

518. $\frac{2x}{a}, \frac{3y}{1-a}, \frac{4z}{a^2}$.

519. $\frac{3}{1+x}, \frac{3}{1-x}$.

520. $\frac{2}{c+z}, \frac{c-z}{cz}$.

521. $\frac{2}{m^2}, \frac{a}{m+n}$.

522. $\frac{1}{3}, \frac{a}{2}, \frac{-x^2}{a-x}$.

523. $\frac{a-b}{a}, \frac{ax}{a-b}, \frac{b}{c}$.

524. $\frac{2}{3}, \frac{a}{x^2}, \frac{p+q}{p-q}$.

525. $\frac{1}{1+c}, \frac{1}{1-c}, \frac{1}{1-c^2}$.

526. $\frac{a+x}{a-x}, 1, \frac{a-x}{a+x}$.

527. $\frac{2}{a-1}, \frac{3}{a+1}, \frac{4}{a^2-1}$.

528. $\frac{a}{x+2}, \frac{b}{x-2}, \frac{c}{x^2-4}$.

529. $\frac{3}{n-3}, \frac{4ac}{n^2-9}, \frac{5}{3+n}$.

530. $\frac{m}{x+1}, \frac{n}{2x+2}, \frac{p}{4x+4}$.

531. $\frac{3a}{6c^2+2c}, \frac{4a}{9c^2-1}$.

532. $\frac{2a^2}{3(x+y)}, \frac{mn}{6(x^2-y^2)}$.

533. $\frac{1}{a-x}, \frac{2}{x^2-a^2}$.

534. $\frac{c}{c+1}, \frac{1}{c-1}, \frac{1}{1-c^2}$.

535. $\frac{x}{1-x}, \frac{2x+1}{x^2-1}, \frac{1}{x^2-x}$.

536. $\frac{2}{x}, \frac{xy}{x^2+xy}, \frac{y^2}{x^2-y^2}$.

537. $\frac{x}{1-a}, \frac{y}{2a(1-a)}, \frac{z}{3a^2}$.

538. $\frac{8n+2}{n-2}, \frac{2n-1}{3n-6}, \frac{3n+2}{5n-10}$.

539. $\frac{2}{a}, \frac{3}{2a-1}, \frac{2a-3}{4a^2-1}$.

540. $\frac{a}{5(a-b)}, \frac{b^2}{(a-b)^2}, \frac{c^3}{(a-b)^3}$.

541. $\frac{a}{x^2-a^2}, \frac{b}{a^2-x^2}, \frac{-c}{x-a}, \frac{d}{x+a}$.

542. $\frac{1}{(a-b)(b-c)}, \frac{1}{(a-b)(a-c)}$.

543. $\frac{1}{ab(a-b)(a-c)}, \frac{1}{ac(a-c)(b-c)}.$

544. $\frac{m^2}{a^2-x^2}, \frac{n^2}{a^2+x^2}, \frac{m-n}{a^4-x^4}.$

545. $\frac{2x}{x^2-y^2}, \frac{4y^2}{x^2+y^2}.$

546. $\frac{a}{b(c-d)}, \frac{e}{a(d-c)}, \frac{1+a}{ab}.$

547. $\frac{ac-x}{ac+ax}, \frac{a-cx}{cx+x^2}, \frac{1}{a^2x^2}.$

548. $\frac{1}{m^2+7m+10}, \frac{1}{m^2+m-20}.$

549. $\frac{c}{c^2-1}, \frac{d}{c^2+2c-3}.$

550. $\frac{x-2}{x^2+x-6}, \frac{x+2}{x^2+5x+6}.$

551. $\frac{3}{2a-2}, \frac{4}{a^2-2a+1}, \frac{a}{1-a^2}.$

552. $\frac{1}{a-c}, \frac{3ac}{c^3-a^3}, \frac{c-a}{c^2+ca+a^2}.$

553. $\frac{a+b}{am-an-bm+bn}, \frac{m-n}{a^2-2ab+b^2}.$

554. $\frac{3}{2x^3-8x^2}, \frac{x}{x^2-7x+12}, \frac{x^2}{x^3-27}.$

555. $\frac{x+2}{x^2-x-12}, \frac{x+3}{x^2-6x+8}, \frac{x+4}{x^2+x-6}.$

ADDITION AND SUBTRACTION OF FRACTIONS.

Simplify the following expressions :

556. $\frac{1}{2}+\frac{2}{3}+\frac{3}{5}$.

557. $\frac{3}{4}+\frac{5}{6}+\frac{7}{8}$.

558. $\frac{9}{16}+7\frac{11}{12}+5+3\frac{3}{8}$.

559. $4\frac{1}{2}+\frac{2}{3}-2\frac{3}{4}$.

560. $-\frac{7}{8}+\frac{7}{4}-\frac{7}{2}+1$.

561. $\frac{x}{2}+\frac{x}{3}+\frac{x}{4}$.

562. $\frac{c}{2}+\frac{5}{6}c+\frac{3c}{8}$.

563. $\frac{3m}{4}+\frac{n}{5}+\frac{p}{3}$.

564. $\frac{-4}{2}+\frac{-16}{7-3}$.

565. $\frac{4d}{7}+\frac{d-2}{5}$.

566. $a+\frac{a+x}{2}+\frac{a-x}{2}$.

567. $\frac{a}{3}+\frac{a}{4}+\frac{a}{12}+\frac{a}{18}+\frac{a}{6}+\frac{a}{9}$.

568. $\frac{a+b}{2}+\frac{a-b}{2}$. What results from adding half the sum of two quantities to half their difference?

569. $\frac{x}{7}+\frac{5x}{9}+\frac{4x}{11}$.

570. $\frac{x+y}{3}+\frac{x-y}{6}+\frac{x+y}{12}$.

571. $\frac{2b+c}{d}+\frac{3b-c}{d}$.

572. $\frac{3a+4c}{x}+\frac{4a-2c}{x}$.

573. $\frac{a+b}{a^2}+\frac{a-b}{a\cdot a}$.

574. $\frac{3ax}{2mn}+\frac{5ax}{2mn}+\frac{8ax}{2mn}+\frac{7ax}{2mn}$.

575. $\frac{a}{c} + \frac{x}{-y}$

576. $5x + \frac{a}{b} + 4x + \frac{c}{b}$.

577. $\frac{1}{x} + \frac{1}{2x} + \frac{1}{3x}$.

578. $\frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3}$.

579. $5a + \frac{x}{y} + \frac{-z}{2}$.

580. $\frac{a}{xy} + \frac{b}{xz} + \frac{c}{yz}$.

581. $\frac{1}{a} + \frac{1}{b} + \frac{1}{c}$.

582. $\frac{h}{k} + \frac{m}{n} + \frac{p}{q}$.

583. $\frac{x}{2y} + \frac{a-b}{3y} + \frac{b-a}{4y}$.

584. $\frac{k-l}{kl} + \frac{l-m}{lm} + \frac{m-k}{mk}$.

585. $\frac{e}{ax} + \frac{f}{ay} + \frac{g}{xy}$.

586. $\frac{1+x}{x} + \frac{1-x}{y}$.

587. $\frac{a}{b^2c} + \frac{b}{c^2d} + \frac{c}{cd^2}$.

588. $\frac{4}{ef} + \frac{5}{fg} + \frac{6}{eg}$.

589. $\frac{y}{z^2} + \frac{z}{x^2} + \frac{x}{y^2}$.

590. $\frac{a-b}{ab} + \frac{b-c}{bc} + \frac{c-a}{ca}$.

591. $\frac{1}{x} - \frac{1}{y}$.

592. $\frac{3a}{4} - \frac{3a}{8}$.

593. $\frac{a}{x} - \frac{b}{y}$.

594. $\frac{k}{z} - l$.

595. $3bc - \frac{5bd}{c}$.

596. $a - \frac{b}{c} - \frac{3d}{2}$.

597. $\frac{a}{c} - \left(-\frac{c}{a}\right)$.

598. $\frac{acx^3}{ac^3x^3} - \frac{a^2c^4x}{ac^2mx^5}$.

599. $\frac{m-n}{2} - \frac{n-m}{2}$.

600. $\frac{a+x}{2x} - \frac{a-x}{2x}$.

601. $\frac{5x-2y}{2} - \frac{3x-5y}{3}$.

602. $\frac{6a-5b}{3} - \frac{4a-3b}{2}$.

603. $\frac{x-y}{x \cdot x} - \frac{x+y}{x^2}$.

604. $\frac{5c}{x} - \left(\frac{c-x}{x} + \frac{3c+2x}{x}\right)$.

605. $\frac{3p-5q}{pq} + \frac{-p+2q}{pq} - \frac{p-4q}{pq}$.

606. $\frac{6xy-2xz}{w} - \left(\frac{3xy-5xz}{w} - \frac{7xz-2xy}{w}\right)$.

607. $\frac{a-b}{y} - \left(-\frac{a}{y}\right)$.

608. $a + \frac{c}{2} - \frac{b-c}{3}$.

609. $4x + \frac{m}{n} - \left(3x - \frac{n}{m}\right)$.

610. $x - \frac{y-c}{2} - \left(\frac{x-y}{3} - x\right)$.

611. $-\frac{a-x}{3c} - 1 - \frac{x-a}{2c}$.

612. $\frac{7a-10}{5} - \frac{3a-7}{6} - \frac{27a-30}{30}$.

613. $2y + \frac{2+7y}{8} - \left(y - \frac{5y-6}{21}\right)$.

614. $\frac{1}{2x} - \frac{1}{3x} - \frac{1}{4x} + \frac{1}{5x}$.

615. $\frac{7}{12}a + \frac{10}{21}c - \frac{1}{7}c - \frac{1}{4}a$.

616. $\frac{4}{5}x + \frac{1}{14}z - \frac{3}{10}x - \frac{4}{7}z$.

617. $2\frac{a-c}{3} - 3\frac{a+c}{4}$.

618. $5\frac{m-2n}{6} - 3\frac{n-2m}{4}$.

619. $\frac{3a-2c}{3} - \frac{22c-9a}{15} - \frac{4c+2a}{5}$.

620. $\frac{3p+2q}{p} + \frac{2p^2-2q^2}{pq} - \frac{2p+3q}{q}$.

621. $\frac{2}{3a} - \frac{1}{2b} - \frac{2a+3}{6a^2} + \frac{1}{2x^2} + \frac{3a-2b}{6ab}$.

622. $\frac{3(2a^3+1)}{2a^3c^4} - \frac{2(3a^3x^4+2)}{5a^3x^5} - \frac{3(5x-2c^4)}{5c^4x}$.

623. $\frac{1}{x-y} - \frac{1}{x+y}$. 624. $\frac{a}{x} - \frac{a}{x+1}$. 625. $\frac{1}{c-1} - 1$.

626. $\frac{1}{1-a^2} - \frac{1}{1-a}$. 627. $\frac{a}{2a-2b} - \frac{b}{2(a-b)}$.

628. $\frac{a-x}{(a+x)^2} - \frac{1}{a+x}$. 629. $\frac{a+c}{a-c} - \frac{a-c}{a+c}$.

630. $\frac{a}{m-n} - \frac{b}{p+q}$. 631. $1 - \frac{a+x}{x+a}$.

632. $\frac{x}{x-1} - \frac{1-2x}{x^2-x}$. 633. $\frac{p^2+pq+q^2}{p^2-q^2} - \frac{p-q}{p+q}$.

634. $\frac{c}{c-2} - \frac{c+2}{c}$. 635. $\frac{1}{e-f} - \frac{1}{(e-f)^2}$.

636. $\frac{a^2-b^2}{a-b} - \frac{(a+b)^2}{a+b}$. 637. $\frac{1+2x}{1-2x} - \frac{1-2x}{1+2x}$.

638. $\frac{x}{2x-2y} - \frac{y}{2y-2x}$. 639. $\frac{2a+3c}{2a-3c} - \frac{2a-3c}{3c-2a}$.

640. $\frac{x+a}{x-2a} - \frac{x^2+2a^2}{x^2-4a^2}$. 641. $\frac{a-4}{a-2} - \frac{a-7}{a-5}$.

642. $\frac{1}{2c+x} - \frac{(2c-x)^2}{8c^3+x^3}$.

643. $\frac{p^2+pq+q^2}{p+q} - \frac{p^2-pq+q^2}{p-q}$.

644. $\frac{a^3+x^3}{a^2-ax+x^2} - \frac{a^3-x^3}{a^2+ax+x^2}$.

645. $\dfrac{m^2+3n^2+4mn}{m^2+n^2+2mn}-2.$ 646. $\dfrac{x}{x-1}-1-\dfrac{1}{x(x-1)}.$

647. $\dfrac{1}{c+d}-\dfrac{c}{c^2+d^2}+\dfrac{d}{c^2-d^2}.$

648. $\dfrac{x^2}{x^2-1}+\dfrac{x}{x+1}-\dfrac{x}{1-x}.$

649. $\dfrac{1}{2-a}-\dfrac{2}{2+a}+\dfrac{3-a}{4-a^2}.$

650. $\dfrac{a^2}{a^2-1}-\dfrac{a}{a+1}+\dfrac{a}{a-1}.$

651. $\dfrac{3+2k}{2-k}+\dfrac{3k-2}{k+2}+\dfrac{16k-k^2}{4-k^2}.$

652. $\dfrac{x-a}{x-b}+\dfrac{x-b}{x-a}-\dfrac{(a-b)^2}{(x-a)(x-b)}.$

653. $\dfrac{3}{1+2x}-\dfrac{4(1-5x)}{4x^2-1}-\dfrac{7}{2x-1}.$

654. $\dfrac{a^2+c^2}{ac}-\dfrac{a^2}{ac+c^2}-\dfrac{c^2}{a^2+ac}.$

655. $\dfrac{m(16-m)}{m^2-4}+\dfrac{2m+3}{2-m}-\dfrac{2-3m}{m+2}.$

656. $\dfrac{a-1}{a-2}+\dfrac{a-3}{a^2-4}-\dfrac{a+1}{a+2}.$

657. $\dfrac{x+10}{8-2x^2}-\dfrac{1}{x+2}+\dfrac{3}{2x-4}.$

658. $\dfrac{5}{1+2k}-\dfrac{3k}{1-2k}-\dfrac{4-13k}{1-4k^2}.$

659. $\dfrac{a+b}{a-b}-\dfrac{a-b}{a+b}+\dfrac{4b^2}{a^2-b^2}.$

660. $\frac{x^2+7}{x+5} - \frac{3x^2+20}{3x+15} - \frac{x-5}{9}$.

661. $\frac{3-a}{1-3a} - \frac{3+a}{1+3a} - \frac{1-16a}{9a^2-1}$.

662. $\frac{x^2-2x}{x^2-1} - \frac{x+3}{x+1} - \frac{4x}{1-x}$.

663. $\frac{q+2}{q-2} - \frac{q-2}{q+2} - \frac{16}{q^2-4}$.

664. $\frac{1}{x+1} - \frac{1}{x+2} - \frac{3}{(x+1)(x+2)}$.

665. $\frac{1}{(x+1)(x+2)} - \frac{1}{(x+1)(x+2)(x+3)} - \frac{1}{(x+1)(x+3)}$.

666. $\frac{1}{(x-2)(x-3)} + \frac{1}{(x-1)(x-2)} + \frac{2}{(x-1)(3-x)}$.

667. $\frac{bc}{(c-a)(a-b)} + \frac{ab}{(b-c)(c-a)} + \frac{ac}{(a-b)(b-c)}$.

668. $\frac{1}{(x-3)(x-4)} - \frac{2}{(x-2)(x-4)} + \frac{1}{(x-2)(x-3)}$.

669. $\frac{a^2}{(a-b)(a-c)} + \frac{b^2}{(b-c)(b-a)} + \frac{c^2}{(c-a)(c-b)}$.

670. $\frac{a^2-b}{(a-b)(a-1)} + \frac{a+b^2}{(b-a)(b+1)} - \frac{ab+1}{(a-1)(b+1)}$.

671. $\frac{a^2}{(a^2-b^2)(a^2-c^2)} + \frac{b^2}{(b^2-c^2)(b^2-a^2)} + \frac{c^2}{(c^2-a^2)(c^2-b^2)}$.

672. $\frac{x^2-yz}{(x-y)(x-z)} + \frac{y^2+xz}{(y-x)(y+z)} + \frac{z^2+xy}{(z-x)(z+y)}$.

673. $\dfrac{a^2+ac+c^2}{(a-b)(b-c)} - \dfrac{a^2+ab+b^2}{(a-c)(c-b)} - \dfrac{b^2+bc+c^2}{(a-c)(b-a)}$.

674. $\dfrac{a-2}{a-3} - \dfrac{a-3}{a-2} + \dfrac{1}{a^2-5a+6}$.

675. $\dfrac{a+5}{a+3} - \dfrac{a-3}{a-5} + \dfrac{a^2-a-20}{a^2-2a-15}$.

676. $\dfrac{x+1}{x-2} - \dfrac{x-3}{x+4} - \dfrac{18}{x^2+2x-8}$.

677. $\dfrac{x-5}{x+4} - \dfrac{x^2+47}{x^2+x-12} + \dfrac{x+4}{x-3}$.

678. $\dfrac{1}{x^2+4x-12} - \dfrac{1}{x^2-3x-54}$.

679. $\dfrac{3}{a^2+7a+12} - \dfrac{2}{a^2+2a-8}$.

680. $\dfrac{x}{x^2-7x-18} - \dfrac{x}{x^2+7x+10}$.

681. $\dfrac{m}{a^2-6ac+9c^2} - \dfrac{m}{a^2+4ac-21c^2}$.

682. $\dfrac{c+1}{c^2+3c+2} + \dfrac{c+2}{c^2+5c+6}$.

683. $\dfrac{1}{x^2-7x+12} + \dfrac{2}{x^2-4x+3} - \dfrac{3}{x^2-5x+4}$.

684. $\dfrac{m-1}{m^2-7m+10} - \dfrac{m+2}{m^2-9m+14} - \dfrac{m-3}{m^2-12m+35}$.

685. $\dfrac{a+2}{a^2+4a+3} - \dfrac{2(a-1)}{a^2+a-6} + \dfrac{a-3}{a^2-a-2}$.

686. $\dfrac{4-e}{e^2-7e+10} - \dfrac{e}{e^2-8e+15} + \dfrac{3e+2}{e^2-5e+16}$.

687. $\frac{2}{a+c}+\frac{3c}{(a+c)^2}-\frac{2a-3c}{a^2-2ac-3c^2}$.

688. $\frac{1}{m-n}+\frac{mn}{m^3-n^3}-\frac{3m}{m^2-n^2}$.

689. $\frac{a}{a-x}+\frac{a^2}{a^2+ax+x^2}-\frac{2x^3}{a^3-x^3}$.

690. $\frac{2}{n+2}-\frac{n-3}{n^2-2n+4}-\frac{14}{n^3+8}$.

691. $\frac{2}{x+4}-\frac{x-3}{x^2-4x+16}+\frac{x^2}{x^3+64}$.

692. $\frac{2rs^2}{r^3+s^3}+\frac{s}{r+s}-\frac{s^2}{r^2-rs+s^2}$.

693. $\frac{p}{p+q}-\frac{p^2}{p^2-pq+q^2}+\frac{2p^2q}{p^3+q^3}$.

694. $\frac{1}{x+1}-\frac{x}{x^2-x+1}+\frac{x^2-4}{x^3+1}$.

695. $\frac{d}{c-d}+\frac{c^3}{c^2+cd+d^2}-\frac{c^3+d^3}{c^3-d^3}$.

696. $\frac{1}{c-3}+\frac{c-1}{c^2+3c+9}-\frac{2c^2-c+12}{c^3-27}$.

697. $\frac{x^2+2x+28}{8-x^3}-\frac{x+6}{x^2+2x+4}+\frac{3}{x-2}$.

698. $\frac{1}{(c-1)^2+3c}-\frac{1}{1-c^3}-\frac{1}{c-1}$.

699. $\frac{1}{8-8a}-\frac{1}{8+8a}+\frac{a}{4+4a^2}-\frac{a}{2+2a^4}$.

700. $\frac{2x}{x^4-x^2+1}-\frac{1}{x^2-x+1}+\frac{1}{x^2+x+1}$.

701. $\dfrac{b+1}{b^2+b+1}+\dfrac{b-1}{b^2-b+1}+\dfrac{2}{b^4+b^2+1}$.

702. $\dfrac{p+q}{2p+2q+4}-\dfrac{2}{p^2+2pq+2p+2q+q^2}$.

703. $\dfrac{a+3}{2a-1}-\dfrac{a^2-5}{4a^2-4a+1}-\dfrac{2a^3-a(1-5a)-1}{8a^3-12a^2+6a-1}$.

704. $\dfrac{a^2-(b-c)^2}{(a+c)^2-b^2}+\dfrac{b^2-(a-c)^2}{(a+b)^2-c^2}+\dfrac{c^2-(a-b)^2}{(b+c)^2-a^2}$.

Multiplication of Fractions.

705. $\frac{3}{16}\times 5$.

706. $\frac{3}{20}\times 4$.

707. $\frac{6}{7}\times 21$.

708. $\frac{3}{10}\times 15$.

709. $(3:7)\times 2$.

710. $(11:15)\times 5$.

711. $3\times\frac{4}{5}$.

712. $\frac{3}{7}\times\frac{5}{8}$.

713. $\frac{4}{9}\times\frac{5}{16}$.

714. $7\times\dfrac{8}{21}$.

715. $\dfrac{1}{a}\times\dfrac{1}{x}$.

716. $\dfrac{4cn}{15}\times 5$.

717. $\dfrac{4}{3y}\times\dfrac{6}{y}$.

718. $\dfrac{2r}{s^2}\times 3t$.

719. $\dfrac{z}{w}\times\dfrac{p}{q}$.

720. $\dfrac{hk}{x}\times\dfrac{a}{k}$.

721. $\dfrac{ax}{cz}\times z$.

722. $p^2q\times\dfrac{1}{mp}$.

723. $\dfrac{3}{a^3}\times\dfrac{b^2}{u}$.

724. $2rs\times\dfrac{2s}{r}$

725. $\dfrac{9}{2c}\times 4cd$.

726. $(2:3)\times(7:9)$.

727. $(m:n)\times(p:q)$.

728. $\dfrac{ay^2}{6x}\times 3x$.

729. $\dfrac{3m}{cx}\times\dfrac{c}{3}$.

730. $\dfrac{bcd}{axy}\times\dfrac{xdy}{bd}$.

731. $\dfrac{amx}{bny}\times\dfrac{bcy}{adx}$.

732. $\dfrac{2uv}{5k^2l^3}\times kl^2$.

733. $\dfrac{3a^2}{10x}\times\dfrac{5x^3}{9ay^2}$.

734. $\dfrac{2ab}{6cd}\times\dfrac{cm}{ax}$.

735. $\left(\dfrac{2a}{3x}\right)^2$.

736. $\left(\dfrac{2a}{5c}\right)^3$.

737. $\left(\frac{5ax}{7cy}\right)^2$.

738. $\left(\frac{2a^2b^3c^4}{3x^5}\right)^2$

739. $\frac{2a^2}{41c^3x} \times \frac{11cx^3}{5a^2z}$

740. $-\frac{3cx}{5ay} \times -\frac{2c}{3y^3}$.

741. $-\frac{2ax}{3by} \times -\frac{5a^2}{7by^2}$

742. $\frac{2mx}{3a^2b^3} \times 3a$.

743. $\frac{5c}{8x^{-3}} \times \frac{2a}{ac^{-4}}$.

744. $\frac{8x^4y}{15ab^3} \times \frac{2x^3}{3ab^2}$.

745. $\frac{8a^2b^3}{45x^2y} \times \frac{15xy^2}{24a^3b^2}$.

746. $\frac{9x^2y^2z}{10a^2b^2c} \times -\frac{20a^3b^2c}{18xy^2z}$.

747. $\frac{x}{2} \times \frac{4x}{5} \times \frac{10x}{21}$.

748. $\frac{2x}{a} \times \frac{3ab}{c} \times \frac{3ac}{2b}$.

749. $\frac{a}{b^2} \times \frac{b^3}{c^3} \times \frac{c^3}{d^4}$.

750. $\frac{5a}{12} \times p \times \frac{4y}{7}$.

751. $\frac{2a}{x} \times \frac{3b}{c} \times -\frac{4ac}{d}$.

752. $\frac{2a}{3b} \times \frac{4b}{3c} \times \frac{5c}{6a}$.

753. $\frac{3x^2y}{4xz^2} \times \frac{5y^2z}{6xy} \times -\frac{12x^3}{2xy^2}$.

754. $\frac{9k^2l^2}{8p^3q^3} \times \frac{5p^2q}{2xy} \times \frac{24x^2y^2}{90kl}$.

755. $\frac{23a^2c^3}{51x^3y^4} \times \frac{17x^2y^2}{ac^4} \times -cx$.

756. $\frac{21c^4m^3}{13x^2y^3} \times \frac{18y^4c^2}{49x^3y^3} \times 91$.

757. $\frac{18a^2bcx}{35c^3mn} \times \frac{21c^5abm}{27a^3c^3x} \times \frac{25a^5c^2x^3}{30a^8cx^3}$.

758. $-\frac{12m^2c^3x}{y^2} \times \frac{15y^2}{m^2cx^3} \times \frac{x^2y^3}{144a^4c^2x}$.

759. $\frac{24c^3m^2n}{132abx^2y} \times \frac{121a^3m^2x}{55m^3y^3} \times \frac{5y^3x^2}{12mm}$.

760. $-\frac{25x^3y^2z}{132m^4c^2q} \times \frac{11u^2zy^6}{50x^3y} \times \frac{24c^2ux}{18y^2q}$.

761. $\frac{1}{4}(6a - 10)$.

762. $\frac{2}{5}\left(\frac{3c + 4}{6}\right)$

763. $\frac{2}{3}\left(\frac{3m + 18}{2}\right)$.

764. $\frac{4}{9}\left(\frac{6 + 3e}{8}\right)$.

765. $\frac{8x}{2 + x} \times (x + 2)^2$.

766. $\frac{4}{3 - a}(a - 3)(2 - a)$.

767. $\frac{1}{2x - 1}(2 - x)(1 - 2x)$.

768. $\frac{a + c}{c} \times \frac{c^2x}{a^2 - c^2}$.

769. $\frac{a}{m + n} \times \frac{m^2 - n^2}{ay}$.

770. $\frac{3c + 3d}{x} \times \frac{x^2}{c + d}$.

771. $\frac{p^2 - 4}{6} \times \frac{3}{2 - p}$.

772. $\frac{ax + ay}{bx - by} \times \frac{b}{a}$.

773. $\left[a + \frac{2a^2}{ab}\right] \times \frac{2ab}{a^2}$.

774. $\left[a - \frac{x^2}{a}\right] \times \frac{a^2}{a + x}$.

775. $[k^4 - 1] \times \frac{c}{k^2 - 1}$.

776. $[e^6 + f^6] \times \frac{1}{e^2 + f^2}$.

777. $\frac{3x + 3y}{bc} \times \frac{ab}{lx + ly}$.

778. $\left(\frac{x + y}{x - y}\right)^2$.

779. $\left[h + \frac{g}{h}\right]^2$.

780. $\frac{x(a - x)}{(a + x)^2} \times \frac{a(a + x)}{(a - x)^2}$.

781. $\frac{(x + y)^3}{3a} \times \frac{3a^2}{x + y}$.

782. $\frac{m^4 - n^4}{m^2 - 2mn + n^2} \times \frac{m - n}{m^2 + mn}$.

783. $\frac{a^2 + ax + x^2}{a^2 - ax + x^2} \times \frac{a - x}{a + x}$.

784. $\frac{(a + c)^5}{(x + z)^4} \times \frac{(x + z)^5}{(a + c)^6}$.

785. $\frac{x^2 + xy}{x - y} \times \frac{(x - y)^2}{x^4 - y^4}$.

786. $\frac{a^2 - b^2}{r^3 + s^3} \times \frac{r + s}{b - a}$.

787. $\frac{2x-y}{4x} \times \frac{6x-2y}{y^2-2xy}$.

788. $\frac{4a-2b}{b^2-2ab} \times \frac{2a-b}{6a}$.

789. $\left[x-\frac{y^2}{x}\right] \times \left[\frac{x}{y}+\frac{y}{x}\right]$.

790. $\frac{(a+b)^2}{a-b} \times \frac{1}{a+b}$.

791. $\frac{a-b}{a} \times \frac{a+b}{b} \times \frac{a^2}{a^2-b^2}$.

792. $\frac{1-x^2}{1+y} \times \frac{1-y^2}{x+x^2} \times \left[1+\frac{x}{1-x}\right]$.

793. $\frac{c^2}{d^2} \times \frac{a^2-b^2}{c} \times \frac{d}{a+b}$.

794. $[x^2+x-2] \times \frac{x+1}{x+2}$.

795. $[y^2-7y+10] \times \frac{2y+5}{y-2}$.

796. $\left[1+\frac{x-y}{x+y}\right] \times \left[1-\frac{y^2}{x^2}\right]$.

797. $\left[\frac{y}{x+y}\right]\left[\frac{1}{x-y}\right]\left[\frac{x^2-y^2}{x}\right]$.

798. $\frac{a+x}{a^2+ax+x^2} \times \frac{a^3-x^3}{a^2-x^2} \times \frac{a+x}{a-x}$.

799. $\frac{k^2-ks+s^2}{k^3-3k(k-s)s-s^3} \times \frac{k^2-s^2}{k^3+s^3}$.

800. $\frac{3x^2}{5x-10} \times \frac{30x-60}{4x}$.

801. $\frac{3x^2-x}{5} \times \frac{10}{2x^2-4x}$.

802. $\frac{x^2+4x}{x^2-3x} \times \frac{4x^2-12x}{3x^2-12x}$.

803. $\left(-\frac{a}{x^2-y^2}\right)\left(-\frac{(x+y)^2}{a^5}\right)\left(-\frac{3b}{x+y}\right)$.

804. $\frac{c^2-4}{c^2+5c} \times \frac{c^2-25}{c^2+2c}$.

805. $\dfrac{m^2+3m+2}{m^2-5m+6}\times\dfrac{m^2-7m+12}{m^2+m}$.

806. $\dfrac{x^3-4x}{x^3+7x^2}\times\dfrac{x^3+7x}{x-4}$.

807. $\dfrac{a^2-b^2}{(a+b)^2}\times\dfrac{a^2+ab}{4(a^2-ab)}$.

808. $\dfrac{z^2+4z+3}{z^2+4z+4}\times\dfrac{z^2+7z+10}{z^2+8z+15}$.

809. $\dfrac{n^2+3n+2}{n^2+9n+20}\times\dfrac{n^2+7n+12}{n^2+5x+6}$.

810. $\dfrac{a^2+5a+6}{a^2-1}\times\dfrac{a^2-2a-3}{a^2-9}$.

811. $\dfrac{p^3-6p^2+36p}{p^3-49}\times\dfrac{p^2-p-42}{p^4+216p}$.

812. $\dfrac{x^2-2x-3}{(x+3)^2}\times\dfrac{x^2-9}{x^2-x-12}$.

813. $\dfrac{a^2-2ac-3c^2}{(a+c)^3}\times\dfrac{a^3+c^3}{a^2-4ac+3c^2}$.

814. $\dfrac{a^2-4a+3}{a^2-5a+4}\times\dfrac{a^2-9a+20}{a^2-10a+21}\times\dfrac{a^2-7a}{a^2-5a}$.

815. $\dfrac{x^3-7x+6}{x^3+3x-4}\times\dfrac{x^2\times10x+24}{x^2-14x+48}\times\dfrac{x^3-8x^2}{x^2+6x}$.

816. $\dfrac{a^2-x^2}{a+b}\times\dfrac{a^2-b^2}{ax+x^2}\times\left(a+\dfrac{ax}{a-x}\right)$.

817. $\left(\dfrac{1-b^2}{1+c}\right)\left(\dfrac{1-c^2}{1+b}\right)\left(1+\dfrac{b}{1-b}\right)$.

818. $\left[1+\dfrac{x}{a}\right]\left[\dfrac{a^2}{a^2-x^2}\right]\left[\dfrac{a}{x}-1\right]$.

819. $\left(a+\dfrac{3x^2}{a}\right)\left(\dfrac{a^2}{3x^2}-1\right)\left(\dfrac{x^2}{a}\right)$.

820. $\frac{p^4-1}{16p^4-9p^2} \times \frac{4p+3}{2p^2+2} \times \left(4p \times \frac{p}{p-1}\right)$.

821. $\frac{2(x-2a)^3}{x(x^2+4a^2)} \times \frac{x^3+8a^3}{3a(x^2-4a^2)} \times \frac{x^4+5x^2a^2+4a^4}{x^4-4ax^3+4a^2x^2}$.

822. $\frac{x^4+9x^3-36x^2}{x^2a^2-10xa^2+9a^2} \times \frac{x^4-4x^2+3}{x^4-7x^2-18} \times \frac{a^2x^2+2a^2}{x^4+x^3}$.

823. $\frac{x^2+2xy+y^2-z^2}{x^2-2xy+y^2-z^2} \times \frac{x-y+z}{x+y+z}$.

824. $\frac{x^2-16y^2-9z^2+24yz}{x^2+16y^2-9z^2+8xy} \times \frac{x^2-16y^2-9z^2-24yz}{x^2+16y^2-9z^2-8xy}$.

Division of Fractions.

825. $5 \div \frac{1}{2}$.

826. $\frac{2}{3} \div 4$.

827. $\frac{2}{3} \div \frac{4}{5}$.

828. $4 \div 5\frac{1}{3}$.

829. $2\frac{3}{4} \div 11$.

830. $3\frac{1}{2} \div 2\frac{1}{3}$.

831. $5 \div (2:3)$.

832. $(5:7) \div 3$.

833. $(5:6) \div (4:3)$

834. $1 \div \frac{4a}{5x}$.

835. $\frac{5x}{7} \div 2$.

836. $\frac{2a}{3x^2} \div \frac{1}{3x}$.

837. $\frac{2m}{n} \div \frac{3b}{5c}$.

838. $\frac{a}{3} \div \frac{2a}{9}$.

839. $\frac{6p^2q}{t} \div 3pq$.

840. $\frac{15rst}{mnp} \div 3rs$.

841. $\frac{3m^2x}{4a^2b} \div 3x$.

842. $aby \div \frac{cx}{dm}$.

843. $\frac{35bcd}{27y} \div 9xy$.

844. $\frac{14x^2}{27y^2} \div \frac{7x}{9y}$.

845. $\frac{5a^2c^2y}{22a^5c^2x} \div \frac{15a^3cy^2}{11a^4x^2}$.

846. $\frac{13a^2bcd^2}{21a^4c^3x} \div 121a^5c^3xy^2$.

847. $\frac{25a^2bcx^2}{15c^2dx} \div 5abcx$.

848. $-\frac{33m^2npq^3}{125x^2yu} \div 121m^2npq$.

849. $183c^2dx^3y \div \frac{61am^2dx}{2a^3c^2dy}$.

850. $\frac{21c^3dax^6}{32abcx^2y} \times \frac{a^2cdx^2}{7abcy} \div \frac{3ac^2dx^2}{16b^3y^2}$.

851. $\frac{225r^3sdx}{51s^3pq} \div \left(\frac{15r^3s^3}{11a^2c} \times \frac{22c^2x}{3cx^2}\right)$.

852. $-\frac{12a^5x}{15a^3y^2} \div \frac{am^3}{c^2x} \times \frac{18ab^2y^3}{72ax^2} \div -\frac{5m^3}{abc^2}$.

853. $\frac{9x^2-3x}{5} \div \frac{x^2}{5}$.

854. $\frac{ab+bx}{x} \div \frac{a}{x}$.

855. $\frac{x^2-xy}{y^2} \div \frac{xy-y^2}{x^2}$.

856. $\frac{2b^2}{a^3-b^3} \div \frac{b}{a-b}$.

857. $\frac{15ab}{a-x} \div \frac{10ac}{a^2-x^2}$.

858. $\frac{a^4-c^4}{(a-c)^2} \div \frac{a+c}{a-c}$.

859. $\frac{15a^2-15x^2}{x-y} \div 5(a+x)$.

860. $\frac{x^2-1}{x^2+3} \div x-1$.

861. $\frac{a^3-n^3}{a+n} \div a-n$.

862. $\frac{a}{m^2-4} \div m^2+4$.

863. $\frac{g^2-2gh+h^2}{g+h} \div g^2-h^2$.

864. $\left[1+\frac{1}{x}\right] \div \left[1-\frac{1}{x}\right]$.

865. $\frac{4a(a^2-x^2)}{3b(c^2-x^2)} \div \frac{a^2-ax}{bc+bx}$.

866. $\frac{x+1}{x-1} \div \frac{x+1}{1-x^2}$.

867. $\frac{x^2+xy}{x-y} \div \frac{x^4-y^4}{(x-y)^2}$.

868. $\frac{x^3+y^3}{x^2-y^2} \div \frac{x^2-xy+y^2}{x-y}$.

869. $\frac{3(x^2-1)}{2(a+b)} \div \left(\frac{x+1}{2a} \times \frac{x-1}{a+b}\right)$.

870. $\frac{a}{c} \times \frac{b}{a} \div \left(\frac{b}{c} \times \frac{x}{y}\right)$.

871. $\left[x - \frac{xy}{x+y}\right] \div \left[x + \frac{xy}{x-y}\right]$.

872. $\frac{(x+3)(x+5)}{(x-1)(x-4)} \div \frac{(x-1)(x+3)}{(x-4)(x+5)}$.

873. $\frac{a^4 - k^4}{a^2 - 2ak + k^2} \div \frac{a^2 + ak}{a-k}$.

874. $\frac{x^4 - y^4}{(x-y)^2} \div \frac{x^2 + xy}{x-y}$.

875. $\left[\frac{a}{a+b} + \frac{b}{a-b}\right] \div \left[\frac{a}{a-b} - \frac{b}{a+b}\right]$.

876. $\left(x^2 + \frac{1}{x^2} + 2\right) \div \left(x + \frac{1}{x}\right)$.

877. $\frac{x^2 - 9}{x^2 - 16} \div \frac{x^2 + x - 6}{x^2 - 3x - 4}$.

878. $\frac{x^2 + x - 6}{x^2 + 11x + 30} \div \frac{x^2 + 8x + 15}{x^2 + 4x - 12}$.

879. $\frac{p^2 + 3p + 9}{p^4 - 3p^2 + 9} \div \frac{p^3 - 27}{p^6 + 27}$.

880. $\frac{x^3 - 8}{x^2 + 7x + 10} \div \frac{x^3 + 2x + 4}{x^2 + 2x}$.

881. $\frac{a^2 + 4ac + 4c^2}{a - c} \div \frac{ac + 2c^2}{a^2 - ac}$.

882. $\frac{y^2 - y}{y^3 + 1} \div \frac{y^2 - 2y + 1}{y^3 - y^2 + y}$.

883. $\frac{p^2 - 5pq - 14q^2}{p^2 + 5pq - 24q^2} \div \frac{p^2 - 3pq - 28q^2}{p^2 - 8pq + 15q^2}$.

884. $\left(5 - \frac{c^2 - 19e^2}{c^2 - 4e^2}\right) \div \left(3 - \frac{c - 5e}{c - 2e}\right)$.

885. $\dfrac{a^2-2bc-b^2-c^2}{a^2-c^2+2bc-b^2} \div \dfrac{a-b-c}{a-c+b}$.

886. $\dfrac{1-a^2}{1+x} \div \dfrac{x^3-1}{a^2} \times \dfrac{1-x^2}{a+a^2}$.

887. $\dfrac{n^2+2n-3}{n^2-2n-3} \times \dfrac{n^3+1}{n^3-1} \div \dfrac{n^2+4n+3}{n^2-4n+3}$.

Complex Fractions.

Simplify the following:

888. $\dfrac{\frac{1}{2}}{\frac{3}{4}}$.

889. $\dfrac{\frac{3}{5}}{6}$.

890. $\dfrac{2}{\frac{7}{8}}$.

891. $\dfrac{2\frac{1}{3}}{3\frac{2}{5}}$.

892. $\dfrac{1+\frac{3}{4}}{3-\frac{2}{3}}$.

893. $\dfrac{\frac{1}{3}}{\frac{2}{x}}$.

894. $\dfrac{\frac{2}{x}}{5}$.

895. $\dfrac{\frac{3}{a}}{7}$.

896. $\dfrac{5\frac{2}{3}}{\frac{a}{4}}$.

897. $\dfrac{\frac{a}{c}}{2\frac{5}{6}}$.

898. $\dfrac{\frac{a}{b}}{\frac{a}{c}}$.

899. $\dfrac{\frac{c}{x}}{\frac{x}{c}}$.

900. $\dfrac{\frac{ax}{c}}{\frac{cx}{a}}$.

901. $\dfrac{\frac{p}{q}}{\frac{h}{k}}$.

902. $\dfrac{4\frac{1}{7}}{\frac{r}{s}}$.

903. $\dfrac{\frac{5}{8}a}{\frac{1}{2}x}$.

904. $\dfrac{2\frac{1}{2}a}{3\frac{2}{5}e}$.

905. $\dfrac{\frac{2x^3y}{3xy^2}}{\frac{6x^2y}{9xy^2}}$.

906. $\dfrac{\dfrac{a^2b}{m^3n}}{\dfrac{ab^3}{mn^3}}$.

907. $\dfrac{\dfrac{225x^3}{4}}{256\frac{1}{8}}$.

908. $\dfrac{1}{x+\frac{1}{3}}$.

909. $\dfrac{1}{a-\dfrac{1}{c}}$.

910. $\dfrac{a+\dfrac{c}{x}}{3}$.

911. $\dfrac{2a}{x+\frac{3}{4}}$.

912. $\dfrac{n+\dfrac{1}{n}}{y}$.

913. $\dfrac{\dfrac{a+b}{c}}{\dfrac{a}{a-b}}$.

914. $\dfrac{\dfrac{x}{a+c}}{\dfrac{a-c}{y}}$.

915. $\dfrac{a+\dfrac{a}{y}}{x+\dfrac{y}{4}}$.

916. $\dfrac{1+\dfrac{1}{a}}{1-\dfrac{1}{a}}$.

917. $\dfrac{x}{n+\dfrac{p}{q}}$.

918. $\dfrac{\dfrac{a+b}{d}}{\dfrac{x-y}{c}}$.

919. $\dfrac{\dfrac{3-m}{n}}{5-\dfrac{p}{q}}$.

920. $\dfrac{\dfrac{4}{e-f}}{e+f}$.

921. $\dfrac{4\frac{3}{5}}{1+\dfrac{h}{k}}$.

922. $\dfrac{a+\dfrac{c}{x}}{\dfrac{d}{y}-b}$.

923. $\dfrac{1-c^2}{1+\dfrac{1}{c}}$.

924. $\dfrac{1+\dfrac{1}{a}}{1+a}$.

925. $\dfrac{\dfrac{22abc}{39klx}}{\dfrac{11ab}{3kx}}$.

926. $\dfrac{\dfrac{x+1}{x-1}}{\dfrac{x-1}{x+1}}$.

927. $\dfrac{\dfrac{m-n}{p+q}}{\dfrac{p-q}{m+n}}$.

928. $\dfrac{\dfrac{a^2-c^2}{a+c}}{a-c}$.

929. $\dfrac{a}{g+h^{-1}}$.

930. $\dfrac{e^{-1}f}{h^{-1}+k}$.

931. $\dfrac{1-\dfrac{m}{m-n}}{1-\dfrac{m}{m+n}}$.

932. $\dfrac{a+\dfrac{1}{a}}{\dfrac{1}{a}\times\dfrac{x}{a}}$.

933. $\dfrac{z+\frac{2}{3}w}{u-\frac{1}{3}y}$.

934. $\dfrac{\dfrac{a-x}{a+x}}{\dfrac{a^2-x^2}{a^3+x^3}}$.

935. $\dfrac{\dfrac{2p+q}{p+q}-1}{1-\dfrac{q}{p+q}}$.

936. $\dfrac{h+k+\dfrac{k^2}{h}}{h+k+\dfrac{h^2}{k}}$.

937. $\dfrac{m+\dfrac{n-m}{1+mn}}{1-m\left(\dfrac{n-m}{1+mn}\right)}$.

938. $\dfrac{\dfrac{r^3-s^3}{x^2-y^2}}{\dfrac{r^2+rs+s^2}{x-y}}$.

939. $\dfrac{\dfrac{p^3-q^3}{p^2-q^2}}{\dfrac{p^2+pq+q^2}{p+q}}$.

940. $\dfrac{\dfrac{x^3-y^3}{x^3+y^3}}{\dfrac{x^2+xy+y^2}{x^2-xy+y^2}}$.

941. $\dfrac{x-2+\dfrac{x^2}{2}}{x-\dfrac{4}{x}+2}$.

942. $\dfrac{m+n-\dfrac{20}{m}}{m-n-\dfrac{12}{m}}$.

943. $\dfrac{x+1+\dfrac{6}{x-6}}{x-2+\dfrac{3}{x-6}}$.

944. $\dfrac{a-1-\dfrac{21}{a+3}}{a-2-\dfrac{14}{a+3}}$.

945. $\dfrac{c-2+\dfrac{6}{c+3}}{c-4+\dfrac{12}{c+3}}$.

946. $\dfrac{m-2-\dfrac{6}{m+3}}{m-4+\dfrac{6}{m+3}}$.

947. $\dfrac{z+5-\dfrac{4z+6}{z-6}}{z+3-\dfrac{8z}{z-6}}$.

948. $\dfrac{2c-3+\dfrac{1}{c}}{\dfrac{2c-1}{c}}$.

949. $\dfrac{2n^2-n-6}{\dfrac{4}{n^2}-1}$.

950. $\dfrac{x^2+\dfrac{16}{x^2}-17}{x+3-\dfrac{4}{x}}$.

951. $\dfrac{\dfrac{1}{a}+\dfrac{1}{x}}{\dfrac{1}{a}-\dfrac{1}{x}}$.

952. $\dfrac{\dfrac{1}{m}+\dfrac{1}{n}}{\dfrac{m}{n}+\dfrac{n}{m}}$.

953. $\dfrac{\dfrac{r}{s}-\dfrac{s}{r}}{r-s}$.

954. $\dfrac{\dfrac{a}{c}-\dfrac{c}{a}}{\dfrac{1}{c}-\dfrac{1}{a}}$.

955. $\dfrac{\dfrac{p}{q}-r}{\dfrac{1}{pq}-\dfrac{r}{p^2}}$.

956. $\dfrac{1-\dfrac{c^3}{z^3}}{\dfrac{1}{c^3}-\dfrac{c}{z^3}}$.

957. $\dfrac{\dfrac{1}{m^2}-\dfrac{1}{n^2}}{\dfrac{n}{m}-\dfrac{m}{n}}$.

958. $\dfrac{\dfrac{h}{k}-\dfrac{p}{q}}{\dfrac{h}{k}+\dfrac{p}{q}}$.

959. $\dfrac{\dfrac{x}{x+a}+\dfrac{x}{x-a}}{\dfrac{2x}{x^2-a^2}}$.

960. $\dfrac{\dfrac{a+c}{c}-\dfrac{c}{a+c}}{1-\dfrac{a}{a+c}}$.

961. $\dfrac{\dfrac{1}{1-a}-\dfrac{1}{1+a}}{\dfrac{a}{1-a}+\dfrac{1}{1+a}}$.

962. $\dfrac{\frac{1}{3}(e-f)}{\frac{2}{3}e+\frac{3}{5}f}$.

963. $\dfrac{2\frac{1}{4}-\frac{3}{5}(c+2)}{1\frac{1}{3}+\frac{1}{2}(c-3)}$.

964. $\dfrac{a-\frac{4}{5}}{2}-\dfrac{a}{1\frac{1}{5}}$.

965. $\dfrac{w+5+\dfrac{6}{w}}{1+\dfrac{6}{w}+\dfrac{8}{w^2}}$.

966. $\dfrac{\dfrac{1}{a}-\dfrac{2}{a^2}-\dfrac{3}{a^3}}{\dfrac{9}{a}-a}$.

967. $\dfrac{\dfrac{1}{a}-\dfrac{3}{a^2}-\dfrac{4}{a^3}}{a-\dfrac{16}{a}}$.

968. $\dfrac{\dfrac{c}{d^2}-\dfrac{d}{c^2}}{\dfrac{1}{c^2}+\dfrac{1}{cd}+\dfrac{1}{d^2}}$.

969. $\dfrac{\dfrac{a}{c}-\dfrac{c}{a}}{\dfrac{(a+c)^2}{ac}-4}$.

970. $\dfrac{\dfrac{2a}{3x}-2+\dfrac{3x}{2a}}{\dfrac{2}{x}-\dfrac{3}{a}}$.

971. $\dfrac{\dfrac{m}{n}+1-\dfrac{20n}{m}}{\dfrac{m}{n}-2-\dfrac{8n}{m}}$.

972. $\dfrac{\dfrac{c^3}{d^2}+\dfrac{8d}{c}}{\dfrac{c}{d}-2+\dfrac{4d}{c}}$.

973. $\dfrac{\dfrac{x+y}{x}+\dfrac{x-y}{y}}{\dfrac{x-y}{x}-\dfrac{x+y}{y}}$.

974. $\dfrac{\dfrac{p}{p+q}-\dfrac{p}{p-q}}{\dfrac{p}{p-q}+\dfrac{p}{p+q}}$.

975. $\dfrac{\dfrac{r}{s}-\dfrac{r-s}{r+s}}{\dfrac{s}{r}+\dfrac{r-s}{r+s}}$.

976. $\dfrac{\dfrac{c+k}{c-k}-\dfrac{c-k}{c+k}}{\dfrac{4ck}{c^2-k^2}}$.

977. $\dfrac{\dfrac{a-v}{a+v}+\dfrac{a+v}{a-v}}{\dfrac{a-v}{a+v}-\dfrac{a+v}{a-v}}$.

978. $\dfrac{\dfrac{1+2x}{1-2x}-\dfrac{1-2x}{1+2x}}{\dfrac{1-2x}{1+2x}+\dfrac{1+2x}{1-2x}}$.

979. $\dfrac{a-c-\dfrac{2c(a-c)}{a+c}}{\dfrac{a^2+c^2}{ac+c^2}-1}$.

980. $\dfrac{\dfrac{a+x}{a-x}-\dfrac{a^2+x^2}{a^2-x^2}}{\dfrac{a^2}{a+x}-\dfrac{a(a^2+x^2)}{(a+x)^2}}$.

981. $1+\dfrac{1}{2+\frac{1}{3}}$.

982. $1+\dfrac{2}{3+\frac{4}{5}}$.

983. $2+\dfrac{3}{4-\frac{5}{6}}$.

984. $2-\dfrac{2}{3-\frac{2}{7}}$.

985. $3-\dfrac{1}{2+\dfrac{2}{3-\frac{7}{8}}}$.

986. $1-\dfrac{1}{1+\dfrac{1}{a}}$.

987. $1+\dfrac{1}{1+\dfrac{2}{c-2}}$.

988. $1-\dfrac{1}{1+\dfrac{3}{2x-3}}$.

989. $\dfrac{1}{1+\dfrac{1}{1+\dfrac{1}{m}}}$.

990. $\dfrac{x}{y-\dfrac{yz}{x+z}}$.

991. $3-\dfrac{2}{5+\dfrac{4}{7+\dfrac{6}{x}}}$.

992. $1-\cfrac{1}{2-\cfrac{2}{1-\cfrac{2}{3y}}}$.

993. $2+\cfrac{1}{1-\cfrac{a}{a+x}}$.

994. $2a-\cfrac{2a}{1-\cfrac{3a}{x+3a}}$.

995. $x-\cfrac{a}{2-\cfrac{4x}{a+2x}}$.

996. $2c-\cfrac{4c}{3-\cfrac{c}{c+2d}}$.

997. $1+\cfrac{x}{1-\cfrac{2x}{1-x}}$.

998. $3-\cfrac{1}{4-\cfrac{8}{2-\cfrac{a}{x}}}$.

999. $\cfrac{x}{1-\cfrac{1-x}{1-\cfrac{x^2}{2-x}}}$.

1000. $\cfrac{1-c^2}{2-\cfrac{c}{1-\cfrac{1}{1+c}}}$.

1001. $\cfrac{4}{1+\cfrac{4}{1+\cfrac{4}{1-a}}}$.

1002. $\cfrac{m-2}{m-2-\cfrac{m}{m-\cfrac{m-1}{m-2}}}$.

1003. $\cfrac{p^2-1}{2p^2-\cfrac{4p^2-1}{1+\cfrac{p}{p-1}}}$.

1004. $\cfrac{3x-2y}{3x-2y-\cfrac{3x}{1-\cfrac{3(x-y)}{3x-2y}}}$.

1005. $3 - \cfrac{1}{2 - \cfrac{1}{1 - \cfrac{2a - \frac{1}{2}}{5a}}}$.

1006. $\cfrac{\cfrac{x}{1 + \cfrac{1}{x - 1}}}{1 + \cfrac{x}{x^2 - \cfrac{1}{1 - \cfrac{x - 1}{x}}}}$.

1007. $(a^6 - x^6)\cfrac{a - \cfrac{x}{1 - \cfrac{a}{x}}}{a + \cfrac{x}{1 + \cfrac{a}{x}}}$.

1008. $\cfrac{\cfrac{\frac{1}{n}}{1 + \frac{1}{n}} + \cfrac{1 - \frac{1}{n}}{\frac{1}{n}}}{\cfrac{\frac{1}{n}}{1 + \frac{1}{n}} - \cfrac{1 - \frac{1}{n}}{\frac{1}{n}}}$.

MISCELLANEOUS EXAMPLES IN FRACTIONS, UNGRADED.

Simplify the following:

1009. $\cfrac{1}{x - 1} - \cfrac{1}{x + 1} + \cfrac{1}{x - 2} - \cfrac{1}{x + 2}$.

1010. $\cfrac{a^4 - 16}{a^4 - 4a^3 + 8a^2 - 16a + 16}$.

1011. $\cfrac{a^{\frac{1}{2}}b^{\frac{1}{2}}}{xy^{\frac{3}{2}}} \div \cfrac{x^2y}{a^2b}$.

1012. $\cfrac{1 - 2c}{1 - c^3} \div \cfrac{1 - 2c + c^2 - 2c^3}{1 + 2c + 2c^2 + c^3}$.

1013. $\cfrac{1}{p - q} - \cfrac{1}{p + q} + \cfrac{2m}{p^2 - q^2} - \cfrac{2m}{p^2 + q^2}$.

1014. $\cfrac{(x + y + z)^2 - (x - y - z)^2}{3x(y^2 + 2yz + z^2)}$.

1015. $\dfrac{b}{x+a^{-1}}$.

1016. $\dfrac{(a+b+c)^2-(a+b-c)^2}{(a-b+c)^2-(a-b-c)^2}$.

1017. $\dfrac{a+x}{a-x}-\left[\dfrac{x-a}{x+a}-\left(\dfrac{a^2+x^2}{a^2-x^2}+\dfrac{4ax}{a^2+x^2}\right)\right]$.

1018. $\dfrac{(a-c)^3+(c-e)^3+(e-a)^3}{(e-a)(a-c)(c-e)}$.

1019. $\dfrac{1}{m-1}-\dfrac{1}{m+1}+\dfrac{3m^2}{m^3+1}-\dfrac{3m^2}{m^3-1}$.

1020. $\dfrac{20a^{\frac{1}{2}}b^{-}}{27xy^2}\times\dfrac{33x^{\frac{1}{2}}y^{-1}}{40a^{\frac{3}{2}}b}$.

1021. $\dfrac{m^2-6mn+9n^2}{m^2-4mn+4n^2}\div\left(\dfrac{m^2-9n^2}{m^2-4n^2}\times\dfrac{m^2-mn-6n^2}{m^2+mn-6n^2}\right)$.

1022. $\dfrac{3a^2y^3}{5b^{\frac{1}{2}}x^{\frac{3}{2}}}\div 5ab^{\frac{1}{2}}x^{\frac{1}{2}}$.

1023. $\dfrac{a^4-2a^2x^2+x^4}{a^4+2a^2x^2+x^4}\div\left(\dfrac{a^2-x^2}{ax}\div\dfrac{(a+x)^2-2ax}{ax}\right)$.

1024. $\dfrac{bc(b-c)+ca(c-a)+ab(a-b)}{(b-c)(c-a)(a-b)}$.

1025. $\left(1-\dfrac{cd}{c^2-cd+d^2}\right)\left(1-\dfrac{cd}{c^2+2cd+d^2}\right)\times\dfrac{c^3+d^3}{c^3-d^3}$.

1026. $\dfrac{x^2y^2}{b^2c^2}+\dfrac{(x^2-b^2)(b^2-y^2)}{b^2(c^2-b^2)}+\dfrac{(c^2-x^2)(c^2-y^2)}{c^2(c^2-b^2)}$.

1027. $\dfrac{2}{a-b}+\dfrac{2}{b-c}+\dfrac{2}{c-a}+\left(\dfrac{(a-b)^2+(b-c)^2+(c-a)^2}{(a-b)(b-c)(c-a)}\right)$.

1028. $\dfrac{ax(x^2-4ax+4a^2)}{6(x^4+7x^2a^2+12a^4)}\times\dfrac{8(x^4+3a^2x^2-4a^4)}{a^2(x^2-4a^2)}\div\dfrac{x^4-x^2a^2}{x^2a^2+3a^4}$.

1029. $\dfrac{a-b}{a+b}+\dfrac{b-c}{b+c}+\dfrac{c-a}{c+a}+\dfrac{(a-b)(b-c)(c-a)}{(a+b)(b+c)(c+a)}$.

1030. $\left(\dfrac{a^6-x^6}{a^2-2ax+x^2}\times\dfrac{1}{a-x}\right)\div\left(\dfrac{a^2+ax+x^2}{a-x}\times[a^2-ax+x^2]\right)$.

1031. $\dfrac{x^2-(y-z)^2}{(x+z)^2-y^2}+\dfrac{y^2-(x-z)^2}{(x+y)^2-z^2}+\dfrac{z^2-(x-y)^2}{(y+z)^2-x^2}$.

1032. $\left(a-\dfrac{2a}{x+\dfrac{1}{x}}\right)\div\left(\dfrac{x}{2}+\dfrac{1}{2x}-1\right)$.

1033. $\dfrac{x^6+64a^3}{(x^2+4a)^3}\times\dfrac{2x^5+16x^4a+32x^3a^2}{6x^4a^2+6x^3a^4}\div\dfrac{x^4-4x^3a+16a^2}{3a^2(x^2+a^2)}$.

1034. $\dfrac{\dfrac{3x}{2}+\dfrac{x-1}{3}}{\dfrac{13}{6}(x+1)-\dfrac{x}{3}-2\frac{1}{2}}$.

1035. $\left(1+\dfrac{8ab+7}{a^2-4ab+4b^2-16}\right)\times\dfrac{a^2-16-2b(a-4)}{a^2-9+2b(a-3)}$
$\div\left(1-\dfrac{1}{a-3}\right)$.

In the next six examples perform the operation as if the quantities were integral.

1036. $\left(\dfrac{x}{a}-\dfrac{a}{x}+\dfrac{y}{b}-\dfrac{b}{y}\right)\times\left(\dfrac{x}{a}-\dfrac{a}{x}-\dfrac{y}{b}+\dfrac{b}{y}\right)$.

1037. $\left(\dfrac{3a^2}{2x^2}+\dfrac{3a}{10x}-\dfrac{4}{15}\right)\div\left(\dfrac{a}{x}-\dfrac{1}{3}\right)$.

1038. $\left(\dfrac{5x^2}{2}+3ax-\dfrac{7a^2}{3}\right)\times\left(2x^2-ax-\dfrac{a^2}{2}\right)$.

1039. $\left(\dfrac{x^3}{3}-\dfrac{17x^2}{36}+\dfrac{x}{3}-\dfrac{1}{8}\right)\div\left(\dfrac{2x}{3}-\dfrac{1}{2}\right)$.

1040. $\left(x^4 - \frac{19}{6}a^2x^2 + \frac{a^3x}{3} + \frac{a^4}{6}\right) \div \left(x^2 - 2ax + \frac{a^2}{2}\right)$.

1041. $\left(\frac{9a^2b^2}{4c^2} - \frac{25f^2m^2}{g^2} + \frac{70dfm}{g} - 49d^2\right) \div \left(\frac{3ab}{2c} + \frac{5fm}{g} - 7d\right)$.

1042. $\dfrac{\frac{1}{a} + \frac{1}{b+c}}{\frac{1}{a} - \frac{1}{b+c}} \left(1 + \frac{b^2 + c^2 - a^2}{2bc}\right)$.

1043. $\dfrac{\frac{m^2+n^2}{n} - m}{\frac{1}{n} - \frac{1}{m}} \times \frac{m^2 - n^2}{m^2 + n^2}$.

1044. $\dfrac{x}{x-y} - \dfrac{x}{x+y} - \dfrac{\frac{x+y}{x-y} - \frac{x-y}{x+y}}{\frac{x+y}{x-y} + \frac{x-y}{x+y}}$.

1045. $\left(\frac{x^{-2}y^3}{x^3y^{-2}} \times \frac{y^3x^{-3}}{x^3y^{-3}}\right)^{-1}$.

1046. $\dfrac{x - 7x^{\frac{1}{2}}}{x - 5x^{\frac{1}{2}} - 14} \div \left(1 + \frac{2}{x^{\frac{1}{2}}}\right)$.

1047. $\left[\left(\frac{4a^{-\frac{1}{4}}x^3}{9b^{-3}y^{\frac{1}{2}}}\right)^{-\frac{1}{3}}\right]^2$.

1048. $\dfrac{1 + \frac{a-x}{a+x}}{1 - \frac{a-x}{a+x}} \div \dfrac{1 + \frac{a^2-x^2}{a^2+x^2}}{1 - \frac{a^2-x^2}{a^2+x^2}}$.

1049. $\left[\left(\frac{a^{-\frac{1}{10}}x^{18}}{b^3y^{-\frac{3}{5}}}\right)^{-\frac{1}{2}}\right]^{-\frac{5}{3}}$.

1050. $\dfrac{x+\dfrac{1}{y}}{x+\dfrac{1}{y+\dfrac{1}{z}}} - \dfrac{1}{y(xyz+x+z)}$.

1051. $\left(\dfrac{y^{-3}}{x^{\frac{2}{3}}z^{-1}}\right)^{-\frac{3}{2}} \times \left(\dfrac{y^{\frac{14}{3}}x^{-1}}{z^{-\frac{21}{4}}}\right)^{\frac{2}{7}}$.

1052. $\dfrac{1-\dfrac{2xy}{(x+y)^2}}{1+\dfrac{2xy}{(x-y)^2}} \div \left(\dfrac{1-\dfrac{y}{x}}{1+\dfrac{y}{x}}\right)^2$.

1053. $\dfrac{1}{x^{\frac{1}{4}}+x^{\frac{1}{2}}+1} + \dfrac{1}{x^{\frac{1}{4}}-x^{\frac{1}{2}}+1} - \dfrac{2x^{\frac{1}{4}}}{x^{\frac{1}{2}}-x^{\frac{1}{4}}+1}$.

1054. $\dfrac{\dfrac{(1-2a)^2+(2a+1)^2}{(1-4a)^2-(1-2a)^2}}{\dfrac{(1+2a)^2-(1-4a)^2}{(1-2a)^2-(2a+1)^2}}$.

1055. $\dfrac{(a+b-c)^2-d^2}{(a+b)^2-(c+d)^2} + \dfrac{(b+c-a)^2-d^2}{(b+c)^2-(a+d)^2} + \dfrac{(c+a-b)^2-d^2}{(c+a)^2-(b+d)^2}$.

EQUATIONS WITH ONE UNKNOWN QUANTITY.

Suppose A has 2 ten-cent pieces and B 4 fives, which has the more money? Now 1 ten equals how many fives? If you add 1 ten to A's and 2 fives to B's, what is the result? If you subtract the same, how much will each have? If you multiply both tens and fives by 2? If you divide each man's number of pieces by 2?

Express on the board A's money and B's as equal to each other and call them "equals." Call things or quantities connected by the sign = "equals."

Beneath what you have written express, also, 1 ten as equal to 2 fives. Add these equals. Result? Subtract them. Result? Multiply the first "equals" by 4. By 7. Result? Divide them by 2. By 5. Result?

Make other sets of "equals" and try the same processes. (For example: $20 = 14 + 6$; $5 + 3 = 17 - 9$.) Read your work. Make your induction and formulate it.

Square $7 = 4 + 3$. Result? Try other "equals." Result?

Cube $3 = 5 - 2$. Result? Try other "equals." Result? Induction?

Square root of $25 = 49 - 28 + 4$? Result? Cube root of $216 = 64 + 96 + 48 + 8$? Result? Try other "equals." Result? Induction? Give all the inductions you have made. These inductions are called "Axioms." Sum all these axioms into one single expression. (Like operations upon equal quantities give equal results.)

When quantities are written as "equals," the resulting expression is called an "Equation," and the "equals," the "Members."

By addition or subtraction make two equations from each of the following :

$16 = 9 + 7$; $8 + 5 = 13$; $15 = 6 + 9$; $23 - 10 = 13$;
$14 = 20 - 6$; $14 - 21 = -7$; $17 - 8 = 9$.

Make as many as you have time for from

$$8 - 5 = 9 - 6. \quad 19 - 7 = 10 + 2.$$

Let a and b represent any two numbers, s their sum and d their difference. Make two equations from these. Subtract a, then b from both members of the first equation.

Add a, then b to both members of the second equation, then examine carefully the results in each case and formulate a rule for transposition.

Transpose

1. $x - b = a.$
2. $5 = 7 - x.$
3. $4x + 2 = 3x + 9.$
4. $7x - 6 = 6x - 4.$
5. $2y - a = b.$
6. $5z + 3 = 2z + 24.$
7. $4x + 2a = 3x + 7b.$
8. $11x + 9 = 6x + 34.$
9. $3b + 2x - 25 = ax.$
10. $2x - 15 = -7x + 30.$
11. $ax + bc = m - 2x.$
12. $3ac - cd + xy = 6ad - 7x.$
13. $3 + 2x - 5 = 12 - 4x.$
14. $4ab - ax - 2c = cx - 3m.$
15. $100 + 4x - 6 = 5x + 5 - 30x.$
16. $4ax - b + 2c = 3x - 2ab - 3nx.$
17. $ab - dx - 2mx = 3ax - 4b.$
18. $-5a - 4b - 3c = -ax - ad.$
19. $x^3 - 13 - 3c = ab - 2x^2 + 4p.$
20. $5x + 2a + 3c - 2x = 4x - 2x + 4a.$
21. $\frac{2x}{3} - 1 = \frac{x}{12} - \frac{4}{3}.$
22. $\frac{3x^2}{a} - \frac{5a}{6} = \frac{13}{3} + 3a - \frac{x^2}{2}.$

23. $\frac{15}{2} - \frac{3}{2}x + 10 - \frac{7}{8} = \frac{1}{2} - \frac{11}{5}x + x.$

24. $3ab - \frac{2a^2}{c} - 4ax + 18bx^2 = \frac{2ax}{5} - x^2.$

25. $x - \frac{a}{c} - \left(\frac{x}{3} - a\right) + \frac{x}{4} - \frac{a}{20} = 0.$

26. $x - a = (a - b)x.$

27. $2x - 5(1 - x) - 8 = 0.$

28. $(x^2 - 16)9 = 10x^2 - 108.$

29. $2x - n = 4(bx + \frac{3}{4}).$

30. $(a + x)(a - x) = b^2 - 24.$

Clearing of Fractions.

Multiply $\frac{3}{4}$ by 4; $\frac{8}{9}$ by 9; $\frac{11}{17}$ by 17; $x/5$ by 5; $3y/16$ by 16.

What is the result of multiplying a fraction by its denominator?

By what must the following fractions be multiplied that the product may equal their numerators?

$$\frac{5a}{x};\quad \frac{4c}{y^2};\quad \frac{xy}{ab};\quad \frac{9z^2}{5x^3};\quad \frac{3x}{9-5}.$$

How, then, can an equation be cleared of fractions? By what must the following expressions be multiplied in order to free them from denominators?

$$\frac{x}{2} + \frac{3x}{4};\quad \frac{5a}{4} - \frac{5a}{12};\quad \frac{3}{ac} - \frac{4}{ab};\quad \frac{a}{c^3x} + \frac{b}{cx^3};\quad \frac{1}{x+1} + \frac{1}{x-1};$$

Clear from fractions the following equations:

31. $\frac{3x}{7} = 6.$

32. $2x + \frac{3x}{4} = 11.$

33. $\frac{2x}{3} + \frac{5x}{6} = 9.$

34. $\frac{x}{2} + \frac{2x}{3} + 9 = 30.$

35. $\frac{x}{2} - \frac{x}{5} = 16.$

36. $\frac{2x}{3} - \frac{x}{6} = 6x - 66.$

37. $3x - \frac{x-2}{5} = 20.$

38. $\frac{3x}{4} - \frac{3x}{5} = 13\frac{1}{2}.$

39. $\frac{2x}{3} - 1 = \frac{x}{12} + \frac{4}{3}.$

40. $\frac{3x}{5} - \frac{2x}{3} = \frac{3}{7}.$

41. $\frac{2x}{5} + \frac{3x}{4} = \frac{7}{10}.$

42. $\frac{x}{2} + \frac{x}{4} + \frac{x}{6} = 10.$

43. $\frac{4x}{7} - \frac{3x}{14} = \frac{8}{21}.$

44. $\frac{3x}{4} + 12 = \frac{5x}{6} + 9.$

45. $\frac{2x}{7} - \frac{3x}{4} + \frac{x}{5} = 6.$

46. $\frac{x}{5} + \frac{x}{4} + \frac{x}{3} = \frac{x}{2} + 17.$

47. $\frac{4a - 5b}{x} = -\frac{3b}{a}.$

48. $3x - \frac{7x}{5} = a - \frac{2b + c}{10}.$

49. $\frac{43 - 7y}{5} = \frac{69 - 9y}{11}.$

50. $\frac{2x-1}{3} - \frac{x+4}{9} = \frac{5x-1}{27}.$

51. $\frac{x-1}{2} - \frac{x-2}{3} + x = \frac{3x-1}{6} + \frac{4-x}{3}.$

52. $\frac{4}{x} + \frac{14}{x} = 2.$

53. $\frac{x}{a} = b + c.$

54. $a + \frac{1}{x} = b + c + \frac{d}{x}.$

55. $m + x^{-1} = q - px^{-1}.$

56. $\frac{3}{x} + \frac{5}{2x} + \frac{5}{8} = \frac{8}{x}.$

57. $\frac{a-x}{b} - \frac{4a-x}{c} = a - b^{-1}.$

58. $\frac{2x}{3a} - \frac{x-1}{2ab} + \frac{3}{a^2} = 1 - x.$

59. $x - 12 = \frac{2}{5}(44 - x + 12).$

60. $\frac{2x}{3} - 6\frac{2}{3} = \frac{4}{9}(30 - x)$.

61. $\frac{3}{2}(x-1) - \frac{2}{3}(x+2) + \frac{1}{4}(x-3) = 4$.

62. $\frac{4}{x-8} = 1$.

63. $\frac{36}{5x+8} = \frac{4}{x}$.

64. $\frac{3}{x-4} = \frac{6}{x+4}$.

65. $\frac{30}{2x+5} = \frac{8}{x-1}$.

66. $\frac{x^2}{x-4} = x + 6$.

67. $\frac{2x+3}{3x+9} = \frac{2x-8}{3x-13}$.

68. $\frac{4}{3+x} - \frac{1}{x} = \frac{9}{7x}$.

69. $\frac{a-b}{x-c} = \frac{a+b}{x+2c}$.

70. $\frac{2x}{a-2b} = 3 + \frac{x}{2a-b}$.

Solve the following equations;

71. $7x = 28$.

72. $4y = 18$.

73. $35 = 5x$.

74. $6w = 25$.

75. $0 = 8x$.

76. $6y = 3$.

77. $0 = -2y$

78. $4z = -12$.

79. $30 = -5v$.

80. $-12 = -4x$.

81. $0x = 15$.

82. $-11x = -4$.

83. $x + 18 = 38$.

84. $x - 2 = 5$.

85. $12 = 20 - x$.

86. $3x = 8 + 2x$.

87. $2y + 3 = 11$.

88. $3z + 7 = 16$.

89. $3w + 5 = 20$.

90. $5u - 5 = 30$.

91. $7x - 8 = 20$.

92. $3x = x + 6$.

93. $3x = 12 - x$.

94. $2x + 6 = 5x$.

95. $12 - 2x = 4x$.

96. $4x = x + 12$.

97. $3y = 21 - 4y$.

98. $7z - 21 = 0$.

99. $0 = 15 - 3x$.

100. $5x - 17 = 2x$.

101. $27 - z = 4z$.

102. $3x + 7 = x + 19$.

103. $5x + 10 = 8x - 5.$ **104.** $4y - 8 = 3y - 1.$

105. $7z - 13 = 5z - 9.$ **106.** $-2x + 48 = -x + 20.$

107. $5x + 11 - 2x = 20.$

108. $12 + 5x = 108 - 5x - 6.$

109. $14 = 60 + 4x - 8x + 2.$

110. $26 - 4x = x + 5 - 2x - 3.$

111. $3x - 30 = 41 + 2x - 2.$

112. $88 - 12 = 14 + 9x - 55.$

113. $22 - 6y + 18 = 100 - 14y + 36.$

114. $6 + 2z - 3 = 5z - 38 + 21.$

115. $10(x - 3) = 40.$

116. $(x + 6)9 - 2 = (x - 1)40 - 1.$

117. $(5 + x)(3 - 7) = -9x + 15.$

118. $2(3x - 10) = 5x - 15.$

119. $(x - 7)(x - 3) = (x - 2)(x - 9) + 1.$

120. $(x + 7)x + 3 = (x - 3)x - 37.$

121. $(5 - x)(6 - x) = (7 - x)(8 - x) - 10.$

122. $\overline{x - 3} \times \overline{x - 20} - x^2 - 15x - 84.$

123. $(x - 4 + 3x)(5 - 2) = 11x - 1.$

124. $x^2 + x + 1 = x^2 - 6x + 8.$

125. $(x + 3)(x - 7) = (x + 13)(x - 12) + 50.$

126. $x^2 + x - 20 = x^2 - 3x + 60.$

127. $3 = 6x^2 - (2x + 2)(3x + 1) + 77.$

128. $(5x + 3)(6x + 2) + 6 = (2x + 9)(15x - 2) + 30.$

129. $(5x + 1)(4x + 1) = (10x + 1)(2x + 1) - 18.$

130. $(x-4)(4x-1)+32=(2x-2)(2x-2)-13$.

131. $3x-(2x+20)=6x-(x+80)-(3x+40)$.

132. $2(x-6)+3(2x+5)=3(3x-2)-1$.

133. $(x+2)3=(x-2)7-(x-5)(x-6)+x^2$.

134. $-2x+(x-1)-(x+1)-(x-1)=5$.

135. $-x(3-6x)+7=9x^2-(3x+2)x-4x$.

136. $(1-3x)^2-(x+5)^2=4(x+1)(2x-3)$.

137. $6(4-x)^2-5(2x+7)(x-2)=5-(2x+3)^2$.

138. $x+2-[x-8-2\{8-3(5-x)-x\}]=0$.

139. $20(2-x)+3(x-7)-2[x+9-3\{9-4(2-x)\}]=22$.

140. $(x+1)^2-(x^2-1)=x(2x+1)-2(x+2)(x+1)+20$.

141. $6(x^2-3x+2)-2(x^2-1)=4(x+1)(x+2)-24$.

142. $84+(x+4)(x-3)(x+5)=(x+1)(x+2)(x+3)$.

143. $(x+1)(x+2)(x+6)=x^3+9x^2+4(7x-1)$.

144. $(x^2-2x+1)^2=x^4-x(2x-2)^2-x(2x+1)+5$.

145. $(x-1)^3+(x-2)^3+(x-3)^3=3(x-1)(x-2)(x-3)$.

146. $2x=6a$.

147. $6x=-18d$.

148. $-7x=21p$.

149. $-8y=-48n$.

150. $0x=24a$.

151. $3z=20e$.

152. $2x-4c=-2c$.

153. $3y+7b=5b$.

154. $8a-4x=2a$.

155. $2x-6a=4$.

156. $3x-9c=2x-5c$.

157. $4z+4a=z-5a$.

158. $20x-44a=36a$.

159. $100x-50c=75x-50c$.

160. $13u + 11k = 20k - 18u.$

161. $2x + a = 3x - 2.$ **162.** $5y - 3 = 2y + 8c.$

163. $x + a = 2x + b.$ **164.** $2 - 5x = 3n - 7x.$

165. $2x - 3a + 7 = x + 35.$ **166.** $apx - k = 0.$

167. $3ax - 4ab = 2ax - 6ac.$ **168.** $x - 3 + c = 2x + a - b.$

169. $2y + bc - ad = y + 2m - 8.$

170. $3ab - y + d = -2y + 17.$

171. $3x + a - 6 = b - 4 + 2x.$

172. $b + c - 4x = 32 + b - 5x + d.$

173. $4cd + 27 - 4x = 28 - d - 3x + 3bh.$

174. $11pq - 15x - 8mn - 7 = 92 - 88pq + 91mn - 16x.$

175. $x + nx = a.$ **176.** $ax + bx = c.$

177. $5ay - c = b - 3ay.$ **178.** $ax + b^2 = bx + a^2.$

179. $4x - 4c = ax - ac.$

180. $bx + 2x = 3x + a - 2c.$

181. $abc - a^2x = ax - a^2b.$

182. $bc + ax - d = a^2b - fx.$

183. $3acx - 6bcd = 12cdx + abc.$

184. $-ac^2 + b^2c + abcx = abc + cdx - ac^2x + b^2c - dc.$

185. $c(3ay - 2c) = a(2c - 3ay).$

186. $(a^2 + x)^2 = x^2 + 4a^2 + a^4.$

187. $[a - x][a + x] = 2a^2 + 2ax - x^2.$

188. $(a^2 - x)(a^2 + x) = a^4 + 2ax - x^2.$

189. $[x + a][x + b] = (x + c)^2.$

190. $(x+a)^2+(b+c)^2=(x-a)^2+(b-c)^2$.

191. $(x+p)(x+q)-(x-p)(x-q)=2(p+q)^2$.

192. $(x-a)(x-b)=(x-a-b)^2$.

193. $2(x-n)(2m-3n-3x)=(2m-3x)(2x+n)$.

194. $(x-2e+3f)^2-(x-2e)(x+3f)=6ef$.

195. $2y+2.5y=900$.

196. $.2x-.325=.15x$.

197. $.5x-2=.25x+.2x-1$.

198. $2.4x-10.75=.25x$.

199. $.2y-.01y+.005y=11.7$.

200. $2.9x-1.98=1.4x-1.845$.

201. $.5x+2-.75x=.4x-11$.

202. $.05z+.117=.186z-.2z-.139$.

203. $3.25x-5.1x+x=3.9+.75x+.5x$.

204. $.125x+.01x=13-.2x+.4$.

205. $.4(1.7w-.6)=.95w+5.16$.

206. $.2x-.5x+.1x=.25x-9$.

207. $.07(x-10)+.54x=.2(.1-.1x)-3(.05-.02)$.

208. $.625x-.25x+.2x=.5(x+2)-.4$.

209. $.08(35y-2.3)=.9(7y+.18)-.997$.

210. $\frac{1}{4}x=5$.

211. $\frac{1}{3}x=-4$.

212. $\frac{1}{5}x=0$.

213. $-8=\frac{1}{2}x$.

214. $-9=-\frac{1}{6}x$.

215. $\frac{1}{3}y+4=13$.

216. $8-3y=\frac{13}{4}y$.

217. $\frac{1}{2}(x+4)=5$.

218. $\frac{7}{4}x=7+2x$.

219. $-\frac{14}{3}w=-4-2w$.

220. $\frac{1}{2}x + 8 = -\frac{1}{2}x.$

221. $36 - \frac{4x}{9} = 8.$

222. $\frac{2x}{3} + 12 = \frac{4x}{5} + 6.$

223. $\frac{7x}{8} - 5 = \frac{9x}{10} - 8.$

224. $x + \frac{x}{2} - \frac{3x}{5} = 9.$

225. $\frac{x}{2} + \frac{x}{3} + \frac{x}{4} = \frac{13}{12}.$

226. $\frac{m}{px} + n = \frac{k}{px} + q.$

227. $\frac{a}{bz} + c = \frac{d}{gz} + 3.$

228. $\frac{ab}{x} = bc + d + \frac{1}{x}.$

229. $ax + b = \frac{x}{a} + \frac{1}{b}.$

230. $3ax + \frac{a}{2} - 3 = bx - a.$

231. $am - b - \frac{ax}{b} + \frac{x}{m} = 0.$

232. $a - cx^{-1} = dx^{-1} - b.$

233. $\frac{5}{6}ab + \frac{4}{5}ac - \frac{2}{3}cx = \frac{3}{4}ac + 2ab - 6cx.$

234. $\frac{b^2}{ax} + \frac{b}{a} - \frac{a}{b} = \frac{a}{x}.$

235. $\frac{2}{ab} - \frac{1}{bx} + \frac{1}{2ax} - \frac{1}{a^2} = 0.$

236. $\frac{a}{b} - \frac{1}{x} + \frac{b}{a} + \frac{1}{x} = 2 + \frac{1}{bx} - \frac{1}{ax}.$

237. $\frac{x}{b} + \frac{x}{q} + \frac{x}{r} = pq + pr + qr.$

238. $\frac{1}{2}x + \frac{1}{3}x + \frac{1}{4}x + \frac{1}{5}x = x - 17.$

239. $\frac{1}{4}(x + 3) = \frac{1}{5}(3x + 21).$

240. $\frac{1}{2}(7 - x) - 3 = \frac{1}{5}(3 - 4x) - 1.$

241. $\frac{1}{8}(x - 2) - \frac{1}{2}(12 - x) = \frac{1}{4}(5x - 36) - 2\frac{1}{4}.$

242. $\frac{1}{10}(1 - x) - \frac{1}{12}(5x - 3) = 1 - \frac{2}{5}(x - 1).$

243. $\frac{1}{12}(7+3x)-\frac{2}{27}(x-5)=1-\frac{1}{18}x.$

244. $\frac{1}{4}(x-15)-\frac{1}{18}(7+2x)=\frac{3}{8}x-\frac{3}{4}\frac{5}{}.$

245. $5x-\dfrac{x+2}{2}=71.$

246. $x-\dfrac{3-x}{3}=5\frac{2}{3}.$

247. $\dfrac{x-1}{5}+\dfrac{x-9}{2}=3.$

248. $\dfrac{5x}{2}-\dfrac{5x}{4}=\dfrac{9}{4}-\dfrac{3-x}{2}.$

249. $\dfrac{3x+9}{4}=\dfrac{5x+16}{7}.$

250. $2x-\dfrac{5x-4}{6}=7-\dfrac{1-2x}{5}.$

251. $x-3-\dfrac{x+2}{8}=\dfrac{x}{3}.$

252. $\dfrac{x+5}{7}-\dfrac{x-2}{3}=\dfrac{x+9}{11}.$

253. $\dfrac{30+x}{x}-5=\dfrac{6}{x}.$

254. $\dfrac{a}{x}+\dfrac{x-b}{x}-\dfrac{a-x}{x}=1.$

255. $\dfrac{b^2-ax}{b}=b-\dfrac{a^2-bx}{a}.$

256. $3\left(\dfrac{x+b}{4}\right)+\dfrac{x-b}{3}=4\left(\dfrac{x-b}{3}\right).$

257. $\dfrac{2x+a}{b}-\dfrac{x-b}{a}=\dfrac{3x}{b}+\dfrac{(a-b)^2}{ab}.$

258. $\dfrac{x^2-a}{bx}-\dfrac{a-x}{b}=\dfrac{2x}{b}-\dfrac{a}{x}.$

259. $\dfrac{3ax-2b}{3b}-\dfrac{ax-a}{2b}=\dfrac{ax}{b}-\frac{2}{3}.$

260. $\dfrac{x}{2}-\dfrac{a-2bcx}{4bc}=\dfrac{5x}{6c}-\dfrac{8ac-8bx-9a}{12bc}.$

261. $.25x+\dfrac{.5-.75x}{4}=\dfrac{.375x+.25}{4}.$

262. $\frac{x+1}{.2} - \frac{.3x}{2.5} - 7.5x = 18.1.$

263. $1.5 = \frac{.36}{.2} - \frac{.09x - .18}{.9}.$

264. $\frac{x+.75}{.125} - \frac{x-.25}{.25} = 15.$

265. $.5x - \frac{.45x - .75}{.6} = \frac{1.2}{.2} - \frac{.03x - .6}{.9}.$

266. $\frac{.2x}{.01} - \frac{x+.25}{.5} - \frac{1.5x + 9.75}{.6} = 12\ 15x.$

267. $\frac{.01x}{.02} - \frac{x}{30} = \frac{.01x}{.5} + 1.34.$

268. $\frac{.03x - .01}{.02} - \frac{.02(x-1)}{.03} = \frac{.01x - .03}{.4} + \frac{.21}{.2}.$

269. $\frac{.25x}{.3} - \frac{.5x - 2.8}{2.5} = \frac{.3x + .34}{.75} + \frac{1.2}{.4}.$

270. $\frac{x-2}{.05} - \frac{x-4}{.0625} = 56.$

271. $\frac{bx + 4a}{4} - \frac{a^2 - 3bx}{a} - bx = ab^2 - \frac{5a^2 - 6bx}{2a} + ax.$

272. $x^2 + a(2a - x) - \frac{3b^2}{4} = \left(x - \frac{b}{2}\right)^2 + a^2(a + b).$

273. $\frac{3(x-3)}{2} - \frac{2(x^2 - 5)}{3x} - \frac{5x^3 - 12}{6x^2} = -\frac{9}{2}.$

274. $3 + \frac{x}{4} = \frac{1}{2}\left(4 - \frac{x}{3}\right) - \frac{5}{6} + \frac{1}{8}\left(11 - \frac{x}{2}\right).$

275. $11 - \left(\frac{3x-1}{4} + \frac{2x+1}{3}\right) = 10 - \left(\frac{2x-5}{3} + \frac{7x-1}{8}\right).$

276. $\frac{(3x-2)(2x-3)}{6} - \frac{6x-8}{5} + \frac{2}{15} = x^2 - \frac{x+10}{3}$.

277. $\frac{11x-13}{25} + \frac{19x+3}{7} - \frac{5x-25\frac{1}{3}}{4} = 28\frac{1}{7} - \frac{17x+4}{21}$.

278. $\frac{x-a}{2bc} + \frac{x-b}{2ac} + \frac{x-c}{2ab} = \frac{1}{a} + \frac{1}{b} + \frac{1}{c}$.

279. $\frac{1}{9}(2x-\frac{1}{5}) - \frac{3}{4}\left(\frac{x}{3} - \frac{2}{9}\right) = \frac{1}{5}\left(\frac{5x}{4} - 1\right) + \frac{1}{15}$.

280. $\frac{2}{5}(x-\frac{1}{3}) + \frac{1}{3}(1 - \{x + \frac{2}{5}\}) - \frac{2}{7}\{x - (1 + \frac{1}{3}x)\}$.

281. $\frac{x}{8}(4x + 2\frac{2}{3}) - \frac{2}{9}\left(\frac{3x}{4} - 4\frac{1}{2}\right) = \frac{x}{6}(x-2) + \frac{x}{5}\left(\frac{5x}{3} + 10\right)$.

282. $(x-\frac{5}{2})(x+\frac{3}{2}) - (x-5)(x+3) = 23\frac{1}{4}$.

283. $(y+\frac{5}{2})(y-\frac{3}{2}) + \frac{3}{4} = (y+5)(y-3)$.

284. $p(p-x) - \frac{p}{q}(x-q)^2 = p(p-q) + pq\left(\frac{x}{q} - 1\right)^2$.

285. $\frac{x}{6} - \frac{x-\frac{1}{2}}{3} - \frac{1}{3}\left(\frac{2}{5} - \frac{x}{3}\right) = 0$.

286. $\frac{5(3x-2)}{4} + \frac{3x}{2} - 23\frac{5}{6} = \frac{x - \frac{4x-9}{3}}{6} + 5$.

287. $\frac{\frac{1}{3}(x-2)}{\frac{2}{3}} + \frac{\frac{3}{4}(6x-7)}{5} = \frac{\frac{2x}{3} - 1}{\frac{1}{6}} - 1\frac{1}{4}$.

288. $\frac{7x + \frac{13}{2}}{10} + \frac{11x - \frac{x-\frac{3}{2}}{2}}{12} = \frac{3x+1}{5} + \frac{43x - \frac{3-8x}{2}}{22}$.

289. $\frac{7ax + 15c}{21cx} = \frac{5ad - 12c}{15cd} + \frac{16b + 5ad}{20bd}$.

290. $\frac{4a+3c}{4c}-\frac{5a-4b}{5b}=\frac{31c+20a}{20c}+\frac{34bdst-3ax}{17bstx}$.

291. $\frac{6bx^2y+5acz}{30bz}-\frac{7fx-8be}{56bf}=\frac{7fx^2y+5cz}{35fz}+\frac{10ax-8b}{80ab}$.

292. $\frac{5ay+4cx}{20cy}-\frac{11bx-9d}{99dx}=\frac{165xy+11x^2+5y}{55xy}+\frac{9dx-b}{9d}$.

293. $\frac{8abc^2-13abdx}{104b^2cd}-\frac{12acgh-13bdef}{132bdgh}=\frac{ef-12gh}{12gh}$.

294. $\frac{17az^2+45x}{153ax}+\frac{13cz^2-54x}{117cx}=\frac{104cmnz^2-216mnx+117cx^2}{468cmnx}$.

295. $\frac{5am+12cmn}{60c}-\frac{54mx+28dm}{126dx}=\frac{2m}{18pq}$
$+\frac{70adm-360cm+168cdmn}{840cd}$.

296. $\frac{5aklq-6px+390qx}{30qx}-\frac{13m+208n}{16n}$
$=\frac{480nq-112klnp-455klmq}{560klnq}$.

297. $\frac{19p^2-3q^2}{57pq}+\frac{5aby-4cdx}{20cdy}-\frac{2x}{3}=\frac{5py-60qy-3qx}{15qy}$
$-\frac{4cdq-19abp}{76cdp}$.

298. $\frac{11axz-117cy}{143cz}+\frac{34a-x^2}{17ax}-\frac{7efn-63am}{21mn}$
$=\frac{693mxz-11efxz-189mxy}{231mxz}$
$+\frac{17a^2nx-13cnx-884a^2c}{221acn}$.

299. $\frac{5}{x} = \frac{1}{x-4}$.

300. $\frac{1}{x-7} = \frac{15}{x}$.

301. $\frac{51}{2x+3} = \frac{21}{x}$.

302. $\frac{1}{3} = \frac{a}{c-x}$.

303. $\frac{9}{x-3} = \frac{a}{b}$.

304. $\frac{5}{x+3} = \frac{7}{x+5}$.

305. $\frac{2}{x-1} = -\frac{6}{x+1}$.

306. $\frac{10}{x+20} = -\frac{7}{3x-14}$.

307. $\frac{m}{m-x} = \frac{n}{n-x}$.

308. $\frac{z}{a+az} = \frac{b}{c+cz}$.

309. $\frac{p}{m(n+x)} = \frac{x}{n+x}$.

310. $\frac{4a}{x-2} = \frac{3c}{x-4}$.

311. $\frac{x}{x+1} - \frac{3x}{x-2} = -2$.

312. $\frac{3x}{x+6} - \frac{x}{x+5} = 2$.

313. $\frac{2x}{x+3} - \frac{4x}{x+7} = -2$.

314. $\frac{ax^2}{b-cx} + a + \frac{ax}{c} = 0$.

315. $\frac{5x}{3a+b} - 2 = \frac{8b}{5a}$.

316. $\frac{x}{a+b} - \frac{x}{a-b} = 2a$.

317. $\frac{3y}{a+3b} + \frac{18by}{a^2-9b^2} = 1$.

318. $\frac{3}{x-\frac{3}{4}} = 60$.

319. $\frac{1}{5-\frac{1}{x}} = \frac{2}{7}$.

320. $\frac{25}{x-\frac{7}{2}} = \frac{10}{3x-4}$.

321. $\frac{a}{x-\frac{b}{a}} = \frac{2}{3}$.

322. $\frac{1}{a-\frac{1}{x}} = \frac{7}{3a}$.

323. $\frac{x-1}{x+1} = \frac{2}{3}$.

324. $\frac{x+a}{x-a} = \frac{5}{4}$.

325. $\frac{5x-5}{x-1} = 3$.

326. $\frac{a-bx}{ax-b}=\frac{3}{4}$. 327. $\frac{a}{b}=\frac{x-b^2}{x-a^2}$. 328. $\frac{1+z}{1-z}=\frac{1}{m}$.

329. $\frac{p(q+x)}{p-x}=q$. 330. $\frac{m+ny}{p+qy}=b$. 331. $\frac{my+n}{my-n}=\frac{a}{b}$.

332. $\frac{(2x-1)(3x+8)}{6x(x+4)}=1$. 333. $\frac{x+a}{2}-\frac{2}{x+a}=\frac{x-a}{2}$.

334. $\frac{2(x+1.5)}{5(.8x-1)}=\frac{15}{19}$. 335. $\frac{.25x-1.5}{.15(x-5)}=\frac{2}{1.5}$.

336. $\frac{\frac{3}{4}(x-1)}{11-\frac{2}{3}x}=\frac{8}{3}$. 337. $\frac{\frac{1}{2}(3x-1)}{\frac{1}{6}(4-x)}=\frac{3}{2}$.

In examples 338–347 find the value of $1/x$ without freeing from fractions.

338. $\frac{2}{x}=4$. 339. $\frac{3}{x}=15$. 340. $\frac{m}{x}=am$.

341. $\frac{c}{dx}=\frac{c^2}{d^2}$. 342. $\frac{m+1}{x}=m$.

343. $\frac{m+n}{x}=m-n$. 344. $\frac{m}{x}+\frac{a}{x}=m^2-a^2$.

345. $\frac{a}{2x}+\frac{b}{2x}=c$. 346. $\frac{k}{ax}+\frac{l}{bx}=a-b$.

347. $\frac{c}{z}-\frac{1}{a}=\frac{1}{b}$.

In examples 348–357, find the value of each letter:

348. $2x-3a=4b$. 349. $5a-4b=2u$.

350. $ax=by$. 351. $7a-14b=21y$.

352. $(x-y)a=b(x+y)$. 353. $\frac{ax}{by}=2$.

354. $\frac{1}{b-c} = \frac{2}{2a-c}$.

355. $a + \frac{1}{x} = b + \frac{1}{y}$.

356. $\frac{x-a}{b} = \frac{x+a}{c}$.

357. $\frac{x-a}{x-b} = \frac{x+c}{x+d}$.

358. $\frac{6x+7}{9} + \frac{7x-13}{6x+3} = \frac{2x+4}{3}$.

359. $\frac{7x+16}{21} - \frac{x+8}{4x-11} = \frac{x}{3}$.

360. $\frac{9x+20}{36} = \frac{4x-12}{5x-4} + \frac{x}{4}$.

361. $\frac{7x-6}{35} - \frac{x-5}{6x-101} = \frac{x}{5}$.

362. $\frac{18x-19}{28} + \frac{11x+21}{6x+14} = \frac{9x+15}{14}$.

363. $\frac{4x+3}{9} + \frac{7x-29}{5x-12} = \frac{8x+19}{18}$.

364. $\frac{6x+1}{15} - \frac{2x-4}{7x-16} = \frac{2x-1}{5}$.

365. $\frac{1}{2} + \frac{2}{x+2} = \frac{x+2}{2x}$.

366. $\frac{9(2x-3)}{14} + \frac{11x-1}{3x+1} = \frac{9x+11}{7}$.

367. $\frac{1}{n} + \frac{n}{n+x} = \frac{n+x}{nx}$.

368. $n + \frac{n-x}{n} = \frac{x^2}{n(n-x)}$.

369. $\frac{20x}{25} + \frac{36}{25} + \frac{5x+20}{9x-16} = \frac{4x}{5} + \frac{86}{25}$.

370. $\frac{9x+4}{5x-48} + \frac{4x-19}{51} = \frac{5x+32}{17} - \frac{11x+13}{51}$.

371. $\frac{ax+b}{c}+\frac{ax+b}{cx+b}=\frac{2ax+d}{2c}+\frac{b}{c}$.

372. $x-\frac{4x-3}{7x+4}-\frac{1-9x}{6}=\frac{4x+3}{8}-\frac{1}{24}+2x$.

373. $\frac{6-5x}{15}-\frac{7-2x^2}{14(x-1)}=\frac{1+3x}{21}-\frac{10x-11}{30}+\frac{1}{105}$.

374. $\frac{ax-b}{bx}+\frac{bx+a}{ax}=\frac{2-x}{x-3}+\frac{a^2+b^2}{ab}-2$.

375. $\frac{x^2-b}{ax}-\frac{b-x}{a}=\frac{2x}{a}+\frac{1-a}{ab-b}-\frac{b}{a}$.

376. $\frac{8x+5}{14}+\frac{7x-3\frac{1}{2}}{6x+2}=\frac{16x+15}{28}+\frac{2\frac{1}{4}}{7}$.

377. $\frac{1}{x^2+9x+20}=\frac{1}{x^2+12x+35}$.

378. $\frac{1}{x^2-13x+42}=\frac{1}{x^2-15x+54}$.

379. $\frac{x}{x^2+3x+2}=\frac{2x}{2x^2+4x+3}$.

380. $\frac{3}{x-4}-\frac{5}{2x-7}=\frac{5}{(x-4)(2x-7)}$.

381. $\frac{4}{2x+1}-\frac{7}{3x+2}=\frac{19}{(2x+1)(3x+2)}$.

382. $\frac{a}{x-a}-\frac{b}{x+a}=\frac{b}{x^2-a^2}$.

383. $\frac{2}{4x-5}=\frac{9}{(4x-5)(2x-1)}+\frac{1}{1-2x}$.

384. $\frac{1}{x+4}=\frac{2}{2x-7}+\frac{15}{(7-2x)(4+x)}$.

385. $\frac{3}{x+1}+\frac{1}{x-1}=\frac{6}{x^2-1}$.

386. $\frac{12}{4x^2-9}-\frac{2}{2x+3}=\frac{1}{2x-3}$.

387. $\frac{2}{x+4}=\frac{11}{x^2-16}+\frac{1}{4-x}$.

388. $\frac{1}{9x+4}-\frac{2}{81x^2-16}+\frac{1}{9x-4}=0$.

389. $\frac{4}{6x-1}=\frac{11}{36x^2-1}+\frac{3}{6x+1}$.

390. $\frac{5}{3x-9}-\frac{2}{x+3}=\frac{11x}{3x^2-27}$.

391. $\frac{2}{x-4}-\frac{1}{2x-9}=\frac{7}{2x^2-17x+36}$.

392. $\frac{1}{2(3x+7)}+\frac{1}{2x+10}=\frac{2}{3x^2+22x+35}$.

393. $\frac{4}{u+2}+\frac{7}{u+3}=\frac{37}{u^2+5u+6}$.

394. $\frac{1}{x^2-13x+42}-\frac{2}{x^2-6x-7}=\frac{5}{x^2-5x-6}$.

395. $\frac{5}{x^2-2x-15}-\frac{7\frac{1}{2}}{x^2+2x-35}=\frac{1}{x^2+10x+21}$.

396. $\frac{10}{x+20}+\frac{15}{x+5}=\frac{20}{x+10}+\frac{5}{x+15}$.

397. $\frac{2}{2x+1}-\frac{3}{3x-1}=\frac{3}{3x-2}-\frac{2}{2x-3}$.

398. $\frac{5}{x-2}-\frac{10}{2x+4}=\frac{2}{x-4}-\frac{2}{x+6}$.

399. $\dfrac{1}{x-2}-\dfrac{1}{x-4}=\dfrac{1}{x-6}-\dfrac{1}{x-8}$.

400. $\dfrac{7}{2x+6}-\dfrac{3\frac{1}{2}}{2x+2}=\dfrac{4}{x+3}-\dfrac{2}{x+1}$.

401. $\dfrac{2x+5}{x+6}=\dfrac{2x-5}{x}$.

402. $\dfrac{2x+7}{x+2}=\dfrac{4x-1}{2x-1}$.

403. $\dfrac{3x-7}{x-9}=\dfrac{6x-4}{2x-5}$.

404. $\dfrac{3-2x}{x-5}=\dfrac{2x-7}{4-x}$.

405. $\dfrac{a-b}{a+u}=\dfrac{a+b}{u+b}$.

406. $\dfrac{3x-4}{3x+4}=\dfrac{5m-2n}{5m+2n}$.

407. $\dfrac{3x+13}{x+4}+\dfrac{4x-3}{x-2}=7$.

408. $\dfrac{7x-12}{x-2}+\dfrac{6x-17}{2x-5}=10$.

409. $\dfrac{4x-11}{x-2}-\dfrac{2x}{x+1}=2$.

410. $\dfrac{x-2}{x+2}-\dfrac{x-1}{x+1}+\dfrac{2x+4}{x^2-1}=0$.

411. $\dfrac{12x+1}{3x-1}+\dfrac{5}{1-9x^2}=\dfrac{11+12x}{1+3x}$.

412. $\dfrac{2+3x}{3-x}-\dfrac{2-3x}{3+x}=\dfrac{36-4x}{x^2-9}$.

413. $\dfrac{ax-b}{x+3}-\dfrac{ax+b}{x-3}=\dfrac{5}{x^2-9}$.

414. $\dfrac{x-b}{x-2a}-\dfrac{x+b}{x+2a}=\dfrac{4a^2-b^2}{x^2-4a^2}$.

415. $\dfrac{2nx-3}{nx-1}=5-\dfrac{9nx+2}{3nx-1}$.

416. $\dfrac{2x^2+3x-1}{2x+1}+\dfrac{2x^2-3x+1}{2x-1}=2x$.

417. $\dfrac{x+4a+b}{x+a+b}+\dfrac{4x+a+2b}{x+a-b}=5.$

418. $\dfrac{x^2+6x+3}{x^2+5x+10}=\dfrac{x^2-2x+11}{x^2-3x+18}.$

419. $\dfrac{x^2-8x+19}{x^2-11x+13}=\dfrac{x^2-3x-6}{x^2-6x-12}.$

420. $\dfrac{x^2+2x-8}{x^2+x+1}=\dfrac{x^2-2x-12}{x^2-3x-3}.$

421. $\dfrac{x+5}{x+4}+\dfrac{x+7}{x+6}=\dfrac{x+4}{x+3}+\dfrac{x+8}{x+7}.$

422. $\dfrac{x-4}{x-5}-\dfrac{x-5}{x-6}=\dfrac{x-7}{x-8}-\dfrac{x-8}{x-9}.$

423. $\dfrac{5x-64}{x-13}-\dfrac{4x-55}{x-14}=\dfrac{2x-11}{x-6}-\dfrac{x-6}{x-7}.$

424. $\dfrac{x-3}{x-5}+\dfrac{x+4}{x+2}=\dfrac{x-5}{x-7}+\dfrac{x+6}{x+4}.$

425. $\dfrac{x+a}{x+b}=\left(\dfrac{2x+a+c}{2x+b+c}\right)^2.$

426. $\dfrac{px-a-b}{qx-c-d}=\dfrac{px-a-c}{qx-b-d}.$

427. $\dfrac{7}{x^2-1}+\dfrac{8}{x^2-2x+1}=\dfrac{37-9x}{x^3-x^2-x+1}.$

428. $\dfrac{1}{x^2+2x+1}+\dfrac{4}{x+2x^2+x^3}=\dfrac{5}{2x+2x^2}.$

429. $\dfrac{a^2+4a}{x^2+x-a^2+a}-\dfrac{a}{x+a}=\dfrac{1}{x-a+1}.$

430. $\dfrac{5}{(x-1)(x+2)}-\dfrac{2}{x^2-x-2}=\dfrac{8}{x^2-1}-\dfrac{5}{x^2-4}.$

431. $\dfrac{a+b}{x-a-b}+\dfrac{a-b}{x-a+b}+\dfrac{2a}{2a-x}=0.$

432. $\dfrac{3def}{d+e}+\dfrac{d^3e^3}{(d+e)^3}+\dfrac{(2d+e)e^3x}{d(d+e)^3}=3f+\dfrac{ex}{d}.$

433. $\dfrac{x-a}{b+c}+\dfrac{x-b}{a+c}+\dfrac{x-c}{a+b}=\dfrac{3x}{a+b+c}.$

434. $\dfrac{k}{z}+\dfrac{z}{k}+\dfrac{k(z-k)}{z(z+k)}-\dfrac{z(z+k)}{k(z-k)}=\dfrac{kz}{k^2-z^2}-2.$

435. $\dfrac{a+x}{a^2+ax+x^2}+\dfrac{a-x}{a^2-ax+x^2}=\dfrac{3a}{x(a^4+a^2x^2+x^4)}.$

436. $\dfrac{c}{a-b}\left(1+\dfrac{1}{x}\right)-\dfrac{b}{a-c}\left(1+\dfrac{1}{x}\right)=\dfrac{a+c}{(a-c)x}+1.$

437. $\frac{3}{4}-\dfrac{\frac{3}{4}x+\frac{3}{4}}{\frac{3}{4}+x}=\dfrac{\frac{3}{4}}{\frac{3}{4}+x}-\frac{3}{4}.$

438. $\dfrac{\frac{x}{6}-3}{2}+\dfrac{\frac{5x}{4}-2}{3}-\dfrac{\frac{2x}{3}-\frac{5}{6}}{2\frac{1}{2}}=\frac{29}{60}.$

439. $\dfrac{\frac{4x}{3}+5}{2}+\dfrac{\frac{2x}{5}-\frac{1}{2}}{6}-\dfrac{x+\frac{1}{4}}{\frac{1}{5}}=5\frac{13}{30}.$

440. $1-\dfrac{1-\frac{1}{a^2}}{\frac{a}{x}\left(1-\frac{1}{a}\right)}=\dfrac{1}{a^3}.$

441. $\dfrac{\frac{1}{8}(x-2)}{\frac{2}{3}}+\dfrac{\frac{3}{4}(6x-7)}{5}=\dfrac{\frac{2x}{3}-1}{\frac{1}{6}}-1\frac{1}{4}.$

442. $\dfrac{x-\frac{1}{2}}{x-\frac{3}{2}}+\dfrac{x-1\frac{3}{2}}{x-1\frac{5}{2}}=\dfrac{x-\frac{3}{2}}{x-\frac{5}{2}}+\dfrac{x-1\frac{1}{2}}{x-1\frac{3}{2}}.$

443. $\dfrac{a+x-\dfrac{a^2}{a+x}}{a+x} = 1 - \dfrac{2ax}{(a+x)^2}$.

PROBLEMS WITH ONE UNKNOWN QUANTITY.

(Note that each problem is a double one, expressing the values first, in figures and second, in letters. Hence two sets of answers are required for each.)

1. What number is that to which if 9, (a), be added the sum will be 23, (b)?

2. The difference between two numbers is 3, (d), and their sum is 21, (s). What are the numbers?

3. If from 3, (a), times a certain number 11, (m), be subtracted, the remainder will be 7, (n). What is the number?

4. If to 5, (b), times a certain number 9, (p), be added, the sum will be 44, (q). What is the number?

5. Divide the number 45, (n), into two parts, one of which shall be 8, (t), times the other.

6. Divide the number 57, (n), into two parts, one of which shall be 23, (d), more than the other.

7. If I spend \$30, (r), for clothing and \$18, (s), for board and then have left one-third of what I had at first, how much have I remaining?

8. John is 3, (n), times as old as James. The difference of their ages plus 30, (e), years is twice the sum of their ages. What is the age of each?

9. A man worked three days for \$10, (k). The second day he received \$1, more than the first; the third day as much as both the first and the second. How much did he receive each day?

10. A man's age is 3, (a), times his son's age. 6, (b), years ago it was 5, (c), times his son's age. Age of each?

11. George is 21, (m), and Thomas 15, (n). How many years ago was George three times as old as Thomas?

12. James is 8, (m), years older than Sam. In 5, (n), years 2 times James' age will equal 3 times Sam's. Age of each?

13. The sixth part of a certain number exceeds its ninth part by 4, (t). What is the number?

14. What number is that the sum of whose half, third and sixth equals itself?

15. What number exceeds the sum of its half, fourth and eighth by 6 (d)?

16. Divide 41, (a), into two parts such that two-fifths of one part may equal five-eighths of the other.

17. Divide \$78, (k), among three persons so that the second shall have two-thirds as much as the first and the third one-half as much as the second.

18. Divide \$78, (k), among three persons so that the second shall have two-thirds as much as the first and the third one-fifth as much as the other two together.

19. If a certain number be divided by 6, (c), the sum of the divisor, dividend and quotient will be 62, (d). What is the number?

20. What number is that which, if multiplied by 6, (a), and the product increased by 5 times the number, and the sum divided by 11, (b), the quotient will be 7, (c)?

21. I paid 375, (a), dollars for two horses, one costing 4, (n), times as much as the other. Price of each?

22. What principal at 3, (r), per cent. interest will yield 108, (i), dollars in 5, (t) years?

23. A man bought a cow and a calf for 100, (a), dollars, and paid 7, (b), times as much for the cow as for the calf. Cost of each?

24. A can do a piece of work in 3, (a), days and C in 6, (c), days. How many days will it take both to do it, working together?

25. What is that number to which, if 20, (m), be added, $\frac{3}{5}$, (p/q), of the sum will be 27, (h)?

26. At what rate per annum will 328, (p), dollars yield 112, (i), dollars in 7, (t), years?

27. The sum of two numbers is 36, (s), and their sum is to their difference as 7, (m), is to 2, (n). What are the numbers?

28. After paying out $\frac{1}{5}$, $(1/n)$, and $\frac{1}{6}$, $(1/m)$, of my money I had left 56, (r), dollars. How much had I at first?

29. In a pen are 4, (n), times as many hens as ducks, and 5, (m), times as many turkeys as hens, and there are 100, (a), in all. How many ducks are there?

30. Divide 96, (e), dollars among A, B and C, giving B 18, (f), dollars more than A, and 18, (f), dollars less than C.

31. The difference between two numbers is 88, (d), and the greater is to the less as 13, (m), is to 5, (n). What are the numbers?

32. A merchant mixes 12, (a), pounds of tea worth 40, (m), cents a pound, 16, (b), pounds worth 50, (n), cents a pound, and 20, (c), pounds worth 80, (p), cents a pound. What is the mixture worth per pound?

33. Divide 300, (a), into two parts such that the larger shall be to the smaller as 7, (m), is to 2, (n).

34. A man owes A a sum of money, B 3, (b), times as much, and C 5, (c), times as much, and D 7, (d), times as much. If he owes 400, (n), dollars in all, how much does he owe A?

35. Find a number such that, if its $\frac{1}{4}$, $(1/m)$, part is added to it, the sum will be equal to the $\frac{1}{3}$, $(1/n)$, part of the number plus 11, (s).

36. A packer engaged to pack 180, (n), tumblers, received 2, (a), cents for every one that arrived safe and forfeiting 10, (b), cents for every one broken. He received 3, (d), dollars. How many were broken?

37. If I divide a certain number by 6, (c), subtract 5, (d), from the quotient, and multiply the remainder by 4, (k), I obtain the same result as if, from the original number I subtract 2, (p), times the $\frac{1}{3}$, $(1/q)$, part. What is the number?

38. Three pipes empty into a cask. The first, alone, fills it in 9, (r), minutes, the second, alone, in 6, (s), minutes and the third, alone, in 2, (t), minutes. In how many minutes will all together fill it?

39. A man engaged to work for 60, (a), days on the conditions that he was to receive 2, (b), dollars for each day he worked and forfeit 3, (c), dollars for each day he was idle. At the end of the 60, (a), days he received 75, (d) dollars. How many days had he worked?

40. My boat broke loose and drifted down stream at the rate of 3, (m), miles per hour. 4, (n), hours afterwards I

started after it, rowing at the rate of 6, (r), miles per hour. In how many hours did I overtake it?

41. Each one of two casks contains the same number of gallons. After drawing 12, (a), gallons from one and pouring them into the other the first contains $\frac{5}{11}$, (m/n), as many as the second. How many gallons in each at first?

42. The denominator of a fraction is 13, (a), more than the numerator, and if 7, (b), be added to the numerator, the value of the fraction will be $\frac{2}{3}$. What is the fraction?

43. Two stoves consume a certain amount of coal in 12, (a), days and the larger stove would consume it in 20, (c), days. In how many days would the smaller stove consume it?

44. 5, (a), lobsters can be bought for a certain sum, but the price having fallen 2, (b), cents each, 6, (c), can now be bought for the same sum. What was the price at first?

45. What two numbers are as 2 : 3, ($a : b$), while, if 6, (c), be added to each, the sums will be as 3 : 4, ($m : n$)?

46. The sum of the digits of a certain number is 12, (s), and if the number be divided by the second digit, the quotient will be 4, (a), and the remainder 3, (b). Find the number.

47. A boy bought marbles at 6, (a), for a cent, and as many more at 8, (b), for a cent, and then sold them at the rate of 12, (c), for 5, (d), cents, thereby gaining 26, (e), cents. How many marbles did he buy?

State problems which will give rise to the following equations. Any number may be stated which will meet the requirements, but the pupil is urged to exercise his ingenuity to give as great a variety as possible.

48. $4x - x = 27$. **49.** $\frac{x}{2} + \frac{x}{3} = 10$. **50.** $\frac{x}{3} - \frac{x}{6} = 1$.

51. $x + x - 6 + x + 11 = 101$.

52. $13 - 4x = 3x - 8$. **53.** $x + \frac{x}{2} + \frac{x}{3} = x + 55$.

54. $(x + 3)(2x - 1) = 99$. **55.** $x + \frac{1}{3}x - 300 = 50$.

56. $x - 2x + 3x + 4x = 7x - 7$.

57. $x - \frac{2x + 4}{3} = 6$. **58.** $\frac{1}{2}x + \frac{1}{3}x + \frac{1}{9} = x + \frac{1}{18}$.

59. $\frac{2}{3}(4x - 5) = \frac{1}{2}(5x - 4)$. **60.** $\frac{2x + 1}{2} = \frac{7x + 5}{8}$.

61. $\frac{2(x - 1)}{3} = \frac{x}{2} + 1\frac{1}{6}$. **62.** $\frac{x}{2} + \frac{x}{3} + \frac{x}{6} = \frac{2x + 3}{6}$.

63. $(x + 2)(x - 2) = x^2 - 2x + 2$.

64. $\frac{16x - 68}{11} = 2x - 10$. **65.** $\frac{3x + 3}{2} - 6 = \frac{5x - 4}{7}$.

66. $\frac{2(3x + 2)}{x - 1} = \frac{12}{x - 4} + 5$. **67.** $\frac{2}{3}\left(\frac{3x}{4} + \frac{1}{4}\right) = \frac{2x - 4}{3}$.

68. $\frac{10(x + 2) + x}{x + 2 + x} = 7$. **69.** $ax : bx = m : n$.

70. $\frac{x}{a} + \frac{x}{b} + \frac{x}{c} = d$. **71.** $\frac{x}{b} = \frac{a - x}{c}$

72. $\frac{b}{c}(x + a) = d$. **73.** $mx - a = n(x - a)$.

74. $mx + nx = an + b$. **75.** $\frac{a - x}{x} = b + c$.

76. $ax + bx = c(x - 1)$.

77. $x - n + x + n + \frac{x}{n} + nx = m.$

78. A man paid \$ 410, (a), for sheep of different grades. For one-sixth of the whole he paid 10, (b), dollars each ; for one-third of the whole he paid 8, (c), dollars each and for the rest 5, (d) dollars each. How many did he buy ?

79. A man has 6 daughters born two years apart and the sum of all their ages is nine times the age of the youngest. What is the age of each ?

80. The sum of \$100 was raised by subscription. Of this sum some subscribed 50 cents each, twice as many 75 cents each and the remainder \$1.00 each. There were 120 subscribers in all. How many were there of each class ?

81. Divide \$550 among A, B, C and D so that A and B will receive \$225, B and C \$275 and D \$25 more than C.

82. Fifteen coins, dollars and quarters, amount to \$7.50. How many coins of each kind are there ?

83. A, B, C and D invest \$4,755, (s), in a speculation. B furnishes 3 (m) times as much as A, C as much as A and B together, and D as much as B and C. How much does each invest ?

84. A certain number consists of three figures whose sum is 9 and the middle figure is one-half the sum of the other two ; and if 198 be added to the number, the order of the figures will be inverted. What is the number ?

85. A certain number consists of two digits, the one in the unit's place being twice that in the ten's place. If the order of the digits be inverted and 3 be added to the resulting number, the new number will be twice the original number. Find the original number.

86. A and B have equal incomes. A lays up $\frac{1}{4}$, $(1/a)$, of his, but B, who spends $\frac{1}{5}$, $(1/b)$, more than A, saves a sum equal to his income in 10, (c), years. What is their income? What does your attempt to solve this problem show?

87. A and B have equal incomes. A saves $\frac{1}{4}$, $(1/a)$, of his, but B, who spends $\frac{1}{2}$, $(1/b)$, more than A, finds himself in debt 450, (c), dollars in 4, (d), years. Find the amount of their income.

88. The deposits in a bank during three days amounted to 6,650, (s), dollars. If the deposit each succeeding day was $\frac{2}{3}$, (a/b), of that of the previous day, how many dollars were deposited each day?

89. Bought a certain number of apples at 2, (a), cents each, $\frac{4}{9}$ as many peaches at 3, (b), cents each, and $\frac{1}{3}$ as many bananas at the rate of 3, (c), for 5, (d), cents. The whole cost \$1.40, (m). How many of each were there?

90. I start business with a certain sum which I double the first year, but my expenses are 750, (a), dollars. The second year I double the money left at the end of the first year, but my expenses are 1,000, (b), dollars. The third year I again double my money and my expenses are 1,000, (c), dollars. I now find I have just doubled my original capital. How much had I?

91. A boy sold such a part of his apples that the number sold was to the number left as 3 : 4, $(a : b)$. Another boy stole 16 of the remainder, and then the number left was to the number sold as 2 : 3, $(c : d)$. How many had he originally?

92. My window is a yard wide. From a brass curtain rod I cut off one-sixth of its length, and finding it still too long, cut off one-fifth of the length remaining. But I had to cut it

the third time, taking off one-tenth its length to make it fit. Find length of the rod.

93. My room is 3, (a), feet longer than it is wide. By a mistake of the measurer the carpet sent was 6, (b), feet too short and 5, (c), feet too wide, yet, to my surprise, when the error was corrected, the bill for the second carpet was exactly the same as for the first. What were the dimensions of the two carpets?

94. A courier left a place 3, (n), days ago and travels 40, (a), miles each day. He is pursued by another who goes 60, (b), miles daily. In how many days will the second, starting to-day, overtake the first?

95. For one dollar I bought a certain number of five-cent stamps, three times as many two-cent stamps less one, and three times as many one-cent stamps plus four. How many of each kind did I buy?

96. The paving of a square court with stone at 40, (a), cents per square yard will cost as much as enclosing it with an iron fence at 1, (b), dollar per yard. How many yards square is the court?

97. What is the distance from A to B, if it takes a freight train running 20, (a), miles an hour 3, (b), hours longer to run it than it takes an express running 40, (c), miles an hour?

98. A furniture dealer bought 200 chairs at one dollar each. He sold one-fourth of them at a profit of 50 per cent., two-fifths at a profit of 25 per cent, and the remainder at cost. How much did he gain by the transaction?

99. In the morning I spent one dollar more than one-half my money, in the afternoon one-half a dollar more than half

of what remained, and in the evening one-half a dollar more than half of what still remained, when I found I had but one dollar left. How much had I at first?

100. In my right hand pocket I have as many dollars as I have cents in my left hand pocket. If I transfer \$6.93, (a), from my right pocket to my left I shall have as many dollars in my left pocket as I shall have cents in my right. How much money have I in my right pocket?

101. A and B together can do a piece of work in $4\frac{1}{10}$ days, A and C in $5\frac{1}{12}$ days and B and C in $4\frac{4}{5}$ days. How many days will it take each working alone?

Substitute a, b and c for the number of days, and find how many days it will take all, working together, to do it.

102. A man sold 60, (a), pounds of tea, part of it at 80, (b), cents a pound and part of it at 50, (c), cents a pound. The average price was 62, (d), cents a pound. How many pounds did he sell at each price?

103. A man invested some money as follows: one-third at 3 per cent., one-fourth at 4 per cent., one-fifth at 5 per cent., and the remainder at 6 per cent. How much money did he invest, to receive an income of \$258?

104. At what time between 4 and 5 o'clock are the hands of a watch opposite each other?

105. At what times between 4 and 5 are the hands of a watch at right angles to each other?

106. How many times and at what hours will the two hands of a watch be together during the time between midnight and noon?

107. How far can a man ride in 4, (a), hours at the rate of 6, (b), miles per hour and yet have time to return on foot walking 3, (c), miles per hour?

108. A man who can row 9 miles an hour finds it takes him twice as long to row up stream a certain distance as it does to return. At what rate does the stream flow?

109. In a certain family each son has as many brothers as sisters, but each daughter has twice as many brothers as sisters. How many are there of each?

110. A makes \$16,000 per year in his business. Of this sum he sets aside a certain amount for yearly expenses and invests the remainder as follows: Two-fifths of the remainder he invests at 3 per cent., two-thirds of what still remains at 4 per cent., and the other third at 5 per cent.

If his investment yields \$494 per year, how much does it cost him for annual expenses?

111. What must be the value of a in order that $\frac{a+2b}{3a+69b}$ may be equal to $\frac{1}{33}$ when b is $\frac{1}{3}$?

112. A regiment had sufficient to support n men through the campaign. But after 9 days the small pox broke out and s men died daily for 10 days. In consequence of this sickness the campaign lasted three days longer than was expected, when the food gave out. How many days would the food have lasted, had no sickness occurred?

113. The circumference of a fore wheel of a carriage is 9, (f), feet; that of a hind wheel, 12, (h), feet. What distance will the carriage have passed over when a fore wheel has made 10, (t), more revolutions than a hind wheel?

If the circumference of the fore wheel be 6 ft. and that of the hind wheel be 8 ft., how many more revolutions than a hind wheel will a fore wheel make each mile?

114. A grocer has two kinds of tea which he sells, one at p cents, and the other at q cents a pound. He wishes to

make a mixture of m pounds which shall cost on the average n cents a pound. How many pounds of each must he use?

Discuss the question (1) when $p = q$; (2) when p or $q = n$; (3) when $p = q = n$; (4) when $p > q$ and $< n$; (5) when $p > q$ and $q > n$.

115. A rich man at his death bequeaths to Harvard College a sum of money, s, together with the nth part of the remainder of his estate, after it has been turned into money. He bequeaths a second sum, $2s$, together with the nth part of what remains after the legacy to Harvard and $2s$ have been subtracted from the estate. To his heirs he leaves a sum $3s$, together with the nth part of what remains after both legacies and $3s$ have been subtracted. This exactly disposes of all his estate. What was it worth?

116. If the population of Boston increases each year p per cent. of that of the preceding year, what was its population n years ago providing that its population to-day is B?

SIMULTANEOUS EQUATIONS CONTAINING TWO UNKNOWN QUANTITIES.

1. $x + y = 8,\ x - y = 2.$
2. $x + 2y = 17,\ 3x - y = 2.$
3. $x = 4y - 3,\ x = 6y - 7.$
4. $x + 2y = 18,\ 2x - y = 1.$
5. $x + 4y = 16,\ 4x + y = 34.$
6. $x = 4y + 24,\ x = 2y + 14.$
7. $4x + 9y = 31,\ 2x - 6y = 12.$
8. $7x - 4y = 11,\ 3x + 2y = 27.$
9. $3x + 2y = 16,\ x - 4y = 24.$
10. $9x - 5y = 52,\ 8y - 3x = 8.$
11. $3x - 5y = 51,\ 2x + 7y = 3.$
12. $7y + 3x = 78,\ 19y - 7x = 136.$
13. $7x + 5y = 64,\ 4x - 2y = 22.$
14. $8x - 2y = 23,\ 5x + 6y = 18.$
15. $x + 3y = 24,\ 4x + 2y = 56.$
16. $4x + 7y = 19,\ 2x - 4y = -28.$
17. $3x + 4y = 26,\ x + 3y = 17.$
18. $4x + 3y = 13,\ 3x + 2y = 9.$
19. $3x + 2y = 118,\ 8 + 5y = 191.$
20. $14x + 6y = 0,\ 6x - 46 = 4y.$
21. $8x - 7y = -15,\ 3y - 6x = -9.$
22. $4x + 9y = 51,\ 8x + 13y = 9.$

23. $7y - 3x = 139,\ 2x + 5y = 91.$

24. $8x - 21y = 33,\ 6x + 35y = 177.$

25. $21y + 20x = 165,\ 77y - 30x = 295.$

26. $17x + 12y = 59,\ 19x - 4y = 153.$

27. $24x + 7y = 27,\ 8x - 33y = 115.$

28. $x = 3y - 19,\ y = 3x - 23.$

29. $3x - 5y = 51,\ 2x + 7y = 3.$

30. $x + 49y = 693,\ 49x + y = 357.$

31. $17x + 3y = 573,\ -3x + 16y = 23.$

32. $2x - 7y = 58,\ 4y - 9x = 69.$

33. $8x - 7y = 105,\ 7x - 8y = 105.$

34. $5x - 4y = 28,\ 8x + 3y = -21.$

35. $7x - 5y = 24,\ 4x - 3y = 11.$

36. $10x + 9y = 290,\ 12x - 11y = 130.$

37. $5x - 2y = 51,\ 19x - 3y = 180.$

38. $8x + 3y = 3,\ 12x + 9y = 3.$

39. $11x - 14y = 1,\ 11y - 14x = 26.$

40. $x + 19y = 97,\ 7x - 53y = 121.$

41. $29x - 14y = 175,\ 87x - 56y = 497.$

42. $171x - 213y = 642,\ 114x - 326y = 244.$

43. $43x + 2y = 266,\ 12x - 17y = 4.$

44. $5x + 9y = 188,\ 13x - 2y = 57.$

45. $2x - y = 118,\ 480x + 2y = 6.$

46. $\frac{x}{2} + \frac{y}{3} = 7,\ \frac{x}{3} + \frac{y}{2} = 8.$

47. $\frac{x}{2} - y = 1,\ x - \frac{y}{2} = 8.$

48. $\frac{x}{3}+\frac{2y}{5}=6,\ \frac{2x}{3}+\frac{y}{5}=6.$

49. $\frac{x}{7}+\frac{y}{9}=11,\ \frac{x}{9}+\frac{y}{2}=16.$ 50. $\frac{x}{3}+\frac{y}{2}=\frac{4}{3},\ \frac{x}{2}+\frac{y}{3}=\frac{7}{6}.$

51. $x+\frac{1}{3}y=y-2,\ y+\frac{1}{4}x=x+6.$

52. $\frac{x}{2}+\frac{y}{3}=7,\ \frac{x}{3}-\frac{y}{8}=\frac{1}{2}.$ 53. $\frac{x}{4}+\frac{y}{7}=14,\ 2x-y=7.$

54. $\frac{y}{5}-\frac{x}{10}=1,\ \frac{2y}{3}-\frac{x}{4}=5.$ 55. $\frac{x}{4}+\frac{y}{5}=5,\ \frac{2x}{3}+y=18.$

56. $\frac{5x}{6}-y=3,\ x-\frac{5y}{6}=8.$

57. $\frac{2}{5}x-\frac{1}{12}y=3,\ 4x-y=20.$

58. $\frac{1}{2}x-\frac{1}{5}y=4,\ \frac{1}{7}x+\frac{1}{15}y=3.$

59. $\frac{x}{3}+\frac{y}{4}=3x-7y-37=0.$

60. $\frac{x}{3}-\frac{y}{6}=\frac{1}{2},\ \frac{x}{5}-\frac{3y}{10}=\frac{1}{2},$ 61. $\frac{x}{2}+\frac{y}{3}=1,\ \frac{x}{4}-\frac{2y}{3}=3.$

62. $\frac{2x}{3}+\frac{3y}{4}=-\frac{7}{2},\ \frac{x}{4}-\frac{2y}{5}=\frac{11}{2}.$

63. $\frac{x}{6}+\frac{y}{5}=\frac{x}{2}+2,\ \frac{x}{4}+\frac{y}{3}=\frac{3y}{10}+4.$

64. $\frac{3}{8}y-\frac{1}{5}x+24=0,\ \frac{2}{7}y+\frac{1}{3}x+11=0.$

65. $\frac{2x}{3}+\frac{3y}{4}=700,\ y+100=\frac{5x}{6}.$

66. $x+2y=.6,\ 1.7x-y=.58.$

67. $.5x+.2y=1.1,\ .25x+.6y=2.05.$

68. $.5x+.75y=8,\ .2x+.1y=.16.$

69. $\frac{1}{3}(x+y)=\frac{1}{5}(x-y)$, $\frac{1}{7}(x+y)=\frac{1}{61}(x-y)-\frac{1}{7}$.

70. $\frac{3x}{10}-\frac{y}{15}-\frac{4}{9}=\frac{x}{12}-\frac{y}{18}$, $2x-2\frac{2}{3}=\frac{x}{12}-\frac{y}{15}+1\frac{1}{10}$.

71. $x+\frac{1}{2}(3x-y-1)=\frac{1}{4}+\frac{3}{4}(y-1)$, $\frac{1}{5}(4x+3y)=\frac{7}{10}y+2$.

72. $\frac{1}{3}(\frac{1}{4}x-\frac{1}{5}y+\frac{1}{6})=\frac{1}{4}(x-y)$, $\frac{1}{2}(\frac{1}{4}y-\frac{1}{5}x+\frac{1}{3})=\frac{1}{4}(x+y)$.

73. $\frac{1}{3}(x+y)-\frac{1}{2}(x-y)=9$, $\frac{1}{2}x+\frac{1}{3}(x+y)=5$.

74. $\frac{x-2}{8}+\frac{y-2}{8}=\frac{3}{4}$, $\frac{x+2}{3}-\frac{y+2}{3}=2$.

75. $\frac{x+y}{8}+\frac{x-y}{6}=5$, $\frac{x+y}{4}-\frac{x-y}{3}=10$.

76. $x+1:y=5:3$, $\frac{2y}{3}-\frac{5-x}{2}=3\frac{5}{12}-\frac{2y-1}{4}$.

77. $\frac{x}{2}-12=\frac{y}{4}+8$, $\frac{x}{3}-8-\frac{2y-x}{4}=27-\frac{x+y}{5}$.

78. $\frac{2x-y}{3}+6=\frac{2y-x}{2}+\frac{9}{2}$, $\frac{3x+y}{5}+1=\frac{3y+x+13}{10}$.

79. $\frac{2x-3y}{13}-\frac{3x-5y}{10}=\frac{1}{10}$, $\frac{x+4y}{11}+\frac{5x-4y}{7}=2$.

80. $\frac{x+1}{10}=\frac{3y-5}{2}=\frac{x-y}{8}$.

81. $\frac{x}{5}-2y=\frac{xy}{10}$, $\frac{x}{10}\left(3-\frac{y}{2}\right)=5y$.

82. $\frac{x+y}{.8}+\frac{y}{.6}=6.5$, $\frac{3x-.3y}{.2}=1$.

83. $\frac{.4x+2.5y}{2}-\frac{.2x}{.25}=2$, $x-y=1$.

84. $\frac{x}{2}+\frac{y}{2\frac{1}{2}}=1\frac{3}{4}$, $\frac{x-y}{11}=\frac{x}{1\frac{3}{4}}-\frac{3}{14}$.

85. $\frac{\frac{1}{3}(45x+4y)}{33}+2=y+1-\frac{1}{8}(3y+x-3)$,

$$\frac{3x+2y}{6}-\frac{y-5}{4}=\frac{11x+152}{12}-\frac{3y+1}{2}.$$

86. $\frac{1}{x}+\frac{1}{y}=\frac{8}{15}$, $\frac{1}{x}-\frac{1}{y}=\frac{2}{15}$.

87. $\frac{6}{x}-\frac{4}{y}=2$, $\frac{5}{x}-\frac{6}{y}=1$.

88. $\frac{1}{x}+\frac{2}{y}=\frac{5}{24}$, $\frac{1}{x}-\frac{2}{y}=\frac{1}{24}$.

89. $\frac{12}{x}-\frac{2}{y}=1$, $\frac{4}{x}+\frac{3}{y}=2\frac{1}{6}$.

90. $\frac{7}{x}-\frac{5}{y}=\frac{1}{6}$, $\frac{3}{x}+\frac{4}{y}=1\frac{2}{21}$.

91. $\frac{1}{x}+\frac{2}{y}=\frac{11}{15}$, $\frac{3}{x}+\frac{4}{y}=\frac{9}{5}$.

92. $\frac{6}{x}+\frac{8}{y}=5$, $\frac{2}{x}+\frac{4}{y}=2$.

93. $\frac{4}{x}+\frac{5}{y}=16$, $\frac{16}{x}-\frac{1}{y}=1$.

94. $\frac{5}{x}+\frac{3}{y}=2$, $\frac{8}{x}+\frac{6}{y}=3$.

95. $\frac{6}{x}+\frac{9}{y}=3$, $\frac{10}{x}-\frac{3}{y}=2$.

96. $\frac{1}{x}+\frac{2}{y}=10$, $\frac{4}{x}+\frac{3}{y}=20$.

97. $\frac{1}{x}+\frac{2}{y}=4$, $\frac{3}{x}-\frac{2}{y}=4$.

98. $\frac{15}{x}+\frac{21}{y}=10$, $\frac{20}{x}-\frac{6}{y}=2$.

99. $\frac{8}{x}-\frac{5}{y}=\frac{1}{6}$, $\frac{7}{x}-\frac{3}{y}=\frac{5}{6}$.

100. $\frac{3}{x}+\frac{2}{y}=1\frac{3}{32}$, $\frac{5}{x}+\frac{7}{y}=\frac{29}{12}$.

101. $\frac{1}{x}:\frac{1}{y}=13:11$, $\frac{5x+3}{2}=\frac{7y-4}{3}$.

102. $\frac{2}{x}+\frac{5}{3y}=\frac{4}{27}$, $\frac{1}{4x}+\frac{1}{y}=\frac{11}{72}$.

103. $\frac{4}{5x}+\frac{5}{6y}=5\frac{11}{15}$, $\frac{5}{4x}-\frac{4}{5y}=\frac{11}{20}$.

104. $\frac{6}{x}-\frac{1}{y}=.4$, $\frac{11}{x}-\frac{1}{2y}=1$.

105. $\frac{4}{x}-\frac{5}{y}=\frac{1}{2}$, $3x-y=\frac{xy}{20}$.

106. $\frac{1}{2x}+\frac{2}{3y}=3$, $\frac{3}{4x}+\frac{4}{5y}=3\frac{9}{10}$.

107. $\frac{10}{x}-\frac{5}{y}=\frac{10}{3}$, $\frac{10}{3x}-\frac{5}{2y}=\frac{5}{6}$.

108. $\frac{5}{3x}+\frac{3}{2y}=\frac{29}{24}$, $\frac{2}{5x}+\frac{5}{3y}=\frac{37}{60}$.

109. $\frac{2}{x+y}+\frac{2}{x-y}=1$, $\frac{3}{x+y}-\frac{2}{x-y}=0$.

110. $\frac{3}{x+1}:\frac{4}{y+4}=1:\frac{2}{3}$, $\frac{3}{x}=\frac{4}{y}$.

111. $1-\frac{x+y}{x-y}=\frac{3x}{x-y}$, $\frac{7x-3y}{23}=3$.

112. $\frac{1}{2(x+1)}+\frac{4}{3(y+1)}=5$, $\frac{1}{x+1}-\frac{1}{3(y+1)}=1$.

113. $\frac{10}{x+1}+\frac{8}{y-1}=7$, $\frac{10}{x+1}-\frac{8}{y-1}=3$.

114. $\frac{1}{3-x}+\frac{2}{5-y}=20$, $\frac{1}{3-x}-\frac{2}{5-y}=8$.

115. $\frac{5}{x+4}+\frac{6}{y-3}=8$, $\frac{9}{x+4}-\frac{4}{y-3}=7$.

116. $\frac{x+1}{y+1}-\frac{x-4}{y-1}=0$, $\frac{y+4}{x-5}-\frac{y+2}{x-4}=0$.

117. $\frac{6x+9}{4}+\frac{3x+5y}{4x-6}=3\frac{1}{4}+\frac{3x+4}{2}$, $\frac{8y+7}{10}+\frac{6x-3y}{2y-8}=4+\frac{4y-9}{5}$.

118. $x - \frac{2y-x}{23-x} = 20 - \frac{59-2x}{2}$, $y + \frac{y-3}{x-18} = 30 - \frac{73-3y}{3}$.

119. $3y + 11 = \frac{4x^2 - y(x+3y)}{x-y+4} + 31 - 4x$, $(x+7)(y-2) + 3 = 2xy - (y-1)(x+1)$.

120. $\frac{4x+7}{3} + \frac{5x-4y}{2x+1} = \frac{17+8x}{6}$, $\frac{5x-12}{4} - \frac{4x-6y-13}{2x-3y} = \frac{10x-53}{8}$.

121. $\frac{2(5-11x)}{11(x-1)} + \frac{11-7y}{3-y} = 5$, $\frac{7+2x}{3-x} - \frac{125-144y}{36(y+5)} = 2$.

122. $\frac{5x-3y+4}{3x-4y+5} + \frac{31-11y-7x}{12} = \frac{5y-4x}{3} - \frac{31y-9x-79}{12}$, $21x + 11y = 202$.

123. $\frac{12(7+6y)}{1-y} + \frac{432x-125}{3x+5} = 72$, $\frac{6(5-33y)}{3y-1} + \frac{11(11-21x)}{1-x} = 165$.

124. $8x - \frac{16+60x}{3y-1} = \frac{16xy-107}{5+2y}$, $2 + 6y + 9x = \frac{27x^2 - 12y^2 + 38}{3x-2y+1}$.

125. $3x + 6y + 1 = \frac{6x^2 + 130 - 24y^2}{2x-4y+3}$, $3x - \frac{151-16x}{4y-1} = \frac{9xy-110}{3y-4}$.

126. $16x + 6y - 1 = \frac{128x^2 - 18y^2 + 217}{8x-3y+2}$, $\frac{10x+10y-35}{2x+2y+3} = 5 - \frac{54}{3x+2y-1}$.

127. $\dfrac{7-6x}{10y-19}=\dfrac{4-3x}{5y-11}$, $\dfrac{6x-10y-17}{3x-5y+2}=\dfrac{4x-14y-5}{2x-7y+12}$.

128. $x+y=a$, $x-y=b$.

129. $x+y=s$, $x-y=d$.

130. $ax=by$, $x+y=c$.

131. $ax+by=c$, $x+y=1$.

132. $ax-by=m$, $x+y=n$.

133. $ax+by=p$, $ax-by=q$.

134. $cx+dy=m$, $ex+fy=n$.

135. $ax-cy=2$, $cx+ay=2$.

136. $ax+by=l$, $a'x+b'y=l'$.

137. $ax+cy=a^2$, $cx+ay=c^2$.

138. $x+ay=a'$, $ax+a'y=1$.

139. $px+qy=0$, $lx+my=n$.

140. $x+y=a+b$, $ax+by=a^2+b^2$.

141. $x+y=m+n$, $mx-ny=n^2-m^2$.

142. $px+qy=2pq$, $qx+py=-p^2+q^2$.

143. $cx+dy=m$, $c^2x+d^2y=n$.

144. $(a-b)x=(a+b)y$, $x+y=c$.

145. $x+ay+1=0$, $y+c(x+1)=0$.

146. $mx-ny=m^2+n^2$, $x+y=2m$.

147. $x-y=r-s$, $rx-sy=2r^2-2s^2$.

148. $e^2x-f^2y=0$, $ex+fy=f+e$.

149. $p^2x+q^2y=t^2$, $p^3x+q^3y=t^3$.

150. $ay+cx=2xy$, $by+dx=3xy$.

151. $3x-2y=(a+b)^2+3ab$, $3y-2x=(a-b)^2-3ab$.

152. $c(3c+x)=d(d+y)$, $cx+2dy=k$.

153. $x^2 - y^2 = a,\ x - y = b.$ 154. $x^2 - y^2 = c^2,\ x - y = d.$

155. $(m - n)x - (m + n)y = m^2 + n^2,\ my + nx = 0.$

156. $(m + n)x + (m - n)y = 2lm,\ (l + n)x - (l - n)y = 2ln.$

157. $p(x + y) + q(x - y) = 5,\ p(x - y) + q(x + y) = 3.$

158. $rx - sy = 2rs,\ 2sx + 2ry = 3s^2 - r^2.$

159. $(c - d)x + (c + d)y = 2c^2 - 2d^2,\ (c + d)x + (c - d)y = 2(c^2 + d^2).$

160. $\frac{x}{a} + \frac{y}{b} = 1,\ \frac{x}{a} - \frac{y}{b} = \frac{1}{2}.$ 161. $\frac{x}{a} + \frac{y}{b} = c,\ \frac{x}{d} = y.$

162. $\frac{x}{a} + \frac{y}{b} = 2,\ \frac{x}{c} + \frac{y}{c} = 1.$ 163. $\frac{x}{m} + \frac{y}{n} = 2,\ \frac{x}{m} - \frac{y}{n} = 1.$

164. $\frac{x}{p} + \frac{y}{q} = 1,\ x - y = p - q.$

165. $\frac{x}{a} - \frac{y}{c} = \frac{1}{m},\ \frac{x}{a'} + \frac{y}{c'} = \frac{1}{m'}.$

166. $\frac{x}{a} + \frac{y}{a'} = 1,\ \frac{x}{a'} - \frac{y}{a} = 1.$ 167. $\frac{x}{m} + \frac{y}{n} = 2,\ \frac{x}{m'} = \frac{y}{n'}.$

168. $\frac{x}{g} - \frac{y}{h} = 1,\ \frac{x}{h} + \frac{y}{g} = \frac{g}{h}.$

169. $\frac{3x}{a} + \frac{2y}{b} = 3,\ \frac{9x}{a} - \frac{6y}{b} = 3.$

170. $\frac{x}{m} + \frac{y}{n} = 1,\ \frac{x}{3m} + \frac{y}{6n} = \frac{2}{3}.$

171. $ax - bc = d(e - y),\ \frac{ax}{e} + b = d\left(1 + \frac{y}{c}\right).$

172. $\frac{x}{e} + \frac{y}{f} = 1 - \frac{x}{h},\ \frac{y}{e} + \frac{x}{f} = 1 + \frac{y}{h}.$

173. $\frac{mx}{n} + \frac{py}{q} = m + p,\ \frac{x}{3n} + \frac{y}{6q} = \frac{1}{2}.$

174. $\frac{x}{2r} + \frac{y}{3s} = \frac{5}{6},\ \frac{x}{r} + \frac{y}{s} = 2.$

175. $\frac{x}{2p} + \frac{y}{3q} = \frac{5}{6},\ \frac{x}{5p} - \frac{y}{2q} = 1\frac{2}{5}.$

176. $\frac{x}{c} + \frac{y}{d} = 2,\ \frac{x}{d} + \frac{y}{c} = \frac{c^2 + d^2}{cd}.$

177. $\frac{x}{m+n} - \frac{y}{m-n} = 1,\ \frac{x}{m+n} + \frac{y}{m-n} = 10.$

178. $\frac{x + py}{q - 2} = p,\ \frac{qx + py}{q} + p = 0.$

179. $\frac{x}{c+d} + \frac{y}{c-d} = 2c,\ \frac{x-y}{4cd} = 1.$

180. $\frac{x+y}{m} + \frac{x-y}{n} = 1,\ \frac{x-y}{m} - \frac{x+y}{n} - 1.$

181. $\frac{2x - c}{e} = \frac{3x - y}{2c + e} = \frac{e - 2y}{c}.$

182. $2 + \frac{2x - a}{b} = \frac{2y - b}{a},\ \frac{2bx + b^2}{a} + \frac{2ay - a^2}{b} = a + b.$

183. $(a + b)x - (a - b)y = 4ab,\ \frac{x}{a+b} + \frac{y}{a-b} = 2.$

184. $x(p + q) + y(p - q) = 2,\ px + qy = \frac{p^2 + q^2}{p^2 - q^2}.$

185. $\frac{x - n}{m - n} + \frac{y - p}{m - p} = 1,\ \frac{x + n}{m} + \frac{y - n}{n - p} = \frac{n}{m}.$

186. $\frac{x}{a+c} - \frac{y}{a-c} = \frac{1}{a+c},\ \frac{x}{a+c} + \frac{y}{a-c} = \frac{1}{a-c}.$

187. $\frac{1}{x}+\frac{1}{y}=a, \frac{1}{x}-\frac{1}{y}=b.$ 188. $\frac{a}{x}+\frac{b}{y}=1, \frac{b}{x}+\frac{a}{y}=1.$

189. $\frac{a}{x}+\frac{b}{y}=c, \frac{c}{x}+\frac{d}{y}=e.$

190. $\frac{2}{ax}+\frac{3}{by}=5, \frac{5}{ax}-\frac{2}{by}=3.$

191. $\frac{c}{dx}+\frac{d}{cy}=c+d, \frac{d}{x}+\frac{c}{y}=c^2+d^2.$

192. $\frac{a}{x}+\frac{c}{x'}=k, \frac{a'}{x}+\frac{c'}{x'}=k'.$

193. $\frac{m}{x}+\frac{n}{y}=\frac{2}{3}, \frac{n}{x}-\frac{m}{y}=\frac{3}{4}.$

194. $\frac{2}{x}+\frac{c}{y}=m, \frac{3}{x}+\frac{d}{y}=n.$ 195. $\frac{a+b}{x+y}=1, \frac{x+a}{y+b}=\frac{b}{a}.$

196. $\frac{x}{a}+\frac{y}{b}=c, \frac{m}{x}=\frac{n}{y}.$

197. $\frac{mx}{ny}=\frac{p}{q}, mx+ny=p+q.$

198. $\frac{x+a}{y-b}=\frac{c}{d}, dx+cy=e.$

199. $\frac{e}{f+y}=\frac{f}{e-x}, \frac{g}{h-x}=\frac{h}{g+y}.$

200. $\frac{x}{y}=\frac{m}{n}, \frac{x-n}{y-n}=2.$

201. $\frac{a}{bx}+\frac{b}{ay}=a+b, \frac{b}{x}+\frac{a}{y}=a^2+b^2.$

202. $\frac{m+n}{x}+\frac{m-n}{y}=5n-m, \frac{m}{x}+\frac{n}{y}=2m-3n.$

203. $\frac{x+y}{x-y}=\frac{r}{s-t}, \frac{x+t}{y-s}=\frac{r+s}{r+t}.$

204. $\frac{p+q}{x}-\frac{1}{py}+\frac{q}{p}=0, \frac{pq-q^2}{x}+\frac{1}{y}=\frac{p^2+3pq}{p+q}.$

205. $\frac{x+1}{y+1}=\frac{a+b+c}{a-b+c}, \frac{x-1}{y-1}=\frac{a+b-c}{a-b-c}.$

206. $\frac{x+y+1}{x-y+1}=\frac{a+1}{a-1}, \frac{x+y+1}{x-y-1}=\frac{1+b}{1-b}.$

SIMULTANEOUS EQUATIONS WITH MORE THAN TWO UNKNOWN QUANTITIES.

1. $x - y - z = 5$, $x + y - z = 25$, $x + y + z = 35$.

2. $x + y + z = 31$, $x + y - z = 25$, $x - y - z = 9$.

3. $x + y - z = 0$, $x + z - y = 2$, $y + z - x = 4$.

4. $x + y = 1$, $y + z = 9$, $x + z = 5$.

5. $x + y = 19$, $x + z = 18$, $y + z = 17$.

6. $x + y = 30$, $x + z = 25$, $y + z = 15$.

7. $x + y = 1$, $x + z = 8$, $y + z = -3$.

8. $x + y + z = 33$, $y - x + z = 23$, $z - x - y = 1$.

9. $x + y + z = 56$, $x + y - z = 4$, $x - y + z = 28$.

10. $x + y - z = 1320$, $x - y + z = 654$, $y + z - x = -12$.

11. $w + x + z = 10$, $x + y + z = 12$, $w + x + y = 9$, $w + y + z = 11$.

12. $x + y + z = 13$, $u + x + y = 17$, $u + x + z = 18$, $u + y + z = 21$.

13. $u + v + x + y = 16$, $u + v + x + z = 18$, $u + v + y + z = 20$, $u + x + y + z = 22$, $v + x + y + z = 24$.

14. $x + y + z = 26$, $x - y = 4$, $x - z = 6$.

15. $x + y + z + t + u = 25$, $x + y + z + u + v = 26$, $x + y + z + t + v = 27$, $x + y + t + u + v = 28$, $x + z + t + u + v = 29$, $y + z + t + u + v = 30$.

16. $uv = 180$, $vw = 270$, $uw = 216$.

17. $xyz = 60$, $uxy = 30$, $uxz = 6$, $uyz = 20$.

18. $4x - 3y - 2z = 12$, $x + 2y = 32$, $5z = 15$.

19. $x + y = 9$, $3z - y = 3$, $3y + z = 11$.

20. $2x + 2y = 20$, $3x + 3z = 57$, $4y + 4z = 92$.

21. $3x + 4z = 57$, $2y - z = 11$, $5x + 3y = 65$.

22. $x - y - z = 6$, $3y - x - z = 12$, $7z - y - x = 24$.

23. $x - 2y = 2$, $3x + z = 28$, $2y - 3z = 14$.

24. $2x - 3y = 3$, $3y - 4z = 7$, $4z - 5x = 2$.

25. $x - y + z = 0$, $x - 2y + 3z = 1$, $x - 3y + 4z = 0$.

26. $3x - 2y - 2z = 2$, $5x - y + 3z = 23$, $x - 5y + 6z = 1$.

27. $2x + 3y + 4z = 29$, $3x + 2y + 5z = 32$, $4x + 3y + 2z = 25$.

28. $7x - 3y - z = 12$, $x + 2y + 3z = 17$, $4x - y + 2z = 13$.

29. $4x + 2y - z = 26$, $5x + 2y - 3z = 16$, $2x - y + 2z = 23$.

30. $x + y + z = 90$, $2x + 40 = 3y + 20$, $2x - 10 = 4z - 40$.

31. $x + y + z + u = 14$, $3x + 2y - z = 28$, $4x - y = 29$, $3x - z = 22$.

32. $2y + 3z + u = 15$, $x + y + z - u = 8$, $x + y + z - 2u = 7$, $3x + 2y + 2z = 21$.

33. $x - 3y + 2x - u = 6$, $3x + 2y - z + 3u = 13$, $2x - 2y + 2z - 2u = 8$, $x + y + z + u = 10$.

34. $3x + 2y - 4z = 11$, $2x + y + u = 16$, $x + z + u = 10$, $2y + z - u = 9$.

35. $w + 50 = x$, $x + 120 = 3y$, $y + 120 = 2z$, $z + 195 = 3w$.

36. $7x + 2z + 3u = 17$, $4y + 2z + v = 11$, $5y - 3x - 2u = 8$, $4y - 3u + 2v = 9$, $8u - 3z = 33$.

37. $7x - 3y = 1$, $11z - 7v = 1$, $4z - 7y = 1$, $19x - 3v = 1$.

38. $x + 2y = 9$, $3y + 4z = 14$, $7z + u = 5$, $2u + 5x = 8$.

39. $3u - 2y = 2$, $2x + 3y = 39$, $5x - 7z = 11$, $4y + 3z = 41$.

40. $2x - 3y + 2z = 13$, $4u - 2x = 30$, $4y + 2z = 14$, $5y + 3u = 32$.

41. $.5x + .3y = 2.2$, $1.2z - 8.5y = -24.4$, $3.3z + 1.3x = 29$.

42. $2.5x + 1.5y = 6.5$, $3.5x - 1.5z = 4$, $.6y + z = 2.2$.

43. $1.7x - .6y = -4.08$, $2.8x + .6z = -6.72$, $2.8y - 1.7z = 19.04$.

44. $5x - 2(y + z + v) = -1$, $4z - 3(x + y + v) = 2$, $-12y + 3(x + z + v) = 3$, $8v - (x + y + z) = -2$.

45. $x + y + z = 100$, $.7x - 4 = y$, $.3x + 4 = z$.

46. $2x + 3y - z = 130$, $x : y = 1 : 5$, $7y = 5z$.

47. $x + y + z = 24$, $x : y = 7 : 8$, $y : z = 8 : 9$.

48. $x : y = 2 : 3$, $y : z = 3 : 4$, $7x - 5y + 3z = 66$.

49. $x + 2y : 3y + 4z = 7 : 8$, $3y + 4z : 5x + 6z = 8 : 9$, $x + y - z = 126$.

50. $x + y = xy$, $x + z = 2xz$, $2(y + z) = 3yz$.

51. $(4 - x)(244 - y) = z$, $(7 - x)(124 - y) = z$, $(13 - x)(64 - y) = z$,

52. $971x + z = xy - 214$, $965x + 20z = xy - 198$, $940x + 100z = xy - 48$.

53. $-yz + xz + xy = 4xyz$, $3xy + 2yz - 6xz = -xyz$, $4xy + 3yz + 2xz = 19xyz$.

54. $xy + xz + yz = xyz$, $xy + xu + yu = xyu$, $xz + xu + zu = xuz$, $zu + yu + yz = uyz$.

55. $(x + 1)(5y - 3) = (7x + 1)(2y - 3)$, $(4x - 1)(z + 1) = (x + 1)(2z - 1)$, $(y + 3)(z + 2) = (3y - 6)(3z - 1)$,

56. $(2x-9)(3y-7) = (3x-11)(2y-6)$,
$(3x-1)(4z-1) = (4x+2)(3z-2)$,
$(4y-5)(5z-4) = (5y-9)(4z-1)$.

57. $\frac{1}{3}(y-z) = \frac{1}{2}(y-x) = 5z-4x$, $y+z = 2x+1$.

58. $x+y+z = 29$, $x+2y+3z = 62$, $\frac{1}{2}x+\frac{1}{3}y+\frac{1}{4}z = 10$.

59. $\frac{x+y}{11} = 1$, $\frac{y+z}{4} = 2$, $\frac{x+z}{7} = 1$.

60. $\frac{2x+y}{3} = 2z$, $\frac{3y+z}{3} = 3$, $\frac{x+2z}{7} = y$.

61. $\frac{2x-y}{3} = \frac{3y+2z}{4} = \frac{x-y-z}{5} = 4$.

62. $11x-10y = \frac{12y-11z}{3}$, $\frac{x+z-2y}{3} = \frac{x-y-1}{2}$,
$3x = y+z+7$.

63. $\frac{x}{4}+\frac{y}{3}+\frac{z}{2} = 9$, $\frac{2x}{3}-\frac{y}{2}+\frac{3z}{4} = 11$, $\frac{3x}{4}+\frac{2y}{3}-\frac{z}{2} = 9$.

64. $x+\frac{1}{2}y = 100$, $y+\frac{1}{3}z = 100$, $z+\frac{1}{4}x = 100$.

65. $y+\frac{1}{2}x = 41$, $x+\frac{1}{4}z = 20\frac{1}{2}$, $y+\frac{1}{5}z = 34$.

66. $x+\frac{y}{2}+\frac{z}{3} = 6$, $y+\frac{z}{2}+\frac{x}{3} = -1$, $z+\frac{x}{2}+\frac{y}{3} = 17$.

67. $x+\frac{1}{2}y = 10-\frac{1}{3}z$, $\frac{1}{2}(x+z) = 9-y$, $\frac{1}{4}(x-z) = 2y-7$.

68. $\frac{3y-1}{4} = \frac{6z}{5}-\frac{x}{2}+\frac{9}{5}$, $\frac{5x}{4}+\frac{4z}{3} = y+\frac{5}{6}$, $\frac{3x+1}{7}-\frac{z}{14}$
$+\frac{1}{6} = \frac{2z}{21}+\frac{y}{3}$.

69. $\frac{x}{5}+\frac{y}{7}+\frac{z}{9} = 258$, $\frac{x}{7}+\frac{y}{9}+\frac{z}{5} = 304$, $\frac{x}{9}+\frac{y}{5}+\frac{z}{7} = 296$.

70. $\frac{x}{2}+\frac{y}{3}+z=13,\ 2x-3y+\frac{z}{2}=4,\ x+2y : 2y+z=5:8.$

71. $x-\frac{y}{2}-\frac{z}{3}=\frac{25}{3},\ y-\frac{z}{2}-\frac{x}{3}=-2,\ z-\frac{x}{2}-\frac{y}{3}=-\frac{19}{3}.$

72. $x+6=\frac{7}{8}y,\ y+1=3.5z,\ z+8=1.25x.$

73. $\frac{x}{2}-\frac{y}{3}+\frac{z}{4}=4,\ \frac{x}{3}+\frac{y}{4}-\frac{z}{5}=1,\ \frac{z}{4}-\frac{y}{6}-\frac{x}{2}=0.$

74. $x=21-4y,\ z=9-\frac{3}{4}x,\ y=64-7.5z.$

75. $\frac{x}{2}+\frac{y}{3}+\frac{z}{4}=62,\ \frac{x}{3}+\frac{y}{4}+\frac{z}{5}=47,\ \frac{x}{4}+\frac{y}{5}+\frac{z}{6}=38.$

76. $3x-\frac{1}{4}y+z=7.5,\ 2x-\frac{y-3z}{3}=\frac{16}{3},\ 2x-\frac{y}{2}+4z=11.$

77. $x+\frac{1}{2}y=20-\frac{1}{3}z,\ \frac{1}{4}(x+z)=\frac{y}{2}+1,\ \frac{2}{7}(y+z)=\frac{1}{2}(x-y)+1.$

78. $x+\frac{y}{2}-\frac{z}{3}=17,\ x+\frac{y}{7}=6-\frac{z+2}{5},\ \frac{x}{3}-y-\frac{z-13}{2}=\frac{1}{2}.$

79. $2x-\frac{3}{4}y=93-\frac{1}{2}x-\frac{1}{4}y,\ 7x-5z=x+y-86,$
$\frac{1}{2}x+\frac{1}{3}y+\frac{1}{4}z=58.$

80. $2y-327=221-3z,\ 5y+213=3x-246,$
$\frac{7}{8}x+452=\frac{33z}{2}-358.$

81. $2y-x=2u-z+2,\ x-2y+19=2y+19=3u+2z,$
$14-x-u=y+z,\ \frac{x+2z}{4}=\frac{60-5u-3y}{15}.$

82. $\frac{x}{3}+\frac{y}{5}+\frac{2z}{7}=58,\ \frac{x}{2}+\frac{3z}{8}+\frac{u}{5}=79,\ \frac{y}{5}+\frac{z}{2}+\frac{u}{25}=92,$
$\frac{5x}{4}+\frac{y}{6}+\frac{z}{3}=76.$

83. $\frac{4x+3y+z}{10}-\frac{2y+2z-x+1}{15}=5+\frac{x-z-5}{5}$,

$\frac{9x+5y-2z}{12}-\frac{2x+y-3z}{4}=\frac{7y+z+3}{11}+\frac{1}{6}$,

$\frac{5y+3z}{4}-\frac{2x-3y-z}{12}+2z=y-1+\frac{3x+2y+7}{6}$.

84. $\frac{5x-7y+2}{12}-\frac{8x+3z-4}{21}=\frac{11y-5z-4x+18}{14}$,

$\frac{11x-5z+12}{14}-\frac{3y+7z-2x}{18}=\frac{8z-3x+82}{21}$,

$3x-y-2z=16$.

85. $\frac{42-6y}{3}-\frac{5x-28}{4}=\frac{x+1}{7}-\frac{4z-2}{5}$, $\frac{3z-2y+x}{5}+19$

$=\frac{x-y}{6}+\frac{5x+3y+1}{3}$, $2\frac{1}{2}x+3\frac{1}{3}z=\frac{5y+2\frac{1}{4}x+4\frac{2}{3}z-2}{2}$.

86. $\frac{1}{u+v}=1$, $\frac{2}{u+w}=1$, $\frac{3}{v+w}=1$.

87. $\frac{3y-5x}{7x-4z}=1$, $\frac{x+3y}{3z-x}=1$, $\frac{4z+5}{3x+5y}=1$.

88. $\frac{x+y+z}{x-y+z}=-7$, $x-y-z=2$, $\frac{y-x+z}{z-x-y}=\frac{1}{8}$.

89. $\frac{1}{x}+\frac{1}{y}=6$, $\frac{1}{x}+\frac{1}{z}=8$, $\frac{1}{y}+\frac{1}{z}=10$.

90. $\frac{1}{x}+\frac{1}{y}=\frac{5}{6}$, $\frac{1}{y}+\frac{1}{z}=\frac{7}{12}$, $\frac{1}{x}+\frac{1}{z}=\frac{3}{4}$.

91. $\frac{1}{x}+\frac{1}{y}=8$, $\frac{1}{y}+\frac{1}{z}=15$, $\frac{1}{x}+\frac{1}{z}=13$.

92. $\frac{1}{y}-\frac{1}{x}=2$, $\frac{1}{z}-\frac{1}{y}=2$, $\frac{1}{z}+\frac{1}{x}=14$.

93. $\frac{1}{x}+\frac{2}{y}=5, \frac{3}{y}-\frac{4}{z}=-6, \frac{3}{z}-\frac{4}{x}=5.$

94. $\frac{7}{x}+\frac{6}{y}=5, \frac{7}{x}+\frac{27}{z}=8, \frac{1}{y}+\frac{1}{2z}=\frac{1}{3}.$

95. $\frac{1}{x}+\frac{2}{y}-\frac{3}{z}=1, \frac{5}{x}+\frac{4}{y}+\frac{6}{z}=24, \frac{7}{x}-\frac{8}{y}+\frac{9}{z}=14.$

96. $\frac{2}{x}+\frac{3}{y}=\frac{3}{4}, \frac{8}{x}+\frac{6}{z}=3, \frac{2}{y}+\frac{2}{z}=1.$

97. $\frac{1}{x}+\frac{4}{y}+\frac{3}{z}=\frac{8}{3}, \frac{3}{x}+\frac{2}{y}+\frac{2}{z}=2, \frac{2}{x}-\frac{3}{y}+\frac{1}{z}=\frac{1}{12}.$

98. $\frac{2}{x}+\frac{1}{y}+\frac{3}{z}=0, \frac{1}{z}+\frac{2}{y}+\frac{3}{x}=-\frac{4}{5}, \frac{1}{x}+\frac{3}{y}-\frac{2}{z}=0.$

99. $\frac{2}{x}-\frac{3}{y}+\frac{4}{z}=2.9, \frac{5}{x}-\frac{6}{y}-\frac{7}{z}=-10.4, \frac{9}{y}+\frac{10}{z}-\frac{8}{x}=14.9.$

100. $\frac{1}{x}+\frac{3}{2y}=\frac{9}{5}, \frac{1}{y}+\frac{4}{3z}=\frac{5}{3}, \frac{1}{z}+\frac{5}{4x}=\frac{7}{4}.$

101. $\frac{1}{2x}+\frac{1}{2y}+\frac{1}{2z}=\frac{13}{24}, \frac{1}{3x}-\frac{1}{5y}+\frac{1}{z}=\frac{7}{20}, \frac{1}{4x}-\frac{1}{y}-\frac{1}{3z}=\frac{7}{24}.$

102. $\frac{2}{x}-\frac{5}{3y}+\frac{1}{z}=\frac{85}{27}, \frac{1}{4x}+\frac{1}{y}+\frac{2}{z}=\frac{443}{72}, \frac{5}{6x}-\frac{1}{y}+\frac{4}{z}=\frac{433}{36}.$

103. $5x-3y=2xy, yz+2xz+3xy=9xyz, \frac{2}{x}+\frac{1}{z}=\frac{5}{2y}.$

104. $4y+3z=2yz, 2z+4w=3zw, 4w+3x=5wx, \frac{3}{y}+\frac{2}{x}=2.$

105. $\frac{2x-3y+8z}{u+1}=1, \frac{3y-2z}{4x}=\frac{7}{8}, \frac{4u-11}{4z-5}=3, \frac{1}{x}+\frac{1}{y}=\frac{5}{2y}.$

106. $\frac{2x+2}{3y+1}=\frac{2y-5}{3x-11}, \frac{11-3x}{2z-5}=\frac{6x-7}{4z+9}, \frac{4y+3}{4z-3}=\frac{5y+6}{5z-3}.$

107. $\frac{xy}{x+y}=70,\ \frac{xz}{x+z}=84,\ \frac{yz}{y+z}=140.$

108. $\frac{xy}{x+y}=\frac{1}{5},\ \frac{xz}{x+z}=\frac{1}{6},\ \frac{yz}{y+z}=\frac{1}{7}.$

109. $\frac{xy}{4y-3x}=20,\ \frac{xz}{2x-3z}=15,\ \frac{yz}{4x-5z}=12.$

110. $\frac{28}{2x+y}+\frac{12}{3y-z}=6,\ \frac{21}{3y-z}-\frac{35}{3x+z}=0,$
$\frac{100}{3x+z}-\frac{7}{2x+y}=9.$

111. $\frac{12}{2x+3y}-\frac{7.5}{3x+4z}=1,\ \frac{30}{3x+4z}+\frac{37}{5y+9z}=3,$
$\frac{222}{5y+9z}-\frac{8}{2x+3y}=5.$

112. $\frac{8}{2x-3y+z}+5x-8y+38=0,$
$\frac{5}{2x-3y+z}+16y-11x=80\frac{1}{2},\ 3x+2y=36.$

113. $\frac{8}{4x+2y-3z}+8x-6y+2=0,$
$\frac{1680}{4x-2y+3z}+12y-16x-4=0,$
$3x+4y+2z=52,\ x+z=2y.$

114. $\frac{3}{x+y+z}+\frac{6}{2x-y}+\frac{1}{y-3z}=1,\ \frac{6}{x+y+z}+\frac{4}{2x-y}$
$-\frac{1}{y-3z}=3,\ \frac{15}{x+y+z}-\frac{2}{2x-y}-\frac{3}{y-3z}=5.$

115. $x+y=a,\ x+z=b,\ y+z=c.$

116. $y+z=2a,\ z+x=2b,\ x+y=2c.$

117. $x + 2y = a,\ 2x + 4z = 3a,\ 4y + 5z = 5a.$

118. $x + a = y + z,\ y + a = 2x + 2z,\ z + a = 3x + 3y.$

119. $x + y + z = r,\ x - y + z = s,\ x + y - z = t.$

120. $x + 11a = y + z,\ y = 2(x + z) - 11a,\ z = 3(x + y) - 11a.$

121. $x + y + 2z = 3a,\ 2x - y + z = 6b,\ 3x + 2y - z = 6a.$

122. $2x + y = a,\ 2y + z = b,\ 2z + x = c.$

123. $x + 3y + 2z = b,\ 3x + 5y - 2z = m,\ 4x - y + z = n.$

124. $x + y - z = c,\ x + z - y = b,\ y + z - x = a.$

125. $ay + bx = c,\ cx + az = b,\ bz + cy = a.$

126. $ax + by + cz = 3,\ ax + by - cz = 1,\ ax - by + cz = 1.$

127. $mx + y = n,\ z - y = p,\ qx + z = t.$

128. $ax + by = 1,\ by + cz = 1,\ cz + ax = 1.$

129. $ax + by = c,\ bx + cz = a,\ cy + az = b.$

130. $x + y + z = a,\ my = nx,\ qy = pz.$

131. $x + y + z = a + b + c,\ bx + cy + az = cx + ay + bz = a^2 + b^2 + c^2.$

132. $x + y = a,\ y + z = b,\ z + u = c,\ u - x = d.$

133. $x + y = a + b,\ y + z = a - c,\ z + u = a - b,\ u - x = c - a.$

134. $2x + y + z + u = a,\ x + 2y + z + u = b,\ x + y + 2z + u = c,\ x + y + z + 2u = d.$

135. $x + y - u = m,\ y + z - v = n,\ z + u - x = o,\ u + v - y = p,\ v + x - z = q.$

136. $ax + by - cz = a^2 + b^2,\ ax = abz + b^2,\ by = abz + a^2.$

137. $x + y + z = 0,\ ax + by + cz = 0,\ bcx + acy + abz + (b - c)(c - a)(a - b) = 0.$

138. $a(yz - xz - xy) = xyz,\ b(xz - xy - yz) = xyz,$
$c(xy - yz - xz) = xyz.$

139. $ax + by + cz = A,\ a^2x + b^2y + c^2z = A^2,\ a^3x + b^3y + c^3z = A^3.$

140. $\frac{x}{a} + \frac{y}{b} = 2,\ \frac{y}{b} + \frac{z}{c} = 2,\ \frac{z}{c} + \frac{x}{a} = 2.$

141. $\frac{a}{x} + \frac{b}{y} = a + 4,\ \frac{b}{y} + \frac{c}{z} = b + c,\ \frac{c}{z} + \frac{a}{x} = a + 2.$

142. $\frac{1}{x} + \frac{1}{y} = a,\ \frac{1}{x} + \frac{1}{z} = b,\ \frac{1}{y} + \frac{1}{z} = c.$

143. $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = a,\ \frac{1}{x} - \frac{1}{y} + \frac{1}{z} = b,\ \frac{1}{y} + \frac{1}{z} - \frac{1}{x} = c.$

144. $\frac{1}{x} + \frac{1}{y} - \frac{1}{z} = a,\ \frac{1}{x} - \frac{1}{y} + \frac{1}{z} = b,\ \frac{1}{y} + \frac{1}{z} - \frac{1}{x} = c.$

145. $\frac{x}{a} + \frac{y}{b} + \frac{c}{z} = 1,\ \frac{x}{a} + \frac{y}{c} + \frac{z}{b} = 1,\ \frac{x}{b} + \frac{y}{a} + \frac{z}{c} = 1.$

146. $\frac{x}{a} + \frac{y}{b} - \frac{z}{c} = \frac{1}{2},\ \frac{y}{b} + \frac{z}{c} - \frac{x}{a} = \frac{1}{2},\ \frac{z}{c} + \frac{x}{a} - \frac{y}{b} = \frac{1}{2}.$

147. $\frac{a}{x} + \frac{b}{y} + \frac{c}{z} = 3,\ \frac{a}{x} + \frac{b}{y} - \frac{c}{z} = 1,\ \frac{2a}{x} - \frac{b}{y} - \frac{c}{z} = 0.$

148. $\frac{a}{x} + \frac{b}{y} - \frac{c}{z} = ac,\ \frac{b}{y} + \frac{c}{z} - \frac{a}{x} = ab,\ \frac{c}{z} + \frac{a}{x} - \frac{b}{y} = bc.$

149. $\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{1}{a},\ \frac{1}{w} + \frac{1}{y} + \frac{1}{z} = \frac{1}{b},\ \frac{1}{w} + \frac{1}{x} + \frac{1}{z} = \frac{1}{c},$
$\frac{1}{w} + \frac{1}{x} + \frac{1}{y} = \frac{1}{d}.$

150. $\frac{ax}{b} + \frac{by}{c} + \frac{cz}{a} = a + b + c,\ \frac{cx}{b} + \frac{ay}{c} = a + c,\ cy + az = a^2 + c^2.$

151. $\frac{x}{a}+\frac{y}{b}+\frac{2c}{z}=3,\ \frac{2x}{a}+\frac{3y}{b}+\frac{c}{z}=2,\ \frac{3x}{a}-\frac{2y}{b}+\frac{3c}{z}=11.$

152. $\frac{x}{a+b}+\frac{y}{b+c}=b-a,\ \frac{y}{c-a}+\frac{z}{c+a}=c+a,$

$\frac{x}{b-c}-\frac{z}{a-b}=b-c.$

153. $\frac{x+y}{a+b}=\frac{y+z}{a},\ \frac{y-x}{y+x}=\frac{a-b}{a+b},\ x+y+z=a+b.$

154. $\frac{x}{a+b}+\frac{y}{b-c}+\frac{z}{a+c}=2c,\ \frac{x}{a-b}-\frac{y}{b-c}+\frac{z}{c-a}=2a,$

$\frac{x}{a-b}-\frac{y}{b-c}-\frac{z}{b+c}=2a-2c.$

155. $r-\frac{x}{s+t}=\frac{y}{t-r}-s,\ s-\frac{y}{r+t}=\frac{z}{r-s}-t,$

$t-\frac{z}{r-s}=\frac{x}{s-t}-r.$

156. $\frac{e-f}{fx+ey}=\frac{(f-g)(e-g)}{g},\ \frac{f-g}{gy+fz}=\frac{(g-e)(f-e)}{e},$

$\frac{g-e}{ex+gx}=\frac{(e-f)(g-f)}{f}.$

PROBLEMS INVOLVING TWO UNKNOWN QUANTITIES.

1. A farmer sells to one person 9, (a), horses and 7, (b), cows for 300, (m), dollars, and, at the same prices, 6, (a_1), horses and 13, (b_1), cows for the same sum. What was the price of each?

2. A vintner wishes to fill a cask containing 84, (n), gallons with wine, so that it may cost him \$100.80, (s), in all. He draws his wine from two casks which cost \$1.00, (p), and \$1.42, (q), a gallon respectively. How much of each kind must he take?

3. The sum of two digits is 9, (s), and six, (a), times one of the numbers they form is equal to five, (b), times the other. Find the numbers.

4. Find two numbers such that one shall be as much greater than 10, (h), as the other is less than 10, (h), and one-tenth, $(1/k)$, of their sum shall equal one-fourth, $(1/e)$, of their difference.

5. A man pays with a \$5, (d), note two bills, one of which is six-sevenths, (a/c), of the other, and receives back in change seven, (n), times the difference of the bills. Find their amounts.

6. A boy, being asked his age and that of his sister, replied, "If I were 3, (a), years older, I would be 3, (n), times as old as my sister; but, if she were 2, (b), years older, she would be one-half, $(1/e)$, as old as I am." How old was each?

7. Find two numbers whose ratio is that of 3 to 7, (a to b), but, if 8, (k), is added to each of them, the ratio of the results will be that of 5 to 9, (a^1 to b^1).

8. Find two numbers such that, if the first be increased by 8, (a), it will be 2, (m), times the second ; and if the second be increased by 31, (b), it will be 3, (n), times the first.

9. If I mix brandy and wine, putting in twice, (a times), as much of the former as of the latter, the mixture is worth 22, (m), dollars a dozen. But if I put in twice, (b times), as much of the latter as of the former, the mixture is worth 24, (n), dollars a dozen. Find price per dozen of each.

10. A certain fraction would be doubled by adding 14, (a), to its numerator and 6, (c), to its denominator, and it would be trebled by adding 7, (p), to its numerator and taking 4, (q), from its denominator. Find the fraction?

11. A number consists of two digits whose difference is 1 ; if it be diminished by the sum of its digits, the digits will be reversed. What is the number?

Show what the sum of the digits must be.

12. A certain fraction which is equal to $\frac{4}{5}$ is increased to $\frac{5}{6}$ by having the same number added to both its terms, and is multiplied by 2 by having another number taken from both its terms. Find the numbers.

13. A certain sum of money at simple interest amounts to 1,375, (a), dollars in 15, (m), months, and to 1,425, (b), dollars in 21, (n), months. Find the principal and the rate of interest.

14. The smaller of two numbers divided by the larger gives .25 with a remainder of .02 ; the larger divided by the smaller gives 3, with a remainder of 1.44. What are the numbers?

15. The difference of two numbers is 8, (d), and two, (a), times the sum of their reciprocals is equal to 3, (b), times the difference of their reciprocals. Find the numbers.

16. Find two numbers such that, if 1, (e), be subtracted from the first and added to the second, the results will be equal; while if 5, (f), be subtracted from the first and the second be subtracted from 5, (f), these results will also be equal.

17. Find a number such that, if it be multiplied by 4, (m), and the product increased by 3, (a), the result will be the same as if it were increased by 4, (m), and the sum multiplied by 3, (a).

18. If 45, (a), be subtracted from a number, the remainder will be a certain multiple of 5, (m); but if the number be subtracted from 135, (b), the remainder will be the same multiple of 10, (n). What is the number and what multiple of 5, (m), is the first number?

19. A man setting out on a journey drove at the rate of 2, (a), miles an hour to the nearest railway station, distant 6, (b), miles from his house. On arriving at the station he found that the train had left 1, (c), hour before. At what rate per hour should he have driven in order to reach the station just in time for the train?

20. I row 8, (p), miles with the stream in 64, (a), minutes, and return against the stream in $137\frac{1}{7}$, (c), minutes. At what rate would I row in still water and at what rate does the stream flow?

21. A sum of money is divided equally among a certain number of persons. Had there been 56, (m), more each would have received 1, (a), dollar less; if 24, less, each

would have received 1, (b), dollar more. How many persons were there, and how much did each receive?

22. Two persons, 27, (c), miles apart, setting out at the same time, are together in 9, (m), hours, if they walk in the same direction, but in three, (n), hours, if they walk in opposite directions. Find their rates of walking.

23. A man can row down stream a distance of 20, (a), miles, and back again in 10, (b), hours, and finds he can row 2, (d), miles against the current in the same time that he rows 3, (c), miles with it. Find the time in going and returning, also the rate of the current and the rate of rowing.

24. If the number of men engaged upon a certain piece of work be made 5, (c), greater, the work can be done in 4, (a), days; if 5, (d), less, in 12, (b), days. How many men are there and in how many days can they do it?

25. A man and his wife labored 6, (m), days and received 24, ($2a$), dollars for compensation. Had the wife been idle, and on expense at the same daily rate as her wages, they would have received but 12, ($2c$), dollars. What were the daily wages of each?

26. The report of a cannon travels 172.21, (a), yards with the wind toward A in the same time that it travels 167.97, (b), yards against the wind toward B. Three, (n), seconds after it is fired it is heard at A and B, which are 2041.98, (s), yards apart. What is the velocity of the report in still air and what is the velocity of the wind?

27. Two vessels contain mixtures of wine and water. In one there is 3, (p), times as much wine as water, and in the other 5, (q), times as much water as wine. How many gallons must be taken from each to fill a third vessel whose capa-

city is 7, (s), gallons, so that its contents may be half wine and half water?

28. I invest \$12,000, (s), in bonds paying 3, (a), per cent, real estate paying 4, (b), per cent, and stock paying 5, (c), per cent. The sum invested at 5, (c), per cent is half as much as the other two investments. If my income is \$490, (k), per year, how much have I invested in each?

29. A student walking from Cambridge to Boston notices that horse-cars coming out meet him every 3, (h), minutes, and those going in overtake him every 9, (k), minutes. Supposing the cars to be started at regular intervals, so that each car is half a mile ahead of the next one, how many miles an hour is the student walking, and how fast do the cars go?

30. Each of two servants was to receive \$160, a dress and a pair of shoes for one year's services. One servant left after 8 months and received the dress and \$106; the other servant left after $9\frac{1}{2}$ months and received a pair of shoes and \$142. What was the value of the dress and of the pair of shoes?

31. Two points move around a circle whose circumference is 100, (r), feet. When they move in the same direction they are together every 20, (s), seconds, when in opposite directions they meet every 4, (t), seconds. Required their rates.

32. 24 Ovids and 12 Cæsars will just fill a certain shelf; 6 Ovids and 10 Cæsars will fill half of it. How many of each alone will fill it?

33. A publisher sent 7 histories and 12 algebras by mail. Having no scales he made a pair of wooden balances and found that 4 histories just balanced 7 algebras, but that 3 histories just balanced 5 algebras and a hammer whose weight was 7 ounces. What was the weight of each history and algebra?

34. The quantity of water which flows from an orifice is proportional to the product of the area of the orifice and the velocity of the water. There are two orifices in a reservoir, the areas being as 5 : 13, and the velocities as 8 : 7. From one there issues in a certain time 561 cubic feet more than from the other. How much water does each orifice discharge during this time?

35. A starts from Boston to New York and B from New York to Boston, traveling uniformly. After they have met it takes A 16, (e), hours to reach New York and B 36, (f), hours to reach Boston. Find in what time each performed the journey.

36. A fires at a target 500 yards distant and hears the bullet strike $4\frac{1}{3}$ seconds after he fires. B, standing 400 yards from the target and 650 yards from A hears the bullet strike $2\frac{1}{3}$ seconds after he hears the report. Required the velocity of sound and of the bullet, each supposed to be uniform.

37. A and B can do a piece of work in 8, (a), hours. After working together 4, (b), hours, B finishes the work in 12, (c), hours. In how many hours could each alone do it?

38. A and B run a race of 336 yards. The first heat A gives B a start of 28 yards and beats him 2 seconds; the second heat A gives B a start of 12 seconds and is beaten by 48 yards. How many yards can each run in a second?

39. Two trains, 92 feet long and 84 feet long respectively, are moving with uniform velocities on parallel rails. When they move in opposite directions they are observed to pass each other in one second and a half, but when they move in the same direction the faster train passes the other in six seconds. Find the rate at which each train moves.

40. A stage set out from Colorado Springs to Cripple Creek with a certain number of passengers, four more on the outside than inside. The fare of seven outside passengers was half a dollar less than the fare of four inside passengers and the whole fare, at the outset amounted to \$45. At the end of half the journey it took up three more outside and one more inside passenger at proportional rates, in consequence of which the whole fare received was $1\frac{2}{15}$ times what it was before. What was the number of passengers at the start and the fare of each?

41. How long will it take before the hands of a clock assume the same relative position they have at this moment?

42. It is a law of physics that, if a body starts with a velocity of u feet per second and if this velocity increases f feet per second, then at the end of t seconds the body will have passed over $ut + \frac{1}{2}ft^2$. Suppose f is uniform and that in the eleventh and fifteenth seconds the body passes through 24 ft. and 32 ft. respectively, find u and f.

43. A person walking in a thick fog meets one wagon and overtakes another traveling at the same rate as the former. If 100, (a), feet be the greatest distance to which he can see, and 75, (b), and 150, (c), feet the distances which he walks between the times of his first seeing and passing the wagons, what equation results?

44. A train, after traveling an hour from A towards B, meets with an accident which detains it half an hour, after which it proceeds at four-fifths its usual rate and arrives an hour and a quarter late. If the accident had happened 30 miles farther on, the train would have been only an hour late. Find the usual rate of the train.

45. Two trains are running westward on parallel tracks, at the rate of a and b miles an hour respectively. At noon they are m miles apart. When are they together? When is the unknown quantity positive? When does it become zero? infinite? indeterminate? negative? Explain the signification of each result.

PROBLEMS INVOLVING MORE THAN TWO UNKNOWN QUANTITIES.

1. Three children being weighed, taken two together, weighed 76, (a), 90, (b), and 94, (c), pounds. How much did each weigh?

2. Bicycles of three different makes were sold at auction, one of each kind bringing \$41. One of the first kind, two of the second and three of the third brought \$85. There was then left two of the first, one of the second, and one of the third, which brought \$45.50, but a deduction of one-half had to be made in the price of the third, as it was damaged. What price did each kind bring?

3. The number of bones in the fingers exceed in number in the wrist by 6, the number in the wrist is twice as many as in the hand less 2, and the number in the wrist and hand together is 1 less than in the fingers. How many bones are in each?

4. A and B spent 18, (n), dollars, B and C spent 14, (p), dollars, and A and C spent 16, (q), dollars. How much did each spend?

5. The sum of three numbers is 59, (s); one-half the difference of the first and second is 5, (m), and one-half the difference of the first and third is 9, (n). Find the numbers.

6. In a school containing three classes the first and second numbered 10 more than half the whole, the second and third 50 more than half the whole, and the first and third 30 more than half the whole. How many pupils were there in each class?

7. A's money added to three times B's and C's amounts to \$470 ; B's money added to four times A's and C's amounts to \$580 ; C's money added to five times A's and B's amounts to \$630. How much money has each ?

8. If to double the height of Bunker Hill Monument 40 feet be added, the sum will be equal to the height of the pyramid of Cheops. If the monument, starting at one angle of the pyramid, be laid three times along the side of the base of the pyramid, there will be left of the side 6 feet less than half the length of the monument. If the monument be put on top of the pyramid, the total height would be less than a side of the base of the pyramid by 64 feet. Find the heights of the monument and pyramid and the length of the side of the base of the pyramid.

9. Divide the number 130, (s), into three parts such that twice the first part increased by 30, (a), three times the second part increased by 15, (b), and four times the third part increased by 50, (c), shall all equal one another.

10. Divide 235, (m), dollars among three men, giving B 25, (a), dollars more than C, and A 35, (b) dollars more than B.

11. Find three numbers such that the first with $\frac{1}{3}$ of the other two, the second with $\frac{1}{4}$ of the other two, and the third with $\frac{1}{5}$ of the other two, shall be equal to 25.

12. A comparison of the cubic inches in the brains of the European race, of the Bushmen of Africa, and of the gorilla gives the following results : The European's brain contains 7 cubic inches more than 3 times the gorilla's. The Bushman's contains 30 more than half the European's. The Bushman's and gorilla's together are twelve more than the European's. How many cubic inches in the brain of each ?

13. The sum of three fractions is 2. The second fraction is double the first and the third is double the second. What are the fractions?

14. The average age of three persons is 60, (k), years. The average of the first and second is 52, (d), years, and of the second and third is 70, (g), years. Find the age of each.

15. The digits of a number of three figures have equal differences in their order. If the dumber be divided by one-half the sum of its digits, the quotient is 41 ; and if 594 be added to the number, the digits will be inverted. Find the number.

16. Three cities, M, N and Q, are situated at the vertices of a triangle. The distance from M to Q by way of N is 82 miles, from M to N by way of Q is 97 miles, from Q to N by way of M is 89 miles. How far are M, N and Q from one another?

17. Of what three numbers is it true that the sum of the reciprocals of the first and second is $\frac{1}{2}$, $(1/a)$, of the first and third $\frac{1}{3}$, $(1/c)$, of the second and third $\frac{1}{4}$, $(1/n)$?

18. A man worked for a person 10 days, having his wife with him for 8 days and his son for 6 days, and received \$10.30 as compensation for all three. At another time he worked 12 days, his wife 10 days and his son 4 days, and received \$13.20. At another time he worked 15 days, his wife 10 days and his son 12 days, and received \$13.85. What were the daily wages of each?

19. A tank has three pipes. The first and second will empty it in 1 hour ; the second and third in 2 hours ; the first and third in $1\frac{1}{5}$ hours. How many minutes will it take each pipe to empty the tank?

20. Three casks together contain 75, (m), gallons of wine. By pouring 8, (c), gallons from the first into the second and then 7, (d), gallons from the second into the third the quantities in the three casks become equal. How much did each cask contain at first?

21. A and B can do a piece of work in 12, (a), days, B and C in 20, (c), days, and A and C in 15, (b), days. In how many days can each alone do it? All together?

22. My brother and I can do a piece of work in 6 days; you and I would require 9 days to do it. In what time could you and he do it, providing that he works twice as fast as I?

23. In a 10, (p), mile race John can beat James by 2, (q), miles and Charles by 4, (t), miles. By how many miles can James beat Charles?

24. The difference between two fractions which have the same denominator is $\frac{2}{3}$. If 2 be added to the numerator of the smaller, its value will be one half of the larger; if 2 be subtracted from the numerator of the larger, its value will be three times the smaller. Find the fractions.

25. A piece of work can be completed by A working 3 days and B 7 days and C 1 day; or by A working 5 days and B 1 day and C 7 days; or by A working 1 day and B 5 days and C 11 days. In how many days can each alone perform the work?

26. A, B and C together have \$24. If A gives to B and C as much as they already have, and then B gives to A and C as much as they have after the first distribution, and again C gives to A and B as much as they have after the second distribution, they will all have the same sum. How much has each?

27. A sum of money consists of quarter-dollars, dimes and half-dimes. Its value is as many dimes as there are pieces of money; its value is, also, as many quarters as there are dimes; and the number of half-dimes is one more than the number of dimes. Find the number of each coin.

28. A number is expressed by three digits whose sum is 12. Reversing the order of the first two figures diminishes the number by 180, and interchanging the last two increases it by 9. What is the number?

29. There are four men, A, B, C and D, the value of whose estates is a dollars. $\frac{1}{2}$ A's, $\frac{1}{3}$ B's, $\frac{1}{4}$ C's and $\frac{1}{5}$ D's is \$5,830. $\frac{1}{3}$ A's, $\frac{1}{4}$ B's, $\frac{1}{5}$ C's and $\frac{1}{2}$ D's is \$5,000. $\frac{1}{4}$ A's, $\frac{1}{5}$ B's, $\frac{1}{2}$ C's and $\frac{1}{3}$ D's is \$4,760. $\frac{1}{5}$ A's, $\frac{1}{2}$ B's, $\frac{1}{3}$ C's and $\frac{1}{4}$ D's is \$5,200. Find the property of each.

30. There are four numbers such that, by adding to each twice the sum of the remaining three, we obtain 46, 43, 41, and 38 respectively. What are the numbers?

31. A regiment of 400 men is quartered in a four-story building. On the fourth floor are one third as many men as are on the first; on the second and third are as many as are on the first and fourth; and to every 9 men on the third floor there are 11 on the second. How many men are there on each floor?

32. Divide the number 192, (s), into four parts such that if the first is increased by 7, (a), the second diminished by 7, (a), the third multiplied by 7, (a), and the fourth divided by 7, (a), the sum, remainder, product and quotient shall all be equal to one another.

33. Divide 105 into four parts such that the first is to the second as 2 : 3, and the second is to the third as 4 : 5, and the third is to the fourth as 6 : 7.

34. A and B can do a piece of work in 10 days, A and C in 12 days, A and D in 20 days, and B, C and D in $7\frac{1}{2}$ days. In how many days can they do the work separately? All together?

35. The year in which printing was invented is expressed by a number of four digits, whose sum is 14. The ten's digit is one-half the unit's digit, and the hundred's digit is equal to the sum of the thousand's and the ten's digit. If the order of the digits be reversed, the resulting number will be equal to the original number increased by 4,905. In what year was printing invented?

36. Five players, A, B, C, D, E, throw dice upon condition that he who has the lowest throw shall give all the others the sum which they already have. Each loses in turn, beginning with A, and at the end of the fifth game each has the same sum, \$32. How much had each at first?

37. A cistern has three pipes, A, B and C. If A and B run in while C runs out, it will be filled in $28\frac{4}{7}$ minutes. If B and C run in while A runs out, it will be filled in 40 minutes. If A and C run in while B runs out, it will be filled in $66\frac{2}{3}$ minutes. In what time would each fill it alone, the others not running?

38. A invests a sum of money at a certain rate of interest. B invests \$3,000 more than A at 1 per cent. less, and his income is \$45 less. C invests \$2,000 less than A at 1 per cent. more, and his income is \$40 greater. Find each man's capital and the rate at which it is invested.

39. There are two numbers, of three digits each, each the same as the other with the digits inverted. The sum of the two numbers is 584. The sum of the digits of each number

is 13. The difference between the units digits is 2. What are the numbers?

40. Three men travel from Boston to Lexington. The second man travels every 4 hours 1 mile less than the first, and is 4 hours longer in making the journey. The third man travels every 3 hours $1\frac{3}{4}$ miles more than the second, and is 7 hours less in making the journey. How far is it from Boston to Lexington and how many hours does it take each man to make the journey?

41. M, N and P are three towns situated at the apexes of a triangle. A man has to walk from one to the next, ride thence to the next, and drive thence to his starting point. He can walk a mile in m minutes, ride a mile in n minutes, and drive a mile in p minutes. If he starts from N, he takes $m + p - n$ hours; if from P, $m + n - p$ hours, if from M, $n + p - m$ hours. Find the length of the circuit.

42. If the sum of two numbers, each of three digits be increased by 1, the result will be 1,000. If the greater be placed on the left of the less, and a decimal point be placed between them, the resulting number will be six times the number obtained by placing the smaller number on the left of the greater, with a decimal point between them. What are the numbers?

43. There is a certain number of six figures, the figure in the unit's place being 4. If this figure be carried over the other five to occupy the left-hand place, the resulting number is four times the original number. Required the original number.

44. A teacher required each of three pupils to multiply together two given numbers. The first pupil, in adding the partial products, neglected to carry 1 from a certain column.

To check his work he divided his product by the smaller number and obtained a quotient 971 and a remainder 214. The second pupil neglected to carry 2 from the next column (to the left), and obtained by dividing his product by the smaller number a quotient 965 and a remainder 198. The third pupil neglected to carry 1 from the next column (always to the left), and obtained by his division a quotient 940 and a remainder 48. What were the two numbers and from what column did each pupil neglect to carry?

INVOLUTION.

1. 8^2.
2. $(15)^2$.
3. $(297)^2$.
4. $(2)^3$.
5. $(3)^4$.
6. $(4)^5$.
7. $(2)^7$.
8. $(4^2)^3$.
9. $(5^3)^2$.
10. $(2^3)^2$.
11. $(a)^3$.
12. $(c)^7$.
13. $(n^3)^2$.
14. $(x^5)^3$.
15. $(ab)^4$.
16. $(mn^2)^3$.
17. $(x^3y^2)^2$.
18. $(b^2c^3)^5$.
19. $(x^4y^3)^6$.
20. $(ab^2c^3)^4$.
21. $(m^5n^4p^2)^3$.
22. $(a^3c^5x^7)^9$.
23. $(n^3p^4q^6)^8$.
24. $(x^{22}y^{17}z^{14})^3$.
25. $(3a)^2$.
26. $(4x)^3$.
27. $(2c)^5$.
28. $(15n^4y^3)^2$.
29. $\{(x^2)^3\}^4$.
30. $\{(a^3)^4\}^3$.
31. $\{(3a^4)^2\}^2$.
32. $\{(2c^5)^5\}^2$.
33. $\{(4z^4)^3\}^2$.
34. $(2a^2bc^3)^4$.
35. $(5ax^3y^3)^3$.
36. $3(a^2x)^2$.
37. $4c(a^2c^2)^3$.
38. $2(a^2)^2(x^2)^2$.
39. $4(3a)^2(2a)$.
40. $3(a^2)^2(x^2)^3$.
41. $5(a^2x)^5(ax^2)^2$.
42. $2(x^2y^3)(2xy^2)$.
43. $(2^33^3)(a^3x^3)$.
44. $4^3(a^2x)^2(ax^2)^3$.
45. $2^4(c^3n^3)^4(c^3n^2)^23^3$.
46. $(-a)^3$.
47. $(-x)^3$.
48. $(-c)^4$.
49. $(-z)^5$.
50. $(-m^2n^3)^2$.
51. $(-ac^3x^3)^4$.
52. $(-b^5d^7x)^5$.
53. $(-a^2c^3x^4y^5)^6$.
54. $(-3q)^2$.
55. $(-5xy)^3$.
56. $(-11a^3c^3)^3$.
57. $(-5ax^3y^2)^3$.
58. $(-3a^2b^3c)^5$.
59. $(-3xy^3)^6$.
60. $(-7m^3nx^2y^4)^2$.

61. $(-6a^{11}n^{15}t^{22}x^{27}y^{33})^3$. 62. $5(-2a)^2$.

63. $2(-3d)^3$. 64. $-2(-3)^3$. 65. $-4(-4)^2$.

66. $2(-2a)^2(-2a)^3$. 67. $-4(-4x)^2(-2x)^3$.

68. $5a(-2a^3)(a)^4$. 69. $\{(2a^3)^2\}^4$.

70. $\{(-3x^3)^3\}^2$. 71 $-4a^2\{-4a^2)^2\}^2$.

72. $3x\{(-x^2)^3\}^4$. 73. $4\{(3a)^2(-2a)\}^3$.

74. $-5\{(m^2n)^5(mn^2)^2\}^2$. 75. $-5^2\{(-5a^2)^2(-5c)^2$.

76. $(a^n)^2$. 77. $(a^c)^3$. 78. $(x^p)^4$.

79. $(y^q)^5$. 80. $(m^n)^7$. 81. $(ax^n)^3$.

82. $(c^ny)^4$. 83. $(m^3x^r)^4$. 84. $(p^aq^5)^2$.

85. $(ab^nc^2)^5$. 86. $(a^rb^s)^2$. 87. $(x^py^q)^3$.

88. $(w^ay^cz^s)^3$. 89. $(a^xb^yc^z)^4$. 90. $(a^2c^mx^nz^3)^5$.

91. $(2x^ny^a)^4$. 92. $(4b^3c^nd^2)^2$. 93. $(5x^ny^2z^4)^3$.

94. $(2y^nz^nx^a)^8$, 95. $(3^na^tx^p)^9$. 96. $(2x^n3^py^q)^4$.

97. $(4^ma^n5^pc^q)^5$. 98. $(a^2)^n$. 99. $(ax^2)^s$.

100. $(a^2c^4)^x$. 101. $(a^3x^2z)^n$. 102. $(c^3d^4x^2y)^a$.

103. $(m^{12}n^{13}p^{15})^t$. 104. $(b^5c^3d)^n$. 105. $(2a^3)^n$.

106. $(4y^8)^c$. 107. $(2^3z^4)^n$. 108. $(4^m5^n)^3$.

109. $(6^57^6)^n$. 110. $(3^4y^5z)^d$. 111. $(5^2m^3n^4q^6)^a$.

112. $(-3a^3xc^4)^e$. 113. $(-3^6ay^5x^2)^n$.

114. $(-19a^7x^3)^t$ 115. $(-4^7a^6c^5)^x$ 116. $(-8^6c^7x^8)^p$.

117. $(3x^3y^4)^{2n}$. 118. $(4a^5m)^{3n}$. 119. $(5u^2z^7)^{2s}$.

120. $(8a^2c^4x^6)^{2t}$. 121. $(11ab^2c^3d^4)^{5q}$.

122. $(-3^2x^4y^7)^{2a}$. 123. $(-5^3m^3n^6p^2)^{3c}$.

124. $(-4^5a^4n^3x^3)^{2k}$.
125. $(-5^53^3a^2b^4)^n$.
126. $(-4^4y^76^3z^3)^{2m}$.
127. $3(a^3)^2(x^3)^n$.
128. $-7(a^n)^3(c^4)^p$.
129. $\{(-2y^3)^2\}^n$.
130. $5\{(-2a^2)^n\}^4$.
131. $-3^m\{(-7^6x^4)^3\}^{2m}$.
132. $(-1)^{2n}$.
133. $(-1)^{2n+1}$.
134. $(-1)^{2n-1}$.
135. $(-h)^{2n}$.
136. $(-h)^{2n+1}$.
137. $(-h)^{2n-1}$.
138. $(a^{m+1})^2$.
139. $(-4x^my^{2n})^2$.
140. $(x^{n-1}y)^5$.
141. $(3a^{m+n}y^3)^2$.
142. $(-5x^ny^3)^{2m}$.
143. $(-2x^3)^n(-2^3x^n)^2$.
144. $(-a^{n-1}b^{m+1})^3$.
145. $(4a^sy^t)^n$.
146. $(-2a^3m^s)^{3n}$.
147. $(a^{-1})^2$.
148. $(c^{-4})^3$.
149. $(d^{-3})^3$.
150. $(x^{-4})^4$.
151. $(e^3)^{-1}$.
152. $(f^2)^{-2}$.
153. $(g^5)^{-3}$.
154. $(h^3)^{-5}$.
155. $(y^3)^{-7}$.
156. $(a^{\frac{1}{2}})^2$.
157. $(x^{\frac{1}{3}})^3$.
158. $(y^{\frac{1}{4}})^4$.
159. $(z^{\frac{3}{5}})^3$.
160. $(c^{\frac{1}{3}})^2$.
161. $(n^2)^{\frac{1}{3}}$.
162. $(p^3)^{\frac{2}{5}}$.
163. $(q^5)^{\frac{1}{2}}$.
164. $(b^4)^{\frac{3}{4}}$.
165. $(w^7)^{\frac{3}{5}}$.
166. $(x^{-1})^n$.
167. $(a^n)^{-2}$.
168. $(x^{-2})^{\frac{1}{2}}$.
169. $(z^{\frac{2}{3}})^{-3}$.
170. $(s^{\frac{1}{2}})^n$.
171. $(t^m)^{\frac{1}{3}}$.
172. $(a^{-\frac{1}{3}})^n$.
173. $(c^t)^{-\frac{2}{5}}$.
174. $(x^{n/2})^2$.
175. $(y^{3/n})^4$.
176. $(u^4)^{n/2}$.
177. $(a^{-n})^m$.
178. $(m^t)^{-t}$.
179. $(3xy^{-2})^3$.
180. $(-2c^3d^{-2})^4$.
181. $(5a^{-2}b^{-1})^3$.
182. $(-\frac{3}{4}x^{-2})^2$.
183. $(2a^{-1}y^{\frac{3}{2}})^2$.
184. $(\frac{3}{5}b^3y)^2$.
185. $(2a^{\frac{2}{3}})^n$.
186. $(-2a^{\frac{1}{5}})^5$.
187. $(3a^2b^{-4})^2$.
188. $(2am^2)^{-3}$.

189. $(-x^{-2n}y^nz)^7$.

190. $(x^4y^{-3}z^{-1})^{-2}$.

191. $(-6ax^{-2}y^3)^n$.

192. $(4a^{-\frac{1}{2}}y^{-2})^{-4}$.

193. $(-6ab^{-5}x^{-2})^3$.

194. $(-3a^2x^{-3}z^4)^{-3}$.

195. $(3ab^2x^{-1}y^{-2})^{-2}$.

196. $(-4a^2x^{-3}y^n)^{-2}$.

197. $(27a^{10}y^{-12})^{\frac{2}{3}}$.

198. $(12m^{12}y^{-3})^{\frac{2}{3}}$.

199. $(64m^{12}y^6)^{-\frac{5}{6}}$.

200. $(-2ab^{-3}c^2x^{-4}y)^5$.

201. $(-4a^{-3}b^2x^{-4})^{-4}$.

202. $(-m^{-1}n^{-2})^{-3}$.

203. $(-2a^{-3}b^{-2})^{-4}$.

204. $(13x^{-\frac{1}{3}}y^{-3})^{-3}$.

205. $(-\frac{2}{3}a^{\frac{1}{2}}m^{-\frac{1}{3}}x)^2$.

206. $(8a^{12}x^{-6})^{\frac{2}{3}}$.

207. $(32a^{20}y^{15})^{-\frac{4}{5}}$.

208. $(-2x^{m+n-1}y^2)^2$.

209. $(2a^{m/n}c^{1/q}x^{p/2})^2$.

210. $\{(y^{\frac{3}{2}})^{-\frac{1}{2}}\}^{\frac{1}{3}}$.

211. $\left(\frac{a}{c}\right)^2$.

212. $\left(\frac{c^2}{x^4}\right)^3$.

213. $\left(\frac{a^n}{c^2}\right)^2$.

214. $\left(\frac{a^3b^2}{2}\right)^4$.

215. $\left(\frac{3x^2y}{2a^3b^2}\right)^5$.

216. $\left(\frac{4x^2y}{-3z}\right)^2$.

217. $\left(\frac{-3a^2c^3}{xy^2}\right)^3$.

218. $\left(\frac{-6mn}{-4a^2}\right)^4$.

219. $\left(-\frac{2x^3y}{3abc}\right)^5$.

220. $\left(-\frac{3ab^2}{4c^3}\right)^4$.

221. $\left(-\frac{x^2y^3z^4}{2}\right)^7$.

222. $\left(\frac{1}{3ax^3}\right)^3$.

223. $\left(\frac{-a^3x}{b^2d^4}\right)^4$.

224. $\left(-\frac{c^2d^3}{2x^4}\right)^5$.

225. $\left(\pm\frac{a^2b}{4x^3}\right)^4$.

226. $\left(\frac{a^{-n}c^n}{-b^m}\right)^6$.

227. $\left(\frac{x^{-3}}{y^{-4}}\right)^2$.

228. $\left(\frac{2}{xy^{-1}}\right)^2$.

229. $(-\frac{3}{4}x^{-2})^3$.

230. $(-\frac{1}{5}x^{2n-1})^3$.

231. $(\frac{3}{5}b^3y)^2$.

232. $\left(-\frac{a^{-1}x^2}{b^{-2}y}\right)^3$.

233. $\left\{\left(\frac{-ab^2c}{-2mn}\right)^3\right\}^5$.

234. $\left(\frac{5a^{-2}c^{2}d^{n}}{4x^{-2n}y^{2}z^{n}}\right)^{4}$.

235. $\left(\frac{1}{2n^{-3}x^{\frac{1}{2}}}\right)^{2}$.

236. $\left(\frac{c^{n}}{x^{m}}\right)^{3}$.

237. $\left(\frac{x^{-3}}{4z^{-\frac{2}{3}}}\right)^{5}$.

238. $\left(\frac{-3m^{-2}b^{-\frac{1}{3}}}{2x^{-\frac{1}{2}}y^{-1}}\right)^{3}$.

239. $(-\frac{2}{3}a^{\frac{1}{2}}m^{-\frac{1}{3}}x)^{2}$.

240. $\left(\frac{-2a^{\frac{1}{2}}x}{3m^{\frac{1}{3}}}\right)^{2}$.

241. $\left(\frac{x^{2}}{y^{3}}\right)^{n}$.

242. $\left(\frac{m^{2}n}{pq^{2}}\right)^{x}$.

243. $\left(\frac{a^{-1}}{c^{-2}}\right)^{n}$.

244. $\left(\frac{x^{2}}{a^{3}}\right)^{\frac{1}{2}}$.

245. $\left(\frac{2abc}{3m^{2}n^{3}}\right)^{n}$.

246. $\left(\frac{a^{\frac{2}{3}}}{c^{\frac{1}{3}}}\right)^{-2}$.

247. $\left(\frac{m^{n}}{c^{a}}\right)^{-\frac{1}{2}}$.

248. $\left(\frac{2^{n}a^{-\frac{1}{2}}}{3^{\frac{1}{2}}x^{-m}}\right)^{2}$.

249. $\left(\frac{x^{n}}{y^{n}}\right)^{-\frac{2}{3}}$.

250. $\left(-\frac{2a^{5}x^{n}}{3a^{-1}z^{\frac{1}{2}}}\right)^{-3}$.

251. $(x+1)^{3}$.

252. $(x-1)^{4}$.

253. $(1+x)^{5}$.

254. $(1-x)^{6}$.

255. $(x+2)^{3}$.

256. $(a+2)^{4}$.

257. $(x-2)^{4}$.

258. $(x+3)^{5}$.

259. $(x-7)^{4}$.

260. $(1-a^{2})^{5}$.

261. $(3x+5)^{3}$.

262. $(2-3x)^{5}$.

263. $(2m-1)^{3}$.

264. $(3x+1)^{4}$.

265. $(1+3c)^{4}$.

266. $(x+y)^{5}$.

267. $(a-b)^{6}$.

268. $(a^{2}-x)^{3}$.

269. $(2x-a)^{4}$.

270. $(2x+3a)^{4}$.

271. $(2a^{3}+cd)^{3}$.

272. $(4x-5y)^{3}$.

273. $(x^{3}+y^{3})^{6}$.

274. $(y^{3}+y)^{4}$.

275. $(4y+z)^{4}$.

276. $(c+2d)^{5}$.

277. $(x+3y)^{6}$.

278. $(2ax-3b)^{5}$.

279. $(x^{2}+3y^{2})^{5}$.

280. $(5x^{2}-2y^{3})^{3}$.

281. $(4a^{2}+3x)^{4}$.

282. $(a^{3}+3ab)^{6}$.

283. $(2a - 3b)^7$.

284. $(a^2 - ax)^{10}$.

285. $(x^2y - 2xy^2)^6$.

286. $(ab - 3)^7$.

287. $(a + a^{-1})^3$.

288. $(x^{-2} + a^{-3})^5$.

289. $(n^{-1} + z^{-2})^5$.

290. $(p^{-3} + q^{-1})^8$.

291. $(c^{-4} - d^{-1})^7$.

292. $(a^2 - a^{\frac{1}{2}})^4$.

293. $(a^{\frac{2}{3}} + x^{\frac{1}{2}})^4$.

294. $(m^{\frac{1}{5}} + n^{\frac{2}{5}})^5$.

295. $(r^{\frac{1}{2}} + s^{\frac{1}{3}})^6$.

296. $(c^{\frac{2}{3}} - x^{\frac{1}{2}})^5$.

297. $(x^{\frac{1}{2}} - 2)^7$.

298. $(1^{-2} - z^{-\frac{2}{3}})^5$.

299. $(a^{-\frac{1}{2}} + x^{-\frac{1}{3}})^5$

300. $(x^{-\frac{2}{5}} - e^{-\frac{3}{5}})^4$.

301. $(q^{-2} - c^{-\frac{3}{5}})^6$.

302. $\left(2 - \frac{1}{y}\right)^4$.

303. $\left(1 - \frac{2}{3x}\right)^6$.

304. $(x^2 + \frac{1}{2}y)^4$.

305. $(\frac{1}{2}x^2 - 2x)^3$.

306. $\left(2a - \frac{b}{2}\right)^6$.

307. $(x + \frac{1}{4}y)^5$.

308. $(\frac{1}{2}a - 2b)^7$.

309. $\left(2 - \frac{3x}{4}\right)^5$.

310. $(\frac{1}{2}x - \frac{2}{3}y)^3$.

311. $(a^{-\frac{2}{3}} - 4x)^3$.

312. $(2a^{-2} - 3c^{\frac{1}{2}})^4$.

313. $(3x^{-1} + 2y^{\frac{2}{3}})^6$.

314. $\left(\frac{a}{2} - \frac{3}{2}n^{-1}\right)^5$.

315. $\left(\frac{2}{x^{-1}} + \frac{a^{-2}}{b^{-3}}\right)^4$.

316. $\left(\frac{a}{2} - \frac{l^{\frac{1}{3}}}{3}\right)^7$.

317. $\left(\frac{x}{3y} + \frac{2a^{\frac{1}{3}}}{c}\right)^5$.

318. $(1 - 3n)^{-1}$.

319. $(1 - 4q^3)^{-1}$.

320. $(1 + x^3)^{-2}$.

321. $(p - 3q)^{-2}$.

322. $(1 - 2x)^{-3}$.

323. $(2x - z)^{-6}$.

324. $(a^2 - c)^{-3}$.

325. $(m + 2n)^{-4}$.

326. $(1 - 2b^2)^{-7}$.

327. $(1 - \frac{1}{2}a)^{-5}$.

328. $(a + 12)^{\frac{1}{2}}$.

329. $(8 + 12k)^{\frac{2}{3}}$.

330. $(1+x)^{\frac{1}{4}}$. 331. $(1+\frac{1}{5})^{\frac{1}{4}}$. 332. $(1-y^2)^{\frac{7}{4}}$.

333. $(1-3d)^{\frac{2}{3}}$. 334. $(m^5+m^3n^2)^{\frac{2}{5}}$.

335. $(y^2-b^3)^{\frac{1}{3}}$. 336. $(x^2+xy)^{\frac{3}{10}}$. 337. $(1+3a)^{-\frac{3}{4}}$.

338. $(1-2c)^{-\frac{1}{4}}$. 339. $(k^2+4l^2)^{-\frac{5}{2}}$. 340. $(x-3n)^{-\frac{1}{2}}$.

341. $(9-6b)^{-\frac{3}{2}}$. 342. $(a^2+3c^2)^{-\frac{2}{3}}$. 343. $(8x^3-4c)^{-\frac{2}{3}}$.

344. $(a^{\frac{1}{5}}-c^{-\frac{1}{5}})^{-\frac{1}{3}}$. 345. $(a^2b-c^{\frac{1}{3}})^{-\frac{2}{3}}$. 346. $(c^2-\frac{2}{3}h)^{\frac{3}{2}}$.

347. $(m+\frac{4}{3}n)^{\frac{3}{2}}$. 348. $(\frac{2}{3}a-\frac{1}{2}x)^{-2}$. 349. $(\frac{2}{3}a-\frac{3}{2}c)^{\frac{2}{3}}$.

Find the required term of the following:

350. Third term of $(1-2e)^7$.

351. Third term of $(3-2b^2)^5$.

352. Fourth term of $(x-y)^{100}$.

353. Middle term of $(x^{\frac{1}{2}}+z^{\frac{1}{2}})^8$.

354 Middle term of $(p-q)^{16}$.

355. Eighth term of $\frac{4}{3}m+\frac{2}{3}n)^7$.

356. Fifth term of $(3a-4c)^8$.

357. Two middle terms of $(s+t)^{13}$.

358. Sixth term of $(3+2c^2)^9$.

359. Seventh term of $(2+x)^{16}$.

360. Eighth term of $\left(5-\frac{x}{5}\right)^{10}$.

361. Ninth term of $\left(\frac{2}{a}-\frac{a}{4}\right)^{10}$.

362. Eighth term of $\left(2y^{\frac{1}{3}}+\frac{1}{2y}\right)^{11}$.

363. Sixth term of $(1-\frac{2}{3}a)^{-1}$.

364. Sixth term of $\left(\frac{4}{3x} - z^{-\frac{3}{5}}\right)^{-8}$.

365. Fifth term of $\left(n^{\frac{1}{4}} - \frac{c^{\frac{1}{2}}}{2n}\right)^{\frac{2}{3}}$.

366. Third term of $(\frac{1}{8} - a^{\frac{3}{2}})^{-\frac{2}{3}}$.

367. Fifth term of $\left(c^{\frac{1}{2}} + \frac{d^2}{3}\right)^{-\frac{3}{4}}$.

368. Fifth term of $\left(3x - \frac{1}{2^{\frac{1}{2}}}\right)^{-\frac{3}{2}}$.

369. Fourth term of $\frac{2cp}{(c^{-\frac{1}{2}} - q^{-\frac{7}{2}})^{-\frac{2}{3}}}$.

370. Fourth term of $cy^2(c^{-a} + y^{-\frac{2}{3}})^{-\frac{2}{5}}$.

371. $(a^3 - a^2 + a)^3$.

372. $(2x^2 - xy + y^2)^3$.

373. $(p^2 - 2pq + q^2)^3$.

374. $(10m + 3mn - n^2)^3$.

375. $(x + 2yz - z)^3$.

376. $(1 - c + c^2)^4$.

377. $(a^2 - 2a - 2)^4$.

378. $(1 + n - n^2)^5$.

379. $(z^2 - z + 2)^5$.

380. $\left(a + 1 - \frac{1}{a}\right)^3$.

381. $\left(\frac{x}{3} - 4 - \frac{x^2}{3n}\right)^3$.

382. $\left(\frac{2}{3}c + \frac{1}{c} - \frac{1}{4}c^2\right)^3$.

383. $\left(\frac{c}{2x} - \frac{3x^2}{c^2} - \frac{2x}{c}\right)^3$.

EVOLUTION.

1. $\sqrt{9}$.
2. $\sqrt{25}$.
3. $\sqrt[3]{27}$.
4. $\sqrt{64}$.
5. $\sqrt[4]{81}$.
6. $\sqrt[4]{625}$.
7. $\sqrt[5]{32}$.
8. $\sqrt[5]{243}$.
9. $\sqrt[6]{64}$.
10. $\sqrt[6]{4096}$.
11. $\sqrt{\frac{1}{36}}$.
12. $\sqrt[3]{\frac{8}{125}}$.
13. $\sqrt[4]{\frac{16}{81}}$.
14. $\sqrt[5]{\frac{32}{243}}$.
15. $\sqrt[6]{\frac{64}{729}}$.
16. $\sqrt{49x^4}$.
17. $\sqrt{9a^2c^4}$.
18. $\sqrt{16a^4x^6}$.
19. $\sqrt{100a^2b^8}$.
20. $\sqrt{144c^{10}d^{14}}$.
21. $\sqrt[3]{27n^6}$.
22. $\sqrt[3]{-p^3}$
23. $\sqrt[3]{-27x^6}$.
24. $\sqrt[3]{-64a^3b^{12}}$.
25. $\sqrt[3]{-216a^6c^{12}x^{18}}$.
26. $\sqrt[4]{81c^8y^{16}}$.
27. $\sqrt[4]{625z^{20}}$.
28. $\sqrt[5]{32x^{10}y^{15}}$.
29. $\sqrt[5]{-243m^{20}}$.
30. $\sqrt[6]{64a^{12}p^6}$.
31. $\sqrt[6]{729x^{18}}$.
32. $\sqrt[3]{512e^{24}}$.
33. $\sqrt[5]{-32x^5y^{30}z^{25}}$.
34. $\sqrt[3]{-7^3a^6y^{15}}$.
35. $\sqrt[4]{3^8p^8y^{12}}$.
36. $\sqrt[5]{-5^5z^{10}}$.
37. $\sqrt[3]{z^3x^6y^3}$.
38. $\sqrt[4]{16x^2y^8}$.
39. $\sqrt{7^2x^4y^5z^6}$.
40. $\sqrt{\frac{c^6n^4}{16}}$.
41. $\sqrt{\frac{25a^4b^2}{36x^2y^4}}$.
42. $\sqrt{\frac{169x^6}{289y^{14}}}$.
43. $\sqrt{\frac{100p^{12}}{324q^{10}}}$.
44. $\sqrt[3]{\frac{125}{a^9c^{12}}}$.
45. $\sqrt[3]{\frac{8m^9}{216c^6d^{12}}}$.
46. $\sqrt[5]{\frac{m^{10}n^5}{-32x^5a^{25}}}$.
47. $\sqrt[4]{\frac{256m^{16}x^{24}}{81p^{32}q^{44}}}$.
48. $\sqrt[5]{\frac{-243c^{45}}{a^5c^{10}x^{20}}}$.
49. $\sqrt{\frac{400a^6}{25x^{2n}}}$.
50. $\sqrt[3]{\frac{125a^{3n}c^6y^3}{-216b^3z^{6n}}}$.
51. $\sqrt[6]{\frac{64a^{6n}}{p^{12}q^{18m}}}$.

52. $\sqrt[7]{\frac{128}{x^{56}y^{63}}}$. 53. $\sqrt[8]{\frac{256x^{16n}}{a^{64}c^{-8n}}}$. 54. $\sqrt[9]{\frac{-p^{18a}}{c^{27}d^{-27a}}}$.

55. $\sqrt[11]{\frac{a^{11}nx^{55}}{-y^{121q}}}$. 56. $\sqrt{6\frac{1}{4}a^{6n}x^{2p-4}}$.

57. $\sqrt{[64p^4(p^2+q^2)^6]}$. 58. $\sqrt{\frac{2}{3}-\frac{5}{6}}$.

59. $\sqrt{\frac{4}{5}+\frac{16}{25}}$. 60. $\sqrt{\frac{5}{6}-\frac{7}{18}}$. 61. $\sqrt{\frac{7}{8}-\frac{7}{64}}$.

62. $\sqrt{\frac{6}{7}-\frac{6}{49}}$. 63. $\sqrt[3]{\{3\frac{3}{8}m^{3a-9}(m-1)^9\}}$.

64. $\sqrt{(35ab \times 63ac \times 45bc)}$.

65. $\sqrt[3]{(75xy^2z^2 \times 40x^3y^3 \times 72x^2yz)}$.

66. $\sqrt{\sqrt{(256a^4x^8)}}$ 67. $\sqrt[3]{\sqrt{(4096c^{12}e^{24})}}$.

68. $\sqrt{(121x^{2n}y^6)^3}$. 69. $\sqrt{\sqrt[3]{(49^3 \times 64^3)}}$.

70. $\sqrt[3]{\left(\frac{27^{3n}}{64^{-6a}}\right)^2}$. 71. $\sqrt[3]{(27n^9p^6q^3)^5}$.

72. $\sqrt[5]{\sqrt[3]{(-2^{30}x^{15}y^{25a})}}$. 73. $\sqrt{\sqrt{(16^2x^{4c-8})}}$.

74. $\sqrt[3]{\sqrt{(5^{6n}a^{12p}c^{-6q})}}$.

75. $\sqrt{1+4+10+12+9}$.

76. $\sqrt{x^2+2xy+y^2+2xz+2yz+z^2}$.

77. $\sqrt{p^4-2p^3-p^2+2p+1}$.

78. $\sqrt{a^6+2a^5+3a^4+2a^3+a^2}$.

79. $\sqrt{1-4b+4b^2+2y-4by+y^2}$.

80. $\sqrt{1+6s+2t+9s^2+6st+t^2}$.

81. $\sqrt{9+6m+48n+m^2+16mn+64n^2}$.

82. $\sqrt{4a^2+8ab-4a+4b^2-4b+1}$.

83. $\sqrt{8x+4+x^4+4x^3+8x^2}$.

84. $\sqrt{m^4+4m^3-4m^2-16m+16}$.

85. $\sqrt{4k^4 - 12k^3 + 13k^2 - 6k + 1}$.

86. $\sqrt{4x^4 - 12x^3 + 5x^2 + 6x + 1}$.

87. $\sqrt{4q^4 - 4q^3 + 13q^2 - 6q + 9}$.

88. $\sqrt{1 + 4a^2 + x^6 + 4a + 2x^4 + 4ax^4}$.

89. $\sqrt{60p + 100 + 16p^4 + 24p^3 + 89p^2}$.

90. $\sqrt{24z^2 - 16z^3 + 4z^4 - 16z + 4}$.

91. $\sqrt{a^4 - 2a^3x - a^2x^2 - 2ax^3 + x^4}$.

92. $\sqrt{4a^4 + 12a^3x + 13a^2x^2 + 6ax^3 + x^4}$.

93. $\sqrt{49c^4 - 14ac^3 + 15a^2c^2 - 2a^3c + a^4}$.

94. $\sqrt{16a^4 - 40a^3b + 25a^2b^2 - 80ab^3 + 64b^2x^2 + 64a^2bx}$.

95. $\sqrt{4a^2 - 4ab + b^2 - 4ac - 4ad + 2bc + 2bd + c^2 + 2cd + d^2}$.

96. $\sqrt{1 - 4z + 10z^2 - 20z^3 + 25z^4 - 24z^5 + 16z^6}$.

97. $\sqrt{4x^8 - 4x^6 + 4x^5 - 3x^4 - 2x^3 + 3x^2 - 2x + 1}$.

98. $\sqrt{x^8 + 2x^7 - x^6 + 3x^4 - 2x^3 + x^2}$.

99. $\sqrt{a^6 - 6a^5c + 15a^4c^2 - 20a^3c^3 + 15a^2c^4 - 6ac^5 + c^6}$.

100. $\sqrt{9a^{-2} + 12a^{-1}b^2 - 6a + 4b^4 - 4a^2b^2 + a^4}$.

101. $\sqrt{4c^{-4} - 4c^{-3} + 5c^{-2} - 2c^{-1} + 1}$.

102. $\sqrt{49a^{-6} + 42a^{-6} - 19a^{-4} - 12a^{-2} + 4}$.

103. $\sqrt{9x^{-4} + 12x^{-3} - 26x^{-2} - 20x^{-1} + 25}$.

104. $\sqrt{4a^{-2} + 9b^{-2} + 16c^{-2} - 12a^{-1}b^{-1} + 16a^{-1}c^{-1} - 24b^{-1}c^{-1}}$.

105. $\sqrt{q^{-4} - 6p^{-1}q^{-3} + 13p^{-2}q^{-2} - 12p^{-3}q + 4p^{-4}}$.

106. $\sqrt{49c^{-2}z^{-2} + 25z^{-4} - 24c^{-3}z^{-1} - 30c^{-1}z^{-3} + 16c^{-4}}$.

107. $\sqrt{-70x^{-1}y^{-3} - 56x^{-3}y^{-1} + 89x^{-2}y^{-2} + 16x^{-4} + 25y^{-4}}$.

108. $\sqrt{a^{-4}c^{-4} - 4a^{-3}c^{-3} + 6a^{-2}c^{-2} - 4a^{-1}c^{-1} + 1}$.

109. $\sqrt{4x^{-6} + 17x^{-2} - 22x^{-3} + 13x^{-4} - 24x^{-1} - 4x^{-5} + 16}$.

110. $\sqrt{c^2 + 2c - 1 - \frac{2}{c} + \frac{1}{c^2}}$. 111. $\sqrt{n^4 - 2n^3 + 2n^2 - n + \frac{1}{4}}$.

112. $\sqrt{25k^4 - 10k^3 + 6k^2 - k + \frac{1}{4}}$.

113. $\sqrt{9h^2 - 30h + 31 - \frac{10}{h} + \frac{1}{h^2}}$.

114. $\sqrt{36d^4 - 36d^3 + 17d^2 - 4d + \frac{4}{9}}$.

115. $\sqrt{\frac{9}{4} + 6y - 17y^2 - 28y^3 + 49y^4}$.

116. $\sqrt{p^4 - 4p^2 + 12 - \frac{16}{p^2} + \frac{16}{p^4}}$.

117. $\sqrt{a^4 + 8a^2 + 24 + \frac{32}{a^2} + \frac{16}{a^4}}$.

118. $\sqrt{\frac{x^2}{4} + 6ax^2 - 4bx^2 + 36a^2 - 48ab + 16b^2}$.

119. $\sqrt{x^4 - x^3 + \frac{x^2}{4} + 4x - 2 + \frac{4}{x^2}}$.

120. $\sqrt{\frac{a^2}{x^2} - 2 + \frac{x^2}{a^2} + \frac{2a^2}{x} - 2x + a^2}$.

121. $\sqrt{\frac{x^2}{y^2} - xy + \frac{1}{4}x^4 - 2 + \frac{x^3}{y} + \frac{y^2}{x^2}}$.

122. $\sqrt{\frac{4a^2}{9c^2} - \frac{a}{e} - \frac{16a^2}{15ce} + \frac{9c^2}{16e^2} + \frac{6ac}{5e^2} + \frac{16a^2}{25e^2}}$.

123. $\sqrt{\frac{4}{9}a^2x^4 - \frac{4}{3}abx^3z + \frac{8}{3}a^2bx^2z^2 + b^2x^2z^2 - 4ab^2xz^3 + 4a^2b^2z^4}$.

124. $\sqrt{1 + \frac{19}{9}a^2 - \frac{a^5}{2} + \frac{a^6}{16} - \frac{2a}{3} - \frac{7a^3}{6} + \frac{7a^4}{6}}$.

125. $\sqrt{\frac{x^2}{y^2} - \frac{4mx}{ny} + \frac{4m^2}{n^2} + \frac{6ax}{cy} + \frac{9a^2}{c^2} - \frac{12am}{cn}}$.

126. $\sqrt{\frac{4p^2}{25q^2} - \frac{4mp}{5nq} + \frac{m^2}{n^2} - \frac{12kp}{5qz} + \frac{6mk}{nz} + \frac{9k^2}{z^2}}$.

127. $\sqrt{\frac{4a^2}{9c^2} - \frac{a^2}{cx} - \frac{16ac}{15cx} + \frac{6ac}{5x^2} + \frac{9a^2}{16x^2} + \frac{16c^2}{25x^2}}$.

128. $\sqrt{\frac{9}{25} + \frac{a^8}{25} + \frac{m^6}{36} - \frac{6a^4}{25} - \frac{m^3}{5} + \frac{4n^8}{49} + \frac{12n^4}{35} + \frac{a^4m^3}{15} - \frac{4a^4n^4}{35} - \frac{2m^3n^4}{21}}$.

129. $\sqrt{\frac{9m^6n^4}{25p^6q^8} - \frac{12m^5n^5}{35p^7q^9} - \frac{332m^4n^6}{735p^8q^{10}} + \frac{16m^3n^7}{63p^9q^{11}} + \frac{16m^2n^8}{81p^{10}q^{12}}}$.

130. $\sqrt{1 - 6a^{\frac{1}{3}} + 15a^{\frac{2}{3}} - 18a^{\frac{3}{3}} + 9a^{\frac{4}{3}}}$.

131. $\sqrt{4 - 12x^{\frac{1}{3}} - 7x^{\frac{2}{3}} + 24x + 16x^{\frac{4}{3}}}$.

132. $\sqrt{9x - 24x^{\frac{1}{2}}y^{\frac{1}{3}} + 12x^{\frac{1}{2}} + 16y^{\frac{2}{3}} - 16y^{\frac{3}{3}} + 4}$.

133. $\sqrt{2c^{\frac{7}{3}} + c^{\frac{3}{3}} - 4c^{\frac{4}{3}} - 4c^{\frac{5}{3}} + c^{\frac{5}{3}} + 4c}$.

134. $\sqrt{2a + 4a^{\frac{2}{3}} + a^{\frac{1}{3}} - 4a^{\frac{2}{3}} + a^{\frac{1}{3}}}$.

135. $\sqrt{x^{\frac{2}{3}} + 2xy^{\frac{1}{3}} - x^{\frac{2}{3}}y^{\frac{2}{3}} - 2x^{\frac{1}{3}}y + y^{\frac{4}{3}}}$.

136. $\sqrt{a^{\frac{8}{5}} - 2a - a^{\frac{4}{5}} + 3a^{\frac{2}{5}} + 2a^{\frac{1}{5}} + 1}$.

137. $\sqrt{4x^{\frac{2}{3}} + 4x^{\frac{2}{3}} - 3x - 6x^{\frac{2}{3}} - x^{\frac{1}{3}} + 2x^{\frac{1}{3}} + 1}$.

138. $\sqrt{x^{\frac{4}{3}} - 6a^{\frac{1}{3}}x + 13a^{\frac{2}{3}}x^{\frac{2}{3}} - 12a^{\frac{3}{3}}x^{\frac{1}{3}} + 4a^{\frac{4}{3}}}$.

139. $\sqrt{p^{\frac{8}{5}} - 2p^{\frac{1}{5}}q^{\frac{7}{5}} + p^{-\frac{6}{5}}q^{\frac{14}{5}} + 2p^{\frac{4}{5}}q^{\frac{4}{5}} - 2p^{-\frac{3}{5}}q^{\frac{11}{5}} + q^{\frac{8}{5}}}$.

140. $\sqrt{x^4 - 2ax^3 + (a^2 + 4b^2)x^2 - 4ab^2x + 4b^4}$.

141. $\sqrt{x^4 + 2(m+n)x^3 + (m^2+n^2)x^2 - 2mn(m+n)x + m^2n^2}$.

142. $\sqrt{a^4 + (4b - 2c)a^3 + (4b^2 - 4bc + 3c^2)a^2 + (4bc - 2c^2)a + c^4}$.

143. $\sqrt{16a^4(2a-1)^2 + 8a^2(2a-1)^2 + 4a(a-1) + 1}$.

144. $\sqrt{x^4 + 2x^3(y+z) + x^2(y^2+z^2+4yz) + (y+z)2xyz + y^2z^2}$.

Cube Root.

145. $\sqrt[3]{x^6 - 3x^5 + 5x^3 - 3x - 1}$.

146. $\sqrt[3]{q^6 - 6q^5 + 15q^4 - 20q^3 + 15q^2 - 6q + 1}$.

147. $\sqrt[3]{x^6 + 6x^5 - 40x^3 + 96x - 64}$.

148. $\sqrt[3]{21x^4 - 9x^5 - 9x^3 - 36x + x^6 - 8 - 42x^2}$.

149. $\sqrt[3]{1 + 66x^4 - 63x^3 - 9x + 8x^6 - 36x^5 + 33x^2}$.

150. $\sqrt[3]{8a^6 + 12a^5 - 30a^4 - 35a^3 + 45a^2 + 27a - 27}$.

151. $\sqrt[3]{n^6 - 6n^5 + 40n^3 - 96n - 64}$.

152. $\sqrt{39y^2 + 64y^6 + 156y^4 + 1 - 144y^5 - 9y - 99y^3}$.

153. $\sqrt[3]{1 + 9x + 18x^2 - 27x^3 - 54x^4 + 81x^5 - 27x^6}$.

154. $\sqrt[3]{8x^{12} - 36x^{10} + 102x^8 - 171x^6 + 204x^4 - 144x^2 + 64}$.

155. $\sqrt[3]{12k + 30k^2 - 45k^4 + 27k^5 - 8 + 27k^5 - 35k^3}$.

156. $\sqrt[3]{- 60e^5 + 8e^6 - 60e + 174e^4 + 8 - 245e^3 + 174e^2}$.

157. $\sqrt[3]{27 + 108v + 90v^2 + 80v^3 - 60v^4 + 48v^5 - 8v^6}$.

158. $\sqrt[3]{64x^6 - 384x^5 + 864x^4 - 896x^3 + 432x^2 - 96x + 8}$.

159. $\sqrt[3]{108x^5 - 48x^4 + 8x^3 + 54x^7 - 12x^8 + x^9 - 112x^6}$.

160. $\sqrt[3]{x^3 + 6x^2y + 12xy^2 + 8y^3 - 3x^2z - 12xyz - 12y^2z + 3xz^2 + 6yz^2 - z^3}$.

161. $\sqrt[3]{96a^2c^4 + 6a^5c + 64c^6 + 96ac^5 + 24a^4c^2 + c^6 + 56a^3c^3}$.

162. $\sqrt[3]{48cx^5 + 8x^6 + 60c^2x^4 - 27c^6 + 108c^5x - 90c^4x^2 - 80c^3x^3}$.

163. $\sqrt[3]{8 - 48cd^2 + 108c^2d^4 - 112c^3d^6 + 54c^4d^8 - 12c^5d^{10} + c^6d^{12}}$.

164. $\sqrt[3]{8s^6 + 48s^5t + 60s^4t^2 - 80s^3t^3 - 90s^2t^4 + 108st^5 - 27t^6}$.

165. $\sqrt[3]{360m^2np + 1000m^6 - 900m^4n - 27n^3 - 600m^4p - 8p^3 + 270m^2n^2 + 120m^2p^2 - 54n^2p - 36np^2}$.

166. $\sqrt[3]{150c^8y^{11} - 172c^6y^9 + 165c^7y^{10} + 27c^5y^6 - 99c^6y^8 + 54c^4y^7 - 125c^9y^{12}}$.

167. $\sqrt[3]{125a^3x^6 - 192a^7x^{10} - 64a^9x^{12} + 240a^5x^8 - 504a^6x^9 + 450a^4x^7 + 288a^8x^{11}}$.

168. $\sqrt[3]{9x^6 - 300x^{11} + 27x^6 - 15x^{10} + 296x^9 - 108x^7 - 125x^{12}}$.

169. $\sqrt[3]{27a^4bc^3x + 36a^4c - 8a^3 + b^3x^3 - 6ab^2x^2 + 27a^6c^3 - 36a^3bcx - 54a^5c^2 + 9a^2b^2cx^2 + 12a^2bx}$.

170. $\sqrt[3]{8x^3 - 12x^2 + 30x - 25 + 30x^{-1} - 12x^{-2} + 8x^{-3}}$.

171. $\sqrt[3]{27x^{\frac{3}{2}} - 54x^{\frac{7}{2}} + 36x^{\frac{5}{2}} - 8x^{\frac{3}{2}} + 27x - 36 + 12x^{-1} + 9x^{-\frac{5}{2}} - 6x^{-\frac{7}{2}} + x^{-6}}$.

172. $\sqrt[3]{a^3 + 6a^{\frac{5}{3}}k^{-1} + 3a^{\frac{1}{3}}k^{-2} - 28a^{\frac{3}{3}}k^{-3} - 9a^{\frac{1}{3}}k^{-4} + 54a^{\frac{1}{3}}k^{-5} - 27k^{-6}}$.

173. $\sqrt[3]{\frac{6}{x^8} + \frac{75}{x^{11}} + \frac{12}{x^7} + \frac{1}{x^9} - \frac{125}{x^{12}} - \frac{60}{x^9} - \frac{60}{x^8} + \frac{8}{x^6} + \frac{150}{x^{10}} - \frac{15}{x^{10}}}$.

174. $\sqrt[3]{\frac{x^3}{27} - \frac{x^2}{3} + \frac{7x}{3} - 9 + \frac{28}{x} - \frac{48}{x^2} + \frac{64}{x^3}}$.

175. $\sqrt[3]{\frac{3a^2}{b^{14}} - \frac{a^3}{b^9} + \frac{a^9}{b^{15}} - \frac{5a^6}{b^{12}} + \frac{3a^4}{b^{10}}}$.

176. $\sqrt[3]{\frac{125a^6}{c^6} - \frac{150a^5}{c^5} - \frac{165a^4}{c^4} + \frac{172a^3}{c^3} + \frac{99a^2}{c^2} - \frac{54a}{c} - 27}$.

177. $\sqrt[3]{\frac{125a^3}{x^6} - \frac{75a^4}{2x^7} - \frac{24a^8}{x^{11}} + \frac{300a^5}{x^8} - \frac{a^6}{8x^9} - \frac{60a^6}{x^9} + \frac{3a^7}{x^{10}} + \frac{15a^5}{4x^8} + \frac{64a^9}{x^{12}} + \frac{240a^7}{x^{10}}}$.

178. $\sqrt[4]{a^{\frac{3}{2}} - 6a^{\frac{5}{2}} - 9a^{\frac{3}{2}} + 12a^{\frac{7}{2}} + 36a^3 + 19a^{\frac{5}{2}} - 36a^{\frac{5}{2}} - 54a^{\frac{11}{4}} - 27a^3}$.

179. $\sqrt[3]{a^2 - 3a^{\frac{5}{3}}x^{-1} - 3a^{\frac{4}{3}}x^{-2} + 11a^{-3} + 6a^{\frac{2}{3}}x^{-4} - 12a^{\frac{1}{3}}x^{-5} - 8x^{-6}}$.

180. $\sqrt[3]{\frac{1}{8}x^2 - \frac{1}{8}xy^5 + \frac{3}{2}x^{\frac{4}{3}}y^{\frac{5}{3}} + 6x^{\frac{2}{3}}y^{\frac{10}{3}} + 8y^{\frac{15}{3}} - \frac{3}{8}x^{\frac{5}{3}}y^{\frac{5}{3}} - 3xy^{\frac{10}{3}} - 6x^{\frac{1}{3}}y^5 + \frac{3}{8}x^{\frac{4}{3}}y^{\frac{10}{3}} + \frac{3}{2}x^{\frac{2}{3}}y^5}$.

181. $\sqrt[3]{a^9 + 3a^8 + 6a^7 + 10a^6 + 12a^5 + 12a^4 + 10a^3 + 6a^2 + 3a + 1}$.

182. $\sqrt[3]{x^9 - 3x^8 + 8x^6 - 6x^5 - 6x^4 + 8x^3 - 3x + 1}$.

183. $\sqrt[3]{1 + 12a^2 + 27b^2 + 48c^2 - 6a + 9b - 12c - 36ab + 48ac - 72bc - 108b^2c - 64c^3 - 8a^3 - 54ab^2 - 96ac^2 + 36a^2b - 48a^2c + 144abc + 27b^3 + 144bc^2}$.

184. $\sqrt[3]{a^{12} + 6a^{11}b + 21a^{10}b^2 + 50a^9b^3 + 90a^8b^4 + 126a^7b^5 + 141a^6b^6 + 126a^5b^7 + 90a^4b^8 + 50a^3b^9 + 21a^2b^{10} + 6ab^{11} + b^{12}}$.

Approximate Roots.

185. $\sqrt[3]{1 - x}$.

186. $\sqrt{1 - x}$.

187. $\sqrt[3]{1 + x^3}$.

188. $\sqrt{a^2 + b^2}$.

189. $\sqrt[3]{a^3 - b^3}$.

190. $\sqrt{x^2 - xy + y^2}$.

191. $\sqrt[3]{x^3 + x^2y + xy^2 + y^3}$.

Express without fractional exponents

1. $a^{\frac{1}{2}}$.	2. $b^{\frac{1}{4}}$.	3. $2x^{\frac{1}{5}}$.
4. $3y^{\frac{2}{5}}$.	5. $cx^{\frac{3}{4}}$.	6. $a^{\frac{1}{2}}z$.
7. $a^{\frac{3}{4}}x^{\frac{3}{4}}$.	8. $a^{\frac{1}{2}}c^{\frac{7}{8}}$.	9. $5a^{\frac{1}{2}}b$.
10. $m^{\frac{1}{2}}np^{\frac{3}{4}}$.	11. $2a^{1/n}$.	12. $3y^{2/c}$.
13. $x^{1/n}y$.	14. $4^{\frac{1}{2}}x$.	15. $2^{\frac{3}{4}}y^{\frac{1}{4}}$.
16. $5^{n/2}a$.	17. $p^{n/3}q^{2/3}$.	18. $x^{(n+1)/2}$.
19. $a^{(n-1)/2}$.	20. $8^{\frac{3}{4}}a^{2/n}$.	21. $a^{\frac{1}{2}}b^{\frac{1}{3}}c^{\frac{1}{4}}$.
22. $x^{\frac{3}{4}}y^{\frac{1}{2}}z^{\frac{2}{3}}$.	23. $a^{\frac{1}{2}}bc^{\frac{1}{3}}d$.	24. $\frac{a^{2/n}}{c^{n/2}}$.
25. $\frac{m^{3/a}}{n^{c/a}}$.	26. $a^{\frac{4}{5}}x^{n}$.	27. $cd^{\frac{2}{3}}e^{\frac{3}{5}}f$.
28. $4^{n/2}c^{3/m}$.	29. $2^{1/n}ax^{n/2}$.	30. $1^{m/n}a^{x/2}$.
31. $2x^{a/c}y^{1/n}z^{m/3}$.	32. $a^{(m+n)/(m-n)}$.	33. $c^{3/(p+q)}$.
34. $(a-b)^{\frac{3}{4}}$.	35. $(x^{\frac{3}{5}})^{2}$.	36. $\left(\frac{a}{x}\right)^{\frac{4}{5}}$.
37. $\left(\frac{2x}{3y}\right)^{\frac{3}{4}}$.	38. $\frac{2a^{\frac{4}{5}}}{3c^{\frac{1}{2}}}$.	39. $\frac{ax^{m/n}}{by^{p/q}}$.
40. $(b^{\frac{3}{4}})^{\frac{2}{3}}$.	41. $(a^{n}+x^{n})^{\frac{3}{2}}$.	42. $3^{\frac{2}{5}}a^{n/3}x^{2}z^{2/m}$.
43. $3m^{\frac{2}{3}}n^{\frac{5}{7}}$.	44. $4ab^{\frac{1}{2}}c^{\frac{3}{4}}d^{\frac{2}{3}}$.	45. $7^{m}a^{\frac{2}{5}}k^{n/2}x^{1/n}$.
46. $a^{\frac{1}{2}}-b^{\frac{1}{3}}c^{\frac{1}{4}}$.	47. $x^{\frac{3}{4}}+y^{\frac{1}{3}}z^{\frac{1}{2}}$.	48. $m^{\frac{1}{2}}-p^{\frac{1}{3}}q^{\frac{1}{4}}$.
49. $a^{(n+1)/(n-1)}x^{(n-1)/(n+1)}$.	50. $c^{(m+n)/(m-n)}d^{(m-n)/(m+n)}$.	

Express without the radical sign

51. $\sqrt{3}$.
52. $\sqrt[3]{-27}$.
53. $\sqrt[3]{a}$.
54. $\sqrt[4]{x^3}$.
55. $\sqrt[3]{x^4}$.
56. $2\sqrt{n}$.
57. $\sqrt{a^5}$.
58. $\sqrt[3]{ac^2}$.
59. $\sqrt[5]{p^4q}$.
60. $3\sqrt[5]{y^6}$.
61. $\sqrt{(x+y)}$.
62. $\sqrt{(c-d)^3}$.
63. $\sqrt[3]{x^n z^2}$.
64. $\sqrt[5]{m^5 n^2}$.
65. $\sqrt[3]{pq^2r^6}$.
66. $\sqrt[3]{a^2x^4}$.
67. $\sqrt{x^n}$.
68. $\sqrt[n]{x}$.
69. $\sqrt[n]{2c^3}$.
70. $\sqrt[n]{ak^3}$.
71. $\sqrt[4]{a^3b^2c}$.
72. $\sqrt[3]{\frac{a^3}{x^2}}$.
73. $\sqrt[3]{c^{n-1}}$.
74. $\sqrt{(a-x)^n}$.
75. $\sqrt{\frac{ax^3}{cy^5}}$.
76. $\sqrt[4]{\frac{a^8}{x^6}}$.
77. $\sqrt{c^3}$.
78. $\sqrt{a^3}$.
79. $\sqrt[3]{16m^2n^3}$.
80. $\sqrt[n]{5ax^n}$.
81. $\sqrt[n]{3b^2c^{2n}}$.
82. $\sqrt[3n]{2p^3q^n}$.
83. $\sqrt[10]{a^4c^5}$.
84. $\sqrt[12]{a^2b^3c^4d^6}$.
85. $\sqrt{x^3} \times \sqrt[3]{x^2}$.
86. $\sqrt[3]{a^2x^3} + \sqrt{a^2x^5}$.
87. $\sqrt[3]{a^5}\sqrt{y^3}$.
88. $\sqrt[c]{c}\sqrt[4]{n^3}$.
89. $7^3\sqrt[3]{a^2}\sqrt{x^3}$.
90. $3a\sqrt[4]{x^5}\sqrt[7]{x^3}$.
91. $\sqrt[n]{x^{2n}y^{3n^2}}$.
92. $\sqrt{\frac{(a+x)y^3}{(a-x)h^3}}$.
93. $\sqrt{\frac{(a+c)^2e}{(a-c)^3f}}$.
94. $\sqrt[3]{\frac{(a+c)^{3n}x^2}{(a-c)^{3n}y^2}}$.
95. $\sqrt[3]{\sqrt{n^5}}$.
96. $\sqrt[3]{\sqrt[4]{x^6}}$.
97. $(\sqrt[3]{x^2})^4$.
98. $\sqrt[4]{2a\sqrt{\frac{m}{n}}}$.
99. $2\sqrt[n]{a^3}\sqrt{c^3}$.
100. $z\sqrt[n]{p}\sqrt{q^n}$.
101. $7\sqrt[3]{x^n}\sqrt{y^3}$.
102. $6\sqrt[3]{s^n}\sqrt[5]{t^n}$.
103. $\sqrt[m]{p^6}\sqrt[n]{q}\sqrt{r^s}$.
104. $f\sqrt[n]{g^{2m+n}}\sqrt{x^{m^2-n^2}}$.

Write without negative exponents

105. x^{-3}. 106. $a^{2}c^{-2}$. 107. $m^{-2}n$.

108. $p^{-3}q^{-3}$. 109. $x^{-7}y^{-5}$. 110. $6a^{-2}c^{-1}$.

111. $a^{-3}b^{\frac{1}{2}}$. 112. $x^{\frac{3}{2}}y^{-1}$. 113. $3ac^{-4}$.

114. $2a^{-5}e$. 115. $c^{4}d^{-\frac{2}{3}}$. 116. $2^{-2}a^{n}$.

117. $3a^{\frac{3}{2}}x^{-4}$. 118. $2c^{-5}k^{-\frac{5}{7}}$. 119. $c^{n}b^{-n}$.

120. $a^{-n}b^{-1}$. 121. $x^{-n}y^{-m}$. 122. $2a^{-n}y^{-3}$.

123. $a^{-\frac{3}{2}}b^{2}c^{-4}$. 124. $3h^{-2}y^{-\frac{1}{2}}z^{\frac{4}{3}}$. 125. $a^{-\frac{1}{2}}b^{-\frac{3}{2}}$.

126. $a^{-n}b^{2}c^{-m}d^{3}$. 127. $m^{-5}n^{-\frac{3}{2}}p^{-\frac{5}{6}}$. 128. $x^{m-1}y^{-1-m}$.

129. $a^{n}c^{-\frac{m-n}{1}}$. 130. $\dfrac{1}{a^{-2}}$. 131. $\dfrac{1}{x^{-n}}$. 132. $\dfrac{a}{c^{-n}}$.

133. $\dfrac{x^{3}}{x^{-3}}$. 134. $\dfrac{2a^{-2}}{3}$. 135. $\dfrac{x^{-2}}{x^{-1}}$. 136. $\dfrac{a^{-1}}{c^{-1}}$.

137. $\dfrac{1}{e^{-3}}$. 138. $\left(\dfrac{a}{c}\right)^{0}$. 139. $\dfrac{a^{-0}}{b^{-n}}$. 140. $\dfrac{1}{x^{-0}}$.

141. $\dfrac{a^{-0}}{3}$. 142. 3^{-0}. 143. $\dfrac{a^{-4}}{a^{-5}}$. 144. $\dfrac{c^{-2}}{c^{-0}}$.

145. $(-x^{3})^{0}$. 146. $\dfrac{m^{-n}}{x^{0}}$. 147. $(a+x)^{-0}$.

148. $\left(\dfrac{c}{d}\right)^{-0}$. 149. $\dfrac{a^{-n}}{x^{-n}}$. 150. $\dfrac{1}{x^{-1/a}}$.

151. $\dfrac{3^{-2}}{4^{-\frac{1}{2}}}$. 152. $m^{\frac{1}{2}}n^{-\frac{1}{2}}p^{-2}$. 153. $\dfrac{x^{-\frac{1}{2}}}{x^{-2}}$.

154. $(-x)^{-2}$. 155. $(-y)^{-3}$. 156. $(-z)^{-4}$.

157. $\dfrac{a^{-p}}{c^{-q}}$. 158. 0^{-n}. 159. 0^{-3}.

160. $\left(\frac{1}{a^{-1}}\right)^{-2}$.

161. $\left(\frac{x}{y}\right)^{-1}$.

162. $\frac{1}{(x^{-2})^3}$.

163. $\left(\frac{1}{a^2}\right)^{-3}$

164. $s(1-s)^{-1}$.

165. $c^2\left(\frac{c}{d}\right)^{-2}$.

166. $x(1-x)^{-0}$.

167. $\frac{(a+c)^{-2}}{(a-c)^{-3}}$.

168. $\frac{a+x}{(a+x)^{-1}}$.

169. $\frac{a^{-2}x^{-2}}{c^{-3}y^{-3}}$.

170. $\frac{a^2b^{-1}c}{m^{-1}np^{-3}}$.

171. $\frac{ab^{-2}c^3}{x^{-1}y^{-2}z}$.

172. $\frac{a^{-1}b^{-2}c^{-3}}{x^2y^{-1}z^{-2}}$.

173. $\frac{x^{-m}y^n}{c^{-m}d^ne^{-1}}$.

174. $\frac{2^{-3}a^{-3}c^3}{2^4x^{-3}z^3}$.

175. $\frac{4e^{-1}f^2g^{-5}}{5h^{-2}kq^{-5}}$.

176. $\frac{2a^0b^{-1}c^{-2}}{3a^{-1}b^{-2}c^{-3}}$.

177. $\frac{x^{-2}}{y^{n-3}}$.

178. $\frac{x^{n-2}}{x^{n-3}}$.

179. $\frac{y^{m-1}}{y^{m-2}}$.

180. $\frac{a^{n-3}}{a^{-4}}$.

181. $\frac{x^{\frac{1}{3}}y^2z^{-\frac{1}{3}}}{x^{\frac{1}{3}}y^{-2}z^{-\frac{2}{3}}}$.

182. $\frac{(6cx)^{-3}}{(3c^2)^{-2}(2x^3)^{-4}}$.

183. $\frac{2}{a^{1/n}b^{-n}c^{-1}}$.

184. $\frac{a^nc^{n/2}x}{a^{-1/n}c^{-n/2}x^{-2}}$.

185. $\frac{m^{\frac{1}{2}}n^{\frac{1}{3}}q^2}{m^{-\frac{1}{2}}n^{-\frac{1}{3}}q}$.

186. $\frac{c(p-q)^{-1}}{p+q}$.

187. $\frac{(c+x)(c-x)^{-2}}{(c-x)^{-1}(c+x)^{-2}}$.

188. $\frac{4^{-1}x^{-2}y^3z^{-4}}{5^{-1}a^{-2}bc^{-3}}$.

189. a^2+x^{-2}.

190. $a^{-1}+c^{-1}$.

191. $\frac{1}{1+x^{-1}}$.

192. $\frac{2}{p^{-1}+q^{-1}}$.

193. $\frac{a^{-1}}{c^{-1}-x^{-1}}$.

194. $\frac{a^2-x^2}{a^{-2}-x^{-2}}$.

195. $\frac{m^{-\frac{1}{2}}+x}{n^{-3}-x^{-n/2}}$.

Find the values of the following :

196. $4^{\frac{5}{2}}$.

197. $8^{\frac{4}{3}}$.

198. $9^{\frac{3}{2}}$.

199. $16^{\frac{1}{4}}$.

200. $25^{\frac{3}{2}}$.

201. $-27^{\frac{1}{3}}$.

202. $-32^{-\frac{1}{5}}$.

203. $36^{-\frac{1}{2}}$.

204. $-49^{-\frac{1}{2}}$.

205. $-64^{\frac{5}{6}}$.

206. $64^{-\frac{2}{3}}$.

207. $-64^{\frac{4}{3}}$.

208. $64^{\frac{1}{2}}$.

209. $81^{\frac{3}{4}}$.

210. $100^{\frac{3}{2}}$.

211. $-125^{\frac{4}{3}}$.

212. $-128^{-\frac{5}{7}}$.

213. $144^{\frac{3}{2}}$.

214. $-243^{\frac{2}{5}}$.

215. $256^{-\frac{3}{4}}$.

216. $343^{\frac{2}{3}}$.

217. $.16^{\frac{5}{4}}$.

218. $.08^{\frac{4}{3}}$.

219. $1.728^{\frac{2}{3}}$.

220. $64^{-\frac{1}{6}}$.

221. $81^{.75}$.

222. $36^{1.5}$.

223. $2.25^{2.5}$.

224. $256^{.125}$.

225. $(.0001)^{-\frac{1}{4}}$.

226. $(.0625)^{-\frac{3}{4}}$.

227. $\left(\frac{27}{64}\right)^{\frac{4}{3}}$.

228. $\left(\frac{1}{4}\right)^{-2}$.

229. $\frac{3}{2^0}$.

230. $\frac{2}{3^{-1}}$.

231. $\frac{3}{(-2)^3}$.

232. $\frac{2}{(-5)^{-3}}$.

233. $\left(\frac{1}{4}\right)^{-1}$.

234. $-\left(\frac{1}{4}\right)^{-2}$.

235. $\left(\frac{1}{3}\right)^{-2}$.

236. $\frac{2^{-4}}{4^{-2}}$.

237. $\left(\frac{32}{243}\right)^{-\frac{2}{5}}$.

238. $\left(\frac{9}{16}\right)^{1.5}$.

239. $\left(\frac{16}{81}\right)^{1.25}$.

240. $\left(\frac{5^{16}}{5^{12}}\right)^{3.75}$.

241. $(3\frac{3}{8})^{-\frac{2}{3}}$.

242. $(5^{\frac{1}{16}})^{\frac{2}{3}}$.

243. $(15\frac{5}{8})^{-\frac{2}{3}}$.

244. $\frac{2^0}{(-1)^{-9}}$.

245. $\frac{7}{2^{-3}}$.

246. $3^{-1}-3^{-2}$.

247. $16^{-2}-64^{-3}$.

248. $4^{-1.5}+9^{-.5}$.

249. $4^{-3.5}+8^{-2\frac{1}{3}}$.

250. $9^{\frac{1}{2}}\times 27^{-\frac{2}{3}}$.

251. $36^{-\frac{3}{2}}\times 3^2$.

252. $25^{\frac{3}{2}}\div 27^{\frac{2}{3}}$.

253. $9^{-\frac{3}{2}} \times 36^{\frac{3}{2}}$.

254. $\left(\frac{1}{3}\right)^{-3}(243)^{-\frac{4}{5}}$.

255. $81^{-\frac{3}{4}}(.3)^2$.

256. $\frac{1}{5^{-3}} - \frac{1}{4^{-4}}$.

257. $\frac{1}{5^{-4}} - \frac{1}{10^{-2}}$.

258. $\left(\frac{1}{16}\right)^{12} \times 27^{-\frac{2}{3}}$.

259. $1\left(\frac{1}{125}\right)^{-\frac{2}{3}} \times \left(\frac{1}{36}\right)^{\frac{1}{2}}$.

260. $\left(\frac{17^{-8} - \frac{2^5}{5^{-6}}}{\frac{1}{9^{-7}} + 10^{-4}}\right)^0$.

261. $\frac{4^{\frac{3}{2}} \times 9^{-2}}{81^{-\frac{3}{2}} \times 16^{\frac{7}{4}}}$.

262. $\frac{(-27)^{\frac{1}{3}} \times 25^{\frac{5}{2}}}{36^{\frac{3}{2}} \times 16^{-\frac{5}{4}}}$.

263. $25^{\frac{1}{2}} + 25^{-\frac{3}{2}} + 25^0$.

264. $27^{-\frac{2}{3}} - 27^{\frac{1}{3}} + (-27)^{\frac{3}{3}}$.

265. $8^{-\frac{1}{3}} + 8^{-2} - 8^{\frac{2}{3}} + 8^{-\frac{2}{3}}$.

266. $16^{\frac{3}{4}} + 16^{\frac{1}{4}} - 16^{-\frac{1}{4}} - 16^{-\frac{3}{4}}$.

267. $32^{\frac{1}{5}} - 32^{\frac{2}{5}} + 32^{-\frac{3}{5}} + 32^{-\frac{4}{5}}$.

268. $+ 36^{\frac{1}{2}} + 36^{\frac{3}{2}} + 36^{-\frac{5}{2}}$.

269. $3^{-\frac{1}{3}} \times \sqrt[3]{81 \times 5^{-\frac{3}{2}}} \times \sqrt{5}$.

270. $27 \times 3^{-2} \times 32 \times 2^{-3} + 96 \times 2^{-4} + 8 \times 1^{-5} + 4 \times 17^0$.

271. $5^{\frac{3}{3}} \times 5^{\frac{2}{4}} \times 5^{\frac{7}{5}} + 25^{\frac{1}{11}} \times 25^{\frac{4}{11}} \times 25^{\frac{1}{22}}$.

272. $7^{\frac{2}{3}} \times 7^{\frac{3}{2}} \times 7^{\frac{7}{4}} + 16^{\frac{5}{17}} \times 16^{\frac{1}{34}} \times 16^{\frac{8}{17}}$.

273. $16^{\frac{3}{2}} + 8^{\frac{2}{3}} + 16^{\frac{3}{4}} + 125^{\frac{1}{3}} - 512^{\frac{5}{9}} + 100^{.5} - 81^{.75}$.

274. If $17^{.3265} = 4$, what is the value of $17^{.9795}$?

275. If $7^{2.0134} = 50$ and $7^{2.5678} = 55$, what is the value of $7^{4.5812}$?

Find the values of the following:

276. $(x^{-3})^2$.

277. $(y^2)^{-3}$.

278. $(a^{-3})^5$.

279. $(x^{\frac{1}{5}})^5$.

280. $(z^4)^{\frac{1}{4}}$.

281. $(8^{\frac{1}{3}})^2$.

282. $(2a^{\frac{3}{2}})^{-3}$.

283. $(64y^6)^{-\frac{2}{3}}$.

284. $(8^{\frac{3}{2}}x^{-3})^{\frac{2}{3}}$.

285. $(125^{-\frac{3}{2}})^{-\frac{2}{3}}$.

286. $(w^{\frac{4}{5}})^{-3}$.

287. $(x^{-2})^{-\frac{1}{2}}$.

288. $(c^{-3})^{\frac{5}{6}}$.

289. $(u^{-\frac{3}{2}})^{\frac{2}{3}}$.

290. $(c^{m-n})^{m+n}$.

291. $(x^{\frac{5}{6}}y^{\frac{3}{2}})^{\frac{2}{3}}$.

292. $(q^{-n/4})^{2/n}$.

293. $(\sqrt[4]{p})^6$.

294. $(\sqrt[5]{x^4})^{-\frac{5}{6}}$.

295. $\{(-b^2)^3\}^4$.

296. $\{(\sqrt{x^3})^{-\frac{1}{2}}\}^{\frac{1}{3}}$.

297. $(a^{m/n-1})^{m/(m-n)}$.

298. $(a^{u/v})^{s/t}$.

299. $(m^{-\frac{1}{2}}n^{\frac{3}{2}})^{-\frac{2}{3}}$.

300. $(\frac{1}{8}a^{\frac{3}{2}})^{\frac{2}{3}}$.

301. $(64x^{12})^{\frac{5}{6}}$.

302. $(a^{2p/q})^{q/p}$.

303. $(a^{-\frac{4}{5}}b^{\frac{1}{2}})^{\frac{2}{3}}$.

304. $(x^{-n})^{-3}$.

305. $(\sqrt[5]{c^3})^{-\frac{2}{3}}$.

306. $(y^{-a/c})^{b/d}$.

307. $(x^{\frac{1}{2}}y^{-\frac{1}{2}})^{-\frac{1}{2}}$.

308. $(\sqrt[n]{x^{-m}})^{-5}$.

309. $(a^{1/n}x^{2/n})^{n/2}$.

310. $(-x^3)^5$.

311. $(-a^3)^{2n}$.

312. $(-x^{2m})^3$.

313. $(-x^{3c})^2$.

314. $(-y)^{2n-1}$.

315. $(-z^{2n})^{2n-1}$.

316. $(-a^{m+1})^2$.

317. $4(x^{-\frac{2}{3}})^{\frac{3}{2}}$.

318. $(\frac{1}{2}a^{\frac{2}{3}}b^{\frac{1}{2}})^6$.

319. $(243a^{10})^{\frac{2}{5}}$.

320. $(100k^2l^{-4})^{-\frac{1}{2}}$.

321. $(216x^6)^{-\frac{1}{3}}$.

322. $(16m^{-4}n^8)^{-\frac{3}{4}}$.

323. $(32a^5b^{10}c^{20})^{\frac{2}{5}}$.

324. $(a^{-\frac{5}{2}}x^{-\frac{10}{3}}y^{\frac{1}{2}})^{\frac{6}{5}}$.

325. $(e^3f^{\frac{4}{3}}g^{-\frac{2}{5}})^{15}$.

326. $\sqrt[3]{a^2(a+c)^{\frac{1}{2}}}$.

327. $(m^{m/n}n^{2/n})^{n/2}$.

328. $(5x^2y^{n-1}z^{m+2})^3$.

329. $(\sqrt[6]{(a^3bc\sqrt[5]{a^3bc})^5}$.

330. $(6m^an^{a-1}p^{a-2})^3$.

331. $\{(a+x)\sqrt{x}\}^4$.

332. $\{(n^{-\frac{1}{2}})^{-1}\}^{-\frac{2}{3}}$.

333. $(256c^{\frac{3}{4}}d^{-\frac{1}{3}})^{-\frac{3}{4}}$.

334. $\{([a^{-3}]^{-3})^{-3}\}^{-3}$.

335. $\sqrt{(a^{\frac{1}{2}}c^{\frac{1}{3}})m^{-2}n^{-3}}$.

336. $\{(a^{\frac{1}{2}}c^{\frac{1}{3}})a^{-1}c^{-\frac{2}{3}}x^{\frac{1}{2}}\}^4$.

337. $\{(x+y)^{m/n}z^{1/n}\}^{n/m}$.

338. $\{3\sqrt[n]{(x^{-m}y^c)}\}^{-mc}$.

339. $n^{1\frac{7}{4}}\sqrt{\{(m^{\frac{1}{3}}n^{\frac{1}{3}}p)^{-12}m^2n^{-\frac{5}{2}}\}}$.

340. $\{\sqrt[4]{(a^{-\frac{2}{3}}c^{\frac{1}{2}})^3}\}^{-\frac{2}{3}}$.

341. $\left(\frac{1}{x}\right)^{-5}$.

342. $\frac{1}{\sqrt[5]{x^3}}$.

343. $\left(\frac{a^{\frac{3}{2}}}{c^{\frac{3}{2}}}\right)^{-\frac{1}{2}}$.

344. $\left(\frac{p^n}{q^n}\right)^{1/n}$.

345. $\left(\frac{a^{-2}}{b^{-2}}\right)^{-1}$.

346. $\left(\frac{ac^{n/m}}{ef^n}\right)^{m/n}$.

347. $\left(\frac{256}{625}\right)^{-\frac{3}{4}}$.

348. $\left(\frac{\sqrt[5]{p^2q^3}}{p^{-\frac{1}{2}}q^{\frac{1}{2}}}\right)$.

349. $\left(\frac{\sqrt[4]{a}\sqrt{b}}{3\sqrt{ab}}\right)^3$.

350. $\left(\frac{16x^{-4}}{81y^3}\right)^{-\frac{3}{4}}$.

351. $\left(\frac{9a^4}{25n^3}\right)^{-\frac{3}{2}}$.

352. $\left\{\left(\frac{2c^{-\frac{1}{3}}}{m^{-1}n^{-\frac{1}{3}}}\right)^2\right\}^{-3}$.

353. $\left\{\left(\frac{4k^{-\frac{1}{4}}q^2}{9l^{-3}r^{\frac{1}{2}}}\right)^{-\frac{1}{2}}\right\}^2$.

354. $\left\{\left(\frac{\sqrt[4]{x^{-3}}}{2y^{-\frac{1}{2}}z^{\frac{5}{6}}}\right)^{-\frac{2}{3}}\right\}^{-6}$.

355. $\left\{\left(\frac{c^{-\frac{1}{10}}d^{18}}{p^3q^{-\frac{3}{5}}}\right)^{-\frac{1}{2}}\right\}^{-\frac{5}{9}}$.

Find the numerical value of the following expressions when $a = 64$, $b = 16$, $c = 9$, $d = 8$, $e = 4$, $g = 2$, $h = 1$, $n = 3$.

356. $a^{\frac{1}{2}}d^{-\frac{1}{3}}$.

357. $b^{\frac{1}{4}}e^{-\frac{1}{2}}$.

358. $a^{\frac{3}{2}}b^{-\frac{3}{2}}$.

359. $c^{\frac{3}{2}}n^{-3}$.

360. $d^{\frac{4}{3}}a^{-\frac{2}{3}}$.

361. $c^{\frac{1}{2}}e^2$.

362. $d^{\frac{2}{3}}g^{-6}$.

363. $a^{-\frac{5}{6}}e^{\frac{1}{2}}$.

364. $a^{2/n}e^{-\frac{1}{2}}$.

365. $a^{-\frac{1}{2}}b^{\frac{1}{4}}e$.

366. $(ag^3)^{\frac{1}{3}}$.

367. $c(eg)^{\frac{2}{3}}$.

368. $2(ce)^{\frac{1}{2}}$.

369. $a^{-\frac{1}{3}}(cn)^{\frac{1}{3}}$.

370. $(ad)^{1/3n}$.

371. $d^{\frac{1}{3}}h^{-\frac{2}{3}}$.

372. $2c^{-\frac{1}{2}}e^{-\frac{1}{2}}$.

373. $\sqrt[n]{deg}$.

374. $b^{\frac{1}{2}}d^{\frac{1}{3}}e^{\frac{1}{2}}$.

375. $e^{-n}a^{\frac{3}{2}}$.

376. $d^{-\frac{4}{3}}b^{n/3}h$.

377. $a^{\frac{1}{3}}d^{6/n}e^{-2}$.

378. $d(eg)^{-\frac{1}{2}}$.

379. $h(be)^{2/n}$.

380. $(bd)^{-2}eg$.

381. $g^{-4}b^{\frac{1}{2}}n^{-3}d^{\frac{1}{3}}$.

382. $b^{1/e}d^{(n-1)/3}$.

383. $(a^{\frac{1}{2}}e^2)^{(c-d)/(e+n)}$.

384. $(2a^{\frac{1}{6}}b^{-\frac{7}{4}}c^{\frac{5}{2}})^{(g+h)/(3n)}$.

385. $\sqrt[n]{a}\sqrt[n]{d}\sqrt[n]{h}(b^{-\frac{1}{2}}e^{s}g^{-n})$.

386. $c^{-\frac{1}{2}}n + d^{-1/n}e^{\frac{1}{2}} - a^{\frac{3}{5}}g^{-5}$.

387. $d^{\frac{3}{5}}e^{-2}(b^{-\frac{1}{2}}c^{\frac{1}{2}} + g^{5}n^{-2})$.

388. $c^{\frac{1}{2}}d^{\frac{1}{3}} - h^{\frac{1}{2}}e^{\frac{1}{2}} \times a^{1/2n}$.

389. $a^{\frac{1}{2}}c^{\frac{1}{2}}e^{\frac{1}{2}} \div a^{\frac{1}{2}}d^{\frac{1}{3}}h^{\frac{1}{3}} + b^{\frac{1}{2}}d^{-n}$.

390. $g(a+b+h)^{\frac{1}{2}} \div (b+c+g)^{1/n} + a^{-\frac{5}{6}}b^{\frac{2}{3}}c^{\frac{1}{2}} + (b-n)d^{-\frac{1}{3}}$.

391. $\dfrac{e^{n}g^{-4}}{a^{-1/n}g^{2}}$.

392. $\dfrac{c^{\frac{1}{2}}b^{-\frac{2}{3}}}{d^{\frac{1}{3}}e^{-\frac{2}{3}}}$.

393. $\dfrac{(cn)^{1/n}b^{n/3}d^{-n}}{3g^{-n}a^{\frac{1}{6}}}$.

394. $\dfrac{a^{\frac{1}{3}}d^{-\frac{1}{3}}e^{-1}g^{n}}{b^{\frac{2}{3}}d^{1/n}n^{-n}}$.

395. $\dfrac{(ad)^{\frac{2}{3}}c^{(n-1)/n}e^{-2}}{a^{-\frac{1}{6}}b^{-\frac{1}{4}}n^{2}h^{2n}}$.

Perform the following indicated multiplications:

296. $(3-1)^{-3}$.

297. $(1-3)^{-2}$.

398. $(5-2)^{-3}$.

399. $-(2-5)^{-3}$.

400. $2^{-3} \times 2^{-2}$.

401. $3^{-2} \times 2^{-2}$.

402. $4^{-1} \times 5^{-1}$.

403. $5^{-2} \times 3^{-2}$.

404. $4^{-4} \times 2^{-4}$.

405. $3^{-3} \times 2^{-3}$.

406. $.2^{-2} \times .3^{-2}$.

407. $(\frac{2}{3})^{-2} \times (\frac{3}{4})^{-2}$.

408. $a^{2} \times a^{-3}$.

409. $x^{-2} \times x^{-3}$.

410. $n^{-\frac{1}{2}} \times n^{\frac{1}{3}}$.

411. $3a^{\frac{5}{2}} \times a^{-\frac{3}{2}}$.

412. $c \times 2c^{-\frac{7}{6}}$.

413. $3p^{-3} \times 4p^{-4}$.

414. $z^{5} \times \sqrt[6]{z^{-1}}$.

415. $x^{-\frac{3}{2}} \times x^{-\frac{4}{7}}$.

416. $k^{-4}q^{\frac{3}{4}} \times 6k^{5}q^{-3}$.

417. $8\sqrt[7]{m^{2}} \times \sqrt{m^{-3}}$.

418. $a^{0}x^{-n} \times a^{2}x^{p}$.

419. $a^{-2m}a^{-2m}$.

420. $x^{-a} \times x^{-c}$.

421. $m^{-p} \times n^{-q}$.

422. $c^{-2a} \times c^{-3a}$.

423. $x^{-3}(-x)^{-4}$.

424. $(-x^{-5})(-x)^{-6}$.

425. $k^{-2} \times k^{-3n}$.

426. $5\sqrt[3]{a^2} - 2a^{\frac{2}{3}}c^2$.

427. $-17a^{\frac{1}{2}}x^{\frac{1}{3}} \times -2a^{\frac{1}{2}}x^{\frac{1}{3}}$.

428. $2x\sqrt[5]{y^6} \times 4\sqrt[3]{y^5}$.

429. $-m^{-4}n^4p^5q^{-5} \times -m^5n^{-5}p^4q^{-4}$.

430. $a^{1/m} \times a^{1/n}$.

431. $c^{m/n} \times c^{n/m}$.

432. $d^{m/n} \times d^{p/q}$.

433. $x^{a/n} \times x^{-n/a}$.

434. $p^{-a/c} \times p^{-0}$.

435. $q^{-2/x} \times q^{-a/2x}$.

436. $k^{-2m} \times k^{-m/n}$.

437. $x^3y^{\frac{4}{3}} \times (xy)^{15}$.

438. $(c^{\frac{1}{2}})^{-3} \times c^{\frac{3}{2}}$.

439. $(a)^{3n} \times (-a)^6$.

440. $(-p)^4 \times (-p)^{2q}$.

441. $(-x)^{2n-1} \times (-x)^2$.

442. $(a^mc^{-n})^2 \times (-a)^{2n+1}$.

443. $(-a)^3 \times (-a)^{2n+1}$.

444. $(-q)^{2a+1} \times (q)^3$.

445. $(-x)^{-3} \times (-x)^{-2}$.

446. $(-ac)^{2n-2} \times (-ac)^5$.

447. $(-b)^{2m-1} \times (-b)^{2n+1}$.

448. $c^{-n} \times c^m \times c^{-2n}$.

449. $\dfrac{5u^{-3}}{3w^3} \times \dfrac{6u^2}{35w^{-4}}$.

450. $\dfrac{12x^{-3}}{y^{-2}} \times \dfrac{x^2}{9y^3}$.

451. $\dfrac{p^{-n}q^{-n}}{\frac{1}{2}x} \times \dfrac{p^{-2n}q^{-3n}}{x^{-1}}$.

452. $\dfrac{a^{m+n} \times a^{m-n} \times a^{p-2m}}{a^{p-m}}$.

453. $x^{-2} \times \dfrac{12}{x^{-\frac{3}{5}}}$.

454. $\dfrac{1}{c^{-3}y^{\frac{2}{5}}} \times c^{-4}y^{-\frac{1}{10}}$.

455. $\sqrt[3]{xy^{-1}z^{-2}} \times (x^{-1}y^{-2}z^{-4})^{-\frac{1}{3}}$.

456. $\sqrt[6]{m^{4p}n^6} \times (m^{\frac{2}{3}}n^{-1})^{-p}$.

457. $(a \times \sqrt[n]{a^{-1/n}})^{n^2/(1-n)}$.

458. $(a^{-\frac{1}{2}}\sqrt[3]{q})^{-3} \times \sqrt{q^{-2}\sqrt{p^{-6}}}$.

459. $3^{-\frac{1}{2}} \times \sqrt[3]{81 \times 5^{-\frac{3}{2}}} \times \sqrt{5}$.

460. $x^{\frac{1}{2}}y^{-\frac{1}{2}} \times \{x^{\frac{1}{3}}y^{\frac{1}{3}}(x^{\frac{3}{2}})^{\frac{1}{3}}\}^{\frac{1}{2}}$.

461. $\dfrac{2^{n-1} \times (2^{-1})^{n+1}}{(2^{-n})^{-1} \times (2^{n-2})^{-1}}$.

462. $\left(\sqrt[6]{\dfrac{1}{a^3c}}\right)(\sqrt[5]{a^2} \times \sqrt[3]{c})$.

463. $\{-(c^3)^{\frac{1}{2}}\}^{-\frac{1}{3}} \times \{-(-c)^{-3}\}^{\frac{1}{3}}$.

464. $\left\{(m+n)\sqrt{\frac{p^3}{(m+n)^{\frac{1}{3}}}}\right\}^{\frac{1}{3}}$.

465. $\{xy^3\sqrt{xy^3}\sqrt[3]{xy^4}\sqrt[4]{xy^5}\}^{\frac{1}{5}}$.

466. $\left\{\left(\frac{c^{-\frac{1}{2}}x^2}{cx^{\frac{1}{2}}}\right)^{\frac{1}{3}} \times \left(\frac{c^{-1}x}{(cx^2)^{\frac{1}{3}}}\right)^{\frac{1}{3}}\right\}^{-2}$.

467. $\left(\sqrt{\left\{\sqrt{\left(k^{-3} \times \frac{1}{\sqrt{q}}\right) kq}\right\}}\right)^{12}$.

468. $\sqrt[13]{\left(x^3y^2\sqrt[3]{\left\{xyz^4\sqrt[4]{\frac{1}{xy^2z^3}}\right\}}\right)^{12}}$.

469. $\left(\frac{5-x}{y-3}\right)^3 \times \left(\frac{3-y}{5-x}\right)^5$.

470. $\left(\frac{m^2-n^2}{p^4-q^4}\right)^5 \times \left(\frac{p^4-q^4}{n^2-m^2}\right)^7$.

471. $\left(\frac{a-c}{m-n}\right)^{2p} \times \left(\frac{n-m}{a-c}\right)^{2p+1}$.

472. $\left(\frac{x-z}{f-g}\right)^{2n} \times \left(\frac{g-f}{z-x}\right)^{2n-1}$.

473. $\left(\frac{a^8-b^8}{c^5-d^5}\right)^{2n} \times \left(\frac{d^{10}-c^{10}}{b^4-a^4}\right)^n \times \left(\frac{c^5-d^5}{a^4+b^4}\right)^n$

474. $\left(\frac{a+x}{a-x}\right)^{-3} \times \left(\frac{a-x}{a+x}\right)^{-2}$.

475. $\left(\frac{1}{1-k}\right)^{-7} \times \left(\frac{k-1}{k+1}\right)^{-6} \times \left(\frac{1}{1+k}\right)^5$.

476. $\left(\frac{a+c}{s+t}\right)^4 \times \left(\frac{a-c}{s+t}\right)^{-4} \times \left(\frac{a+c}{a-c}\right)^{-5}$.

477. $\left\{\left(\frac{z-w}{g+h}\right)^{-3}\right\}^{2} \times \left\{\left(\frac{g+h}{z-w}\right)^{-2}\right\}^{3}$.

478. $\left\{\left(\frac{p-q}{a+c}\right)^{-2}\right\}^{-5} \times \left\{\left(\frac{a+c}{p-q}\right)^{-5}\right\}^{-3}$.

479. $\left\{\left(\frac{k-g}{e-f}\right)^{-3}\right\}^{-5} \times \left\{\left(\frac{f-e}{q-k}\right)^{-7}\right\}^{2}$.

480. $\left\{\left(\frac{m+n}{x-y}\right)^{-2}\right\}^{7} \times \left\{\left(\frac{x-y}{m+n}\right)^{-6}\right\}^{2}$.

Perform the following multiplications :

481. $a^{\frac{1}{2}} + 1$ by $a^{\frac{1}{2}} - 1$.

482. $b^{\frac{1}{2}} + 3$ by $b^{\frac{1}{2}} - 3$.

483. $c^{\frac{1}{3}} - 2$ by $c^{\frac{1}{3}} - 1$.

484. $a^{\frac{1}{2}} + x^{\frac{1}{2}}$ by $a^{\frac{1}{2}} - x^{\frac{1}{2}}$.

485. $p^{\frac{1}{3}} + q^{\frac{1}{3}}$ by $p^{\frac{1}{3}} - q^{\frac{1}{3}}$.

486. $x^{\frac{3}{2}} + y^{\frac{3}{2}}$ by $x^{\frac{3}{2}} - y^{\frac{3}{2}}$.

487. $m^{\frac{2}{5}} + n^{\frac{2}{5}}$ by $m^{\frac{2}{5}} - n^{\frac{2}{5}}$.

488. $c^{\frac{1}{3}} + c^{-2}$ by $c^{\frac{1}{3}} - c^{-2}$.

489. $d^{\frac{1}{2}} + d^{-\frac{1}{2}}$ by $d^{\frac{1}{2}} - d^{-\frac{1}{2}}$.

490. $g + g^{-1}$ by $g + g^{-1}$.

491. $f^{\frac{1}{2}} + h^{-\frac{1}{2}}$ by $f^{-\frac{1}{2}} + h^{\frac{1}{2}}$.

492. $3^{\frac{1}{2}} + a^{-3}$ by $3^{\frac{1}{2}} + a^{-3}$.

493. $a^{\frac{2}{3}} + a^{\frac{1}{3}}$ by $a^{\frac{1}{3}} - 1$.

494. $x^{\frac{2}{3}} - 2y^{-1}$ by $x^{\frac{2}{3}} - y^{-1}$.

495. $a^{2}x^{-\frac{1}{3}} + a^{\frac{1}{2}}x$ by $a^{-2}x - a^{-1}x^{\frac{1}{3}}$.

496. $a^{m} - c^{-n}$ by $a^{m} + c^{-n}$.

497. $p^{-n} + q^{n/2}$ by $p^{-n} - q^{-n/2}$.

498. $2x^{n} + a^{n-1}$ by $3x^{-n} + 2x^{n}$.

499. $a^{n+1} + a^{n-1}$ by $a^{n+1} + a^{n-1}$.

500. $\left(\frac{5a^{-\frac{5}{6}}}{4c^{-\frac{3}{4}}} + \frac{3m^{-\frac{1}{2}}}{2n^{-\frac{2}{5}}}\right)$ by $\left(\frac{5a^{\frac{5}{6}}}{4c^{-\frac{3}{4}}} - \frac{3m^{-\frac{1}{2}}}{2n^{-\frac{2}{5}}}\right)$.

501. $1 + m^{\frac{1}{2}} + m^{\frac{1}{4}}$ by $1 - m^{\frac{1}{4}}$.

502. $n^{\frac{2}{3}} + n^{\frac{1}{3}} + 1$ by $n^{\frac{1}{3}} - 1$.

503. $8c^{-2} + 7c^{-1} + 6$ by $2c^{-1} - 1$.

504. $a^{\frac{2}{3}} - a^{\frac{1}{3}}x^{\frac{1}{3}} + x^{\frac{2}{3}}$ by $a^{\frac{1}{3}} + x^{\frac{1}{3}}$.

505. $a^{2x} - 1 + a^{-2x}$ by $5a^{x} - 3a^{-x}$.

506. $3a^{-\frac{1}{3}} + a + 2a^{\frac{2}{3}}$ by $a^{\frac{1}{3}} - 2$

507. $x^{\frac{7}{4}} - x^{\frac{3}{2}} + x^{\frac{5}{4}} - x$ by $x^{\frac{3}{4}} + x^{\frac{1}{2}}$.

508. $p^{\frac{1}{2}}q^{-\frac{3}{2}} + pq^{-\frac{1}{2}} + 1$ by $pq^{-\frac{1}{2}} - 1$.

509. $a^{\frac{1}{2}} - a^{\frac{1}{4}}x^{\frac{1}{4}} + x^{\frac{1}{2}}$ by $a^{\frac{1}{4}} + x^{\frac{1}{4}}$.

510. $k^{\frac{5}{4}} - k^{\frac{3}{4}} + k^{\frac{1}{4}} - k^{-\frac{1}{4}}$ by $k^{\frac{3}{4}} + k^{\frac{1}{4}}$.

511. $a^{\frac{3}{2}} + 3ax^{\frac{1}{2}} + 3a^{\frac{1}{2}}x^{\frac{3}{2}} + x$ by $a^{\frac{1}{2}} - x^{\frac{1}{2}}$.

512. $5 + 2c^{2n} + 3c^{-2n}$ by $4c^{n} - 3c^{-n}$.

513. $a^{\frac{7}{6}} - a^{\frac{5}{6}} + a^{\frac{1}{2}} - a^{\frac{1}{6}} + a^{-\frac{1}{6}}$ by $a^{\frac{2}{3}} + a^{\frac{1}{3}}$.

514. $5a^{0}cx^{-5} - 3a^{2}c^{-6}x^{0} + 15c^{0}x^{5}$ by $2a^{-1}c^{3}x^{-3} - 1$.

515. $\frac{1}{8}u^{\frac{3}{2}} - \frac{1}{12}uv^{\frac{1}{2}} + \frac{1}{18}u^{\frac{1}{2}}v - \frac{1}{27}v^{\frac{3}{2}}$ by $\frac{1}{2}u^{\frac{1}{2}} + \frac{1}{3}v^{\frac{1}{2}}$.

516. $x^{3} + x^{\frac{3}{2}} - 1$ by $x^{-3} + x^{-\frac{3}{2}} - 1$.

517. $k + k^{\frac{1}{2}} - 5$ by $k + k^{\frac{1}{2}} + 5$.

518. $z^{n} + z^{n/2} + 1$ by $z^{-n} + z^{-n/2} + 1$.

519. $0^{-7} + 0^{-5} - 0^{-3}$ by $0^{7} + 0^{5} + 0^{3}$.

520. $2 - n^{\frac{1}{5}} + n$ by $2 + n^{\frac{1}{5}} + n$.

521. $2a^{\frac{1}{3}} + 4 + 5a^{-\frac{1}{3}}$ by $2a^{\frac{1}{3}} + 4 - 5a^{-\frac{1}{3}}$.

522. $x^{\frac{2}{3}} + x^{-\frac{2}{3}} - x^{\frac{1}{3}} - x^{-\frac{1}{3}}$ by $x^{\frac{1}{3}} + x^{-\frac{1}{3}} + 1$.

523. $p^{-1} + p^{-\frac{1}{2}}q^{-\frac{1}{2}} + q^{-1}$ by $p^{-1} - p^{-\frac{1}{2}}q^{-\frac{1}{2}} + q^{-1}$.

524. $s^{\frac{3}{2}} + 2s^{\frac{5}{3}} + 3s^{\frac{1}{3}} + 1$ by $s^{\frac{1}{2}} - 2s^{\frac{1}{3}} + 1$.

525. $2\sqrt[3]{x^{5}} - x^{\frac{1}{3}} - \frac{3}{x}$ by $2x - 3\sqrt[3]{\frac{1}{x}} - x^{-\frac{5}{3}}$.

526. $4a - 2a^{\frac{1}{2}}x^{-\frac{1}{2}} + 2a^{\frac{1}{2}}c^{\frac{1}{2}} - x^{-1} + c^{\frac{1}{2}}x^{-\frac{1}{2}}$ by $2a^{\frac{1}{2}} + x^{-\frac{1}{2}} - c^{\frac{1}{2}}$.

527. $a^{m+2} + 2a^{m+1}c^{n} + 3c^{2n}$ by $a^{m+2} - 2a^{m+1}c^{n} + 3c^{2n}$.

528. $m^{a+2} - m^{a+1}n^{c+1} + n^{c+2}$ by $m^{a+2} + m^{a+1}n^{c+1} + n^{c+2}$.

529. $x^{-4n} - 3x^{-3n} + 5x^{-2n} + x^{-1n}$ by $x^{-3n} - 2x^{-2n} - x^{-1n}$.

530. $\frac{1}{2}y^{a} + \frac{1}{3}y^{-2a} - \frac{1}{4}y^{-3a} + \frac{1}{8}y^{-4a}$ by $2y^{-1a} - 3y^{-2a} + 4y^{-3a}$.

Perform the following divisions:

531. 3^{-4} by 3^{-5}.

532. 5^{-4} by 5^{-2}.

533. 3^{-2} by 3^{2}.

534. 4^{2} by 4^{-2}.

535. $(\frac{3}{5})^{-3}$ by $(\frac{3}{5})^{-6}$.

536. $.4^{-1}$ by $.4^{-3}$.

537. $.6^{-6}$ by $.6^{-8}$.

538. $.4^{-7}$ by $.4^{-4}$.

539. $.1^{-5}$ by $.1^{-2}$.

540. 3^{-5} by $.3^{-5}$.

541. 3^{-6} by 6^{-6}.

542. 18^{-3} by 9^{-3}.

543. 12^{-4} by 4^{-4}.

544. 1^{-5} by 2^{-5}.

545. 3^{-5} by 1^{-5}.

546. $.2^{-3}$ by $.5^{-3}$.

547. $(\frac{2}{5})^{-3}$ by $(\frac{4}{15})^{-3}$.

548. $(\frac{3}{4})^{-4}$ by $(\frac{9}{8})^{-4}$.

549. $\left(\frac{3\frac{2}{5}}{4\frac{1}{5}}\right)^{-3}$ by $\left(\frac{8\frac{1}{2}}{10\frac{1}{2}}\right)^{-3}$.

550. $\left(\frac{13\frac{1}{3}}{11\frac{1}{2}}\right)^{-5}$ by $\left(\frac{16\frac{1}{8}}{18\frac{3}{8}}\right)^{-5}$.

551. x by x^{-3}.

552. n by $n^{\frac{1}{3}}$.

553. $x^{-\frac{3}{4}}$ by $x^{\frac{3}{8}}$.

554. $2a$ by $a^{\frac{1}{2}}$.

555. $3c^{\frac{3}{4}}$ by c.

556. $5b$ by $d^{\frac{3}{4}}$.

557. z^{2} by $\frac{1}{\sqrt[3]{z^{2}}}$.

558. $x^{-\frac{3}{2}}$ by $x^{-\frac{1}{2}}$.

559. c^{-2} by $\sqrt{c^{-5}}$.

560. $15g^{-3}k^{-\frac{7}{2}}$ by $5g^{5}k^{-3}$.

561. $1\frac{1}{3}x^{-2}$ by $4x^{2}$.

562. x^{n} by x^{-n}.

563. c^{-a} by c^{a}.

564. a^{m-n} by a^{-n}.

565. b^{-1} by b^{n-1}.

566. $3\frac{1}{2}p^{n}q^{-3}$ by $\frac{1}{8}p^{n}q^{4}$.

567. $a^{-(n+2)}$ by a^{-2n}.

568. x^{-a+1} by $x^{-(a+1)}$.

569. $x^{-(2p+3q)}$ by $x^{-(2p-3q)}$.

570. $(x+y)^{-(m+n)}$ by $(x+y)^{-(m-n)}$.

571. $a^{-1/n}$ by $a^{1/c}$. 572. $x^{1/n}$ by $x^{-1/p}$. 573. $y^{-m/n}$ by $y^{-1/q}$.

574. $a^{-1/x}$ by $a^{p/q}$. 575. $c^{-n/m}$ by $c^{-a/2m}$.

576. $d^{-2/n}$ by $d^{-a/n}$. 577. $f^{a/c}$ by $f^{-1/n}$.

578. $xy^{-p/q}$ by $x^{-\frac{1}{2}}y^{2p/q}$. 579. $2a^{3n/4}x$ by $1\frac{1}{4}a^{-n/2}x^{-\frac{1}{2}}$.

580. $p^{\frac{1}{2}}q^{-\frac{1}{2}}$ by $(p^2q^{-1})^{-\frac{1}{2}}$.

581. $(x^{\frac{7}{3}}y^{\frac{5}{3}} - x^{\frac{2}{3}}y^{\frac{2}{3}} + 3x^{\frac{5}{3}}y^{\frac{5}{3}})$ by $x^{\frac{2}{3}}y^{\frac{2}{3}}$.

582. $(a^{\frac{1}{2}}c^{\frac{3}{2}})^{\frac{1}{2}}$ by $(a^{-\frac{1}{2}}c)^{\frac{1}{2}}$. 583. $\sqrt[a]{x^{a+c}y^{2a-c}}$ by $(x^{1/a}y^{-1/a})^c$.

584. $\{(a-b)^{-3}\}^n$ by $\{(a+b)^n\}^3$.

585. $\dfrac{1}{x^{-\frac{1}{2}}}$ by $\dfrac{3y^{-\frac{3}{2}}}{x^{-\frac{3}{2}}}$.

586. $\left(\dfrac{m^{-2}n}{m^3n^{-4}}\right)^{-3}$ by $\left(\dfrac{mn^{-1}}{m^{-3}n^2}\right)^5$.

587. $\left(\dfrac{x^{\frac{3}{2}}y^{-\frac{1}{2}}}{x^{-\frac{1}{2}}y}\right) \div \left(\dfrac{x^{\frac{1}{2}}y^{-1}}{x^{-\frac{1}{2}}y^{\frac{1}{2}}}\right)^6$. 588. $\dfrac{a^{-m}x^{-n}}{\frac{1}{2}b}$ by $\dfrac{b^{-1}}{a^{-2m}x^{-2n}}$.

589. $\dfrac{x^{-5}y^{-n}}{z^{-5}w^{-4}}$ by $\dfrac{x^{-6}y^{-(n+2)}}{z^{-7}w^{-3}}$. 590. $\left(\dfrac{c^{p+q}}{c^q}\right)^p$ by $\left(\dfrac{c^q}{c^{p+q}}\right)^{q-p}$.

591. $a-1$ by $a^{\frac{1}{2}}-1$. 592. $x-1$ by $x^{\frac{1}{3}}-1$.

593. $a-b$ by $a^{\frac{1}{2}}+b^{\frac{1}{2}}$. 594. $x-y$ by $x^{\frac{1}{3}}+y^{\frac{1}{3}}$.

595. $a-x$ by $a^{\frac{1}{2}}-x^{\frac{1}{2}}$. 596. $p-q$ by $p^{\frac{1}{5}}-q^{\frac{1}{5}}$.

597. $a^{\frac{5}{4}}-c^{\frac{5}{4}}$ by $a^{\frac{1}{4}}-c^{\frac{1}{4}}$.

598. $(2^{n+4}-2\times 2^n)$ by $(4\times 2^{n+2})$.

599. $p^{3n/4}+q^{3n/4}$ by $p^{n/4}+q^{n/4}$. 600. $a^{5n}-x^{5n}$ by $a^{5n/2}-x^{5n/2}$.

601. $8x^{-c}-8x^c+5x^{3c}-3x^{-3c}$ by $5x^c-3x^{-c}$.

602. $1 - \sqrt{x} - \dfrac{2}{x^{-1}} + 2x^2$ by $1 - x^{\frac{1}{2}}$.

603. $a^{\frac{1}{2}} - 3a^{\frac{1}{3}}c^{-\frac{1}{6}} + 3a^{\frac{1}{6}}c^{-\frac{1}{3}} - c^{-\frac{1}{2}}$ by $a^{\frac{1}{3}} - c^{-\frac{1}{6}}$.

604. $a + b + c - 3a^{\frac{1}{3}}b^{\frac{1}{3}}c^{\frac{1}{3}}$ by $a^{\frac{1}{3}} + b^{\frac{1}{3}} + c^{\frac{1}{3}}$.

605. $\frac{2}{5}a^{\frac{8}{15}} + \frac{1}{2}a^{\frac{1}{2}} - \frac{2}{5}a^{\frac{7}{12}} - \frac{3}{5}a^{\frac{7}{10}} + a^{\frac{3}{4}} - \frac{3}{10}a^{\frac{3}{20}}$ by $a^{\frac{1}{4}} - \frac{2}{5}a^{\frac{1}{5}}$.

606. $a^2 - 5a^{4.1} + 10a^{6.2} - 10a^{8.3} + 5a^{10.4} - a^{12.5}$ by $a^{\frac{2}{5}} - a^{\frac{5}{2}}$.

607. $k^{-1} + k^{-\frac{1}{3}} - 1 + k^{\frac{1}{3}} + k$ by $k^{-\frac{1}{3}} + k^{\frac{1}{3}} + 1$.

608. $2a^{\frac{3}{2}}x^{-1} - 2a^{-\frac{3}{2}}x + 32a^{-2}x^3$ by $2 + 6a^{-\frac{3}{2}}x + 8a^{-\frac{4}{3}}x^2$.

609. $6m^{\frac{5}{4}} - 7m - 19m^{\frac{3}{4}} + 2m^{\frac{1}{2}} + 8m^{\frac{1}{4}}$ by $2m^{\frac{3}{4}} - 3m^{\frac{1}{2}} - 4m^{\frac{1}{4}}$.

610. $c^{-1} - 3c^{-\frac{1}{2}} + 3 - 3c^{\frac{1}{2}} + 2c$ by $c^{-\frac{1}{2}} - 2c^{-\frac{1}{4}} + c^{-\frac{1}{4}} - 2$.

611. $xy^{-\frac{1}{3}} - 10y^{\frac{1}{3}} + 9x^{-1}y$ by $x^{\frac{1}{2}} + 2y^{\frac{1}{3}} - 3x^{-\frac{1}{2}}y^{-\frac{1}{3}}$.

612. $p^{\frac{4}{3}} - 2q^{\frac{3}{2}} + p^{-\frac{4}{3}}q^3$ by $p^{-\frac{2}{3}}q^{\frac{3}{2}} - 2p^{-\frac{4}{3}}q^{\frac{3}{2}} + p^{-2}q^{\frac{3}{2}}$.

613. $x^3 - x^{\frac{4}{3}} + 2x^{\frac{1}{3}} - 2 - x^{-\frac{2}{3}} + x^{-1}$ by $x^{\frac{3}{2}} + x^{\frac{2}{3}} - x^{-\frac{1}{3}} - x^{-\frac{2}{3}}$.

614. $4\sqrt[3]{a^2} - 8a^{\frac{1}{3}} - 5 + \dfrac{10}{\sqrt[3]{a}} + 3a^{-\frac{2}{3}}$ by $2a^{\frac{5}{12}} - \sqrt[12]{a} - \dfrac{3}{\sqrt[4]{a}}$.

615. $m + m^{\frac{3}{4}}n^{-\frac{1}{4}} - m^{\frac{1}{4}}n^{-\frac{3}{4}} - n^{-1}$ by $m^{\frac{3}{8}} + m^{\frac{1}{2}}n^{-\frac{1}{8}} + m^{\frac{3}{8}}n^{-\frac{1}{4}} + m^{\frac{1}{4}}n^{-\frac{3}{8}} + m^{\frac{1}{8}}n^{-\frac{1}{2}} + n^{-\frac{3}{8}}$.

616. $\sqrt{a^{2n} - 2a^nb^n + b^{2n} + 2a^nc^p - 2b^nc^p + c^{2p}}$.

617. $\sqrt{4f^{2a} - 12f^ag^a - 16f^ah^a + 9g^{2a} + 16h^{2a} + 24g^ah^a}$.

618. $\sqrt{9a^{4x} + 24a^{2x}c^{3x} - 12a^{2x}b^x + 4b^{2x} + 16c^{6x} - 16b^xc^{3x}}$.

619. $\sqrt{x^{2m} + 2x^{m+n} - 2x^{m+1} + x^{2n} - 2x^{n+1} + x^2}$.

620. $\sqrt{a^{2m} + 2a^{2m-1} + 3a^{2m-2} + 2a^{2m-3} + a^{2m-4}}$.

621. $\sqrt{x^{2n+2} + 2x^{2n-1} - x^{2n} + x^{2n-2} - 2x^{2n+1}}$.

622. $\sqrt{4x^{2c} - 16x^{2c-2} - 12x^{2c+1} + 9x^{2c+2} + 16x^{2c-4} + 24x^{2c-1}}$.

623. $\sqrt{x^{2a} - 4x^a y^{-c} + z + 2x^a z^{\frac{1}{2}} + 4y^{-c} z^{\frac{1}{2}}}$.

624. $\sqrt{9x^{2n} + 12x^{3n/2} + 10x^n + 4x^{n/2} + 1}$.

625. $\sqrt{a^{2m}x^{2n} + 10a^{2m-2}x^{2n+1} - 6a^{m+1}x^{n+1} + 25a^{2m-4}x^{2n+2} - 30a^{m-1}x^{n+2} + 9a^2x^3}$.

626. $\sqrt{a^4 + 16d^{4z} + 9e^{2y} + 9c^{10} - 6a^2c^5 - 8a^2d^{2z} + 24c^5d^{2z} + 6a^2e^y - 18c^5e^y - 24d^{2z}e^y}$.

627. $\sqrt{x^{2a+2} - 2x^{2a} + 3^{2n} - 2x^{a-1}3^n + x^{2a-2} + 2x^{a+1}3^n}$.

628. $\sqrt{\frac{1}{4}a^{2n} + c - \frac{2}{3}a^n x^{-n} + a^n c^{\frac{1}{2}} - \frac{4}{3}x^{-n}c^{\frac{1}{2}} + \frac{4}{9}x^{-2n}}$.

629. $\sqrt{a^{2m}x^{2n} + 10ca^{2m-2}x^{2n+1} - 6a^{m+1}x^{n-1} + 25c^2a^{2m-4}x^{2n+2} - 30ca^{m-1}x^n + \frac{9a^2}{x^3}}$.

630. $\sqrt{\frac{9a^{2m-2}c^3}{4d^{6p}} - \frac{3a^{m+n-1}b^{2n-1}c}{d^{3p-3}} - \frac{2^3a^{m-1}b^x c}{d^{3p}} + a^{2m}b^{4m-2}d^6 + \frac{2^9}{3}a^n b^{x+2n-1}d^3 + \frac{2^{16}b^{2x}}{9}}$.

631. $\sqrt[3]{a^{3m} - 6a^{2m+1}x^n + 12a^{m+2}x^{2n} - 8a^3x^{3n}}$.

632. $\sqrt[3]{\frac{1}{8}a^{3c} - \frac{1}{2}a^{2c}x^n + \frac{2}{3}a^c x^{2n} - \frac{8}{27}x^{3n}}$.

633. $\sqrt[3]{x^{3n} + 3x^{n+4} + 3x^{-n} + 3x^{2n+2} + 3x^n + 6x^3 + x^6 + 3x^{2-2n} + 3x^{4-n} + x^{-3n}}$.

634. $\sqrt[3]{64c^{3a} - 144c^{3a-1} + 12c^{3a-2} + 117c^{3a-c} - 6c^{3a-4} - 36c^{3a-5} + 8c^{3a-6}}$.

635. $\sqrt[3]{(a+1)^{6n}x^3 - 6ca^p(a+1)^{4n}x^3 + 12c^2a^{2p}(a+1)^{2n} - 8c^3a^{3p}}$.

RADICALS.

Simplify the following:

1. $\sqrt{72}$.
2. $\sqrt{80}$.
3. $\sqrt{343}$.
4. $\sqrt{99}$.
5. $3\sqrt{98}$.
6. $\sqrt{2625}$.
7. $\sqrt{2880}$.
8. $\sqrt{127008}$.
9. $\frac{1}{8}\sqrt{176}$.
10. $.002\sqrt{500000}$.
11. $\sqrt[3]{16}$.
12. $\sqrt[3]{135}$.
13. $\sqrt[3]{320}$.
14. $\sqrt[3]{108}$.
15. $\sqrt[3]{-1029}$.
16. $\sqrt[3]{375}$.
17. $\frac{3}{4}\sqrt[3]{192}$.
18. $\sqrt[3]{4116}$.
19. $\sqrt[3]{1944}$.
20. $\sqrt[4]{36}$.
21. $\sqrt[4]{112}$.
22. $3\sqrt[4]{1250}$.
23. $\sqrt[4]{567}$.
24. $\sqrt[4]{7056}$.
25. $\sqrt[5]{1701}$.
26. $\sqrt{125 \times 135}$.
27. $\sqrt{125 \times 147}$.
28. $\sqrt[3]{196 \times 392}$.
29. $\sqrt[3]{40 \times 45 \times 48}$.
30. $\sqrt[4]{49}$.
31. $\sqrt[6]{8}$.
32. $\sqrt[6]{49}$.
33. $\sqrt[8]{16}$.
34. $\sqrt[9]{64}$.
35. $\sqrt[10]{9}$.
36. $\sqrt[12]{16}$.
37. $\sqrt[12]{100}$.
38. $\sqrt[15]{243}$.
39. $\sqrt[-1]{7}$.
40. $\sqrt[-2]{16}$.
41. $\sqrt[-4]{36}$.
42. $\sqrt[-5]{243}$.
43. $(-\sqrt{3})^2$.
44. $(-\sqrt[4]{8})^3$.
45. $(-\sqrt[3]{9})^4$.
46. $(-\sqrt[5]{47})^5$.
47. $(-\sqrt[3]{4})^6$.
48. $\sqrt{\frac{1}{5}}$.
49. $\sqrt{\frac{1}{8}}$.
50. $\sqrt{\frac{5}{9}}$.
51. $\sqrt{\frac{2}{3}}$.
52. $\sqrt{\frac{4}{5}}$.
53. $\sqrt{\frac{9}{8}}$.

54. $\sqrt{\frac{13}{20}}$.

55. $\sqrt{\frac{17}{32}}$.

56. $2\sqrt{\frac{1}{2}}$.

57. $6\sqrt{\frac{2}{3}}$.

58. $5\sqrt{\frac{3}{8}}$.

59. $7\sqrt{\frac{4}{7}}$.

60. $\sqrt[3]{\frac{8}{9}}$.

61. $\sqrt[3]{\frac{3}{2}}$.

62. $\sqrt[3]{\frac{1}{4}}$.

63. $\sqrt[3]{\frac{7}{9}}$.

64. $\sqrt[3]{\frac{8}{25}}$.

65. $\sqrt[3]{\frac{2}{5}}$.

66. $\sqrt[3]{\frac{325}{4}}$.

67. $\sqrt[3]{-10\frac{1}{8}}$.

68. $8\sqrt[3]{\frac{3}{4}}$.

69. $\sqrt[4]{\frac{3}{8}}$.

70. $\sqrt[4]{\frac{5}{9}}$.

71. $\sqrt[4]{\frac{5}{8}}$.

72. $\sqrt[4]{\frac{25}{49}}$.

73. $\sqrt[5]{\frac{1}{3}}$.

74. $\sqrt[5]{\frac{2}{3}}$.

75. $\sqrt[5]{\frac{2}{343}}$.

76. $\frac{\sqrt{1682}}{58}$.

77. $5\sqrt{7.728}$.

78. $\sqrt[-1]{\frac{1}{4}}$.

79. $\sqrt[-3]{\frac{1}{64}}$.

80. $\sqrt[-6]{\frac{64}{15625}}$.

81. $\left(\frac{512}{\sqrt[5]{512}}\right)^{-\frac{3}{5}}$.

82. $\left(\frac{343^{-2}}{81^{-9}}\right)^{\frac{1}{6}}$.

83. $\sqrt{x^3y}$.

84. $\sqrt{4ax^3}$.

85. $\sqrt{p^3q^4}$.

86. $\sqrt{16m^4n^3}$.

87. $2\sqrt{80a^4x}$.

88. $\sqrt{75x^3}$.

89. $\sqrt{384a^5n^2}$.

90. $20m\sqrt{27m^3n^4}$.

91. $\sqrt[3]{u^4v^7}$.

92. $\sqrt[3]{8x^7}$.

93. $\sqrt[3]{27y^6z}$.

94. $\sqrt[3]{222x^4y^3z^3}$.

95. $\sqrt[3]{x^{-6n}}$.

96. $\sqrt[5]{a^{10}c^{-5}x^{5n+1}}$.

97. $\sqrt[3]{1715x^{9m-n}y^{6n}}$.

98. $\sqrt[3]{7290m^{3a}n^{6c+2}}$.

99. $\sqrt[n]{c^{a+n}d^{2n}}$.

100. $\sqrt[a]{x^{3a}y^{2a+3}}$.

101. $\sqrt[2n]{4x^{2n}y^{4n}z^2}$.

102. $\sqrt[p]{a^{3p+2}c^{3p+3}}$.

103. $\sqrt[n-1]{x^{n^2-3n+2}y^{n^2-1}}$.

104. $\sqrt[3]{b^5-3b^4}$.

105. $\sqrt{18a^3c^4-27a^4c^3}$.

106. $\sqrt{pq^2-6pq+9p}$.

107. $\sqrt{5a^3+30a^2+45a}$.

108. $\sqrt{3x^3y-12x^2y^2+12xy^3}$.

109. $\sqrt{(e^2-f^2)(e+f)}$.

110. $\sqrt{(p-q)(p^3-q^3)}$.

111. $\sqrt{(x^2-9)(x^2+x-6)}$.

112. $\sqrt{(x^2+x-6)(x^2-3x+2)}$.

113. $\sqrt[3]{(4+x)^2(64+x^3)}$.

114. $\sqrt[q]{(b+x)^{2q}(b+x)^{q-1}}$.

115. $\sqrt{\frac{x^3}{4}}$.

116. $\sqrt{\frac{9m^2n}{36}}$.

117. $\sqrt{\frac{x^2y^3}{zw^2}}$.

118. $\sqrt{\frac{m^3p}{n^3}}$.

119. $\sqrt{\frac{7ab^2}{8c^3}}$.

120. $\sqrt{\frac{7a^3b^2c^2}{28c^3e^4}}$.

121. $\sqrt{\frac{81c}{64d}}$.

122. $\sqrt[3]{\frac{x}{y^3}}$.

123. $\sqrt[3]{\frac{k^3}{h}}$.

124. $\sqrt[4]{\frac{u^4}{z^3}}$.

125. $\sqrt[4]{\frac{k}{p^3q^4}}$.

126. $\sqrt{\frac{125b^2c^3}{18a^5}}$.

127. $\sqrt[3]{\frac{p^6q^{11}}{128m^5n^3}}$.

128. $\sqrt[3]{\frac{ax}{cy}}$.

129. $\frac{mn}{pq}\sqrt[3]{\frac{125p^6q^6z^3}{343m^6n^6w^6}}$.

130. $\sqrt[3]{\frac{(cy^3-c^2y^7)}{64k^{12}}}$.

131. $(x+y)\sqrt{\frac{x-y}{x+y}}$.

132. $\frac{a}{a^2-b^2}\sqrt{\left(\frac{a^3c-2a^2bc+ab^2c}{b^3}\right)}$.

133. $\sqrt{\frac{(a^2-c^2)^3}{4(a+c)}}$.

134. $(x+y)\sqrt{\frac{1}{x^2-y^2}}$.

135. $\frac{a-1}{2}\sqrt[3]{\frac{8(a+1)}{(a-1)^2}}$.

136. $\frac{x}{y}\sqrt[n]{\frac{y^{n+1}}{x^{n-1}}}$.

137. $m\sqrt[c]{\frac{n^2}{m^{c-2}}}$.

138. $\frac{a}{c^p}\sqrt{\frac{c^{2p+1}}{a^5}}$.

139. $\frac{3a^2}{c^n}\sqrt{\frac{c^{2n+1}}{a}}$.

140. $\frac{p^2}{q^2}\sqrt[n]{\frac{q^{n+1}}{p^{n+1}}}$.

141. $\sqrt[4]{25a^4x^6}$.

142. $\sqrt[4]{81a^2b^4}$.

143. $\sqrt[6]{27a^3c^9}$.

144. $\sqrt[6]{-64x^3}$.

145. $\sqrt[9]{27m^3n^6}$.

146. $\sqrt[10]{729x^{15}y^5}$.

147. $\frac{ab}{ac}\sqrt[4]{\frac{121c^4d^2e^2}{256a^2b^4x^4}}$.

148. $\sqrt[4]{(a-2a^{\frac{1}{2}}b^{\frac{1}{2}}+b)}$.

149. $\sqrt[3n+3]{8a^3c^{3n}}$.

150. $\sqrt[-1]{x}$.

151. $\sqrt[-2]{k}$.

152. $\sqrt[-3]{a^2}$.

153. $\frac{\sqrt{x^3}}{\sqrt[-2]{x^5}}$.

154. $\frac{\sqrt[-3]{x^3}}{\sqrt[-3]{x^5}}$.

155. $\frac{\sqrt{a^5}}{\sqrt[-5]{a}}$.

156. $\frac{\sqrt[-3]{m^3}}{\sqrt[-5]{m^5}}$.

157. $\sqrt[-4]{\frac{p^6}{q^{10}}}$.

158. $\sqrt[-6]{\frac{a^8}{c^{12}}}$.

Express as an entire surd :

159. $3\sqrt{2}$.

160. $5\sqrt{3}$.

161. $2\sqrt[3]{4}$.

162. $4\sqrt[4]{5}$.

163. $2\sqrt[5]{3}$.

164. $6a\sqrt{2}$.

165. $3x\sqrt{5x}$.

166. $4c\sqrt{3b}$.

167. $4y\sqrt[3]{2y^2}$.

168. $\frac{1}{8}\sqrt{3}$.

169. $\frac{4}{8}\sqrt[3]{\frac{9}{16}}$.

170. $\frac{3}{4}\sqrt{\frac{2}{8}}$.

171. $\frac{4}{11}\sqrt{\frac{77}{8}}$.

172. $\frac{3}{2x}\sqrt[3]{\frac{4x^3}{9}}$.

173. $2pq\sqrt[4]{\frac{1}{4p^2q^2}}$.

174. $-3\sqrt{x}$.

175. $-\frac{2}{8}\sqrt[3]{z}$.

176. $-\frac{3}{5}\sqrt[3]{\frac{5}{6}}$.

177. $-\frac{2\frac{1}{2}}{3\frac{1}{3}}\sqrt[3]{\frac{8x}{9y}}$.

178. $.01\sqrt{1000}$.

179. $.25\sqrt[3]{128}$.

180. $.33\frac{1}{3}\sqrt{6x}$.

181. $.625\sqrt[-2]{.125}$.

182. $\sqrt{4}\sqrt[4]{2}$.

183. $\sqrt[3]{2}\sqrt[3]{3}$.

184. $\sqrt{1\frac{7}{9}}\sqrt[3]{\frac{8}{3}\frac{3}{2}}$.

185. $\sqrt[3]{3\frac{3}{8}}\sqrt{\frac{2}{5}}$.

186. $a\sqrt[n]{a}$.

187. $x^{c+1}\sqrt{x^{c-2}}$.

188. $x^2y\sqrt[n-1]{xy}$.

189. $a^2c^2\sqrt[3]{a^4c^2}$. 190. $\frac{1}{2}\sqrt[3]{a}$. 191. $-\frac{x}{y}\sqrt[3]{\frac{x}{y}}$.

192. $-\frac{x-ia}{2}\sqrt{\frac{x-ia}{2}}$. 193. $\frac{1}{2}\left(\frac{m^2}{n}\right)^{\frac{3}{4}}$.

194. $\frac{c+d}{c-d}\sqrt{\frac{c-d}{c+d}}$. 195. $\frac{1}{a-x}\sqrt[3]{a^3-x^3}$.

196. $\frac{1}{x-3}\sqrt{x^2+x-12}$. 197. $(m-n)\sqrt{\frac{m+n}{m^3-n^3}}$.

198. $3x\sqrt[6]{9x^{-2}}$.

Reduce to radicals of the same degree:

199. $\sqrt[3]{3}$, $\sqrt[4]{3}$. 200. $\sqrt[3]{4}$, $\sqrt[4]{6}$. 201. 3, $\sqrt[4]{6}$.

202. $\sqrt[3]{4}$, $\sqrt[4]{8}$. 203. $\sqrt[3]{8}$, $\sqrt{2}$. 204. $\sqrt[12]{15}$, $\sqrt[12]{10}$.

205. $\sqrt[4]{7}$, $\sqrt[5]{5}$, $\sqrt[10]{120}$. 206. $\sqrt{3x}$, $\sqrt[3]{4y}$. 207. $\sqrt{\frac{1}{2}}$, $\sqrt[3]{\frac{1}{4}}$.

208. $\sqrt{\frac{2}{3}}$, $\sqrt[3]{\frac{3}{2}}$. 209. $\sqrt{\frac{1}{2}}$, $\sqrt[3]{\frac{1}{3}}$, $\sqrt[4]{\frac{1}{4}}$. 210. $\sqrt{\frac{3}{5}}$, $\sqrt[3]{\frac{4}{7}}$.

211. $\sqrt{\frac{a}{x}}$, $\sqrt[4]{\frac{c}{z}}$. 212. $\sqrt[5]{\frac{5}{6}}$, $\sqrt[5]{\frac{11}{12}}$, $\sqrt[5]{\frac{13}{14}}$.

213. $2a\sqrt{c}$, $5x\sqrt[3]{2y}$. 214. $\sqrt[3]{a}$, $\sqrt[3]{5c}$, $\sqrt[3]{3x}$.

215. $a^{\frac{1}{3}}$, $c^{\frac{2}{3}}$, $x^{\frac{5}{3}}$. 216. $\sqrt[3]{2a^2}$, $\sqrt[3]{3x}$, $\sqrt{xy}$.

217. $\sqrt{x}$, $\sqrt{y}$, $\sqrt[3]{y}$, $\sqrt[3]{z^2}$, $\sqrt[6]{z^{3a/2}}$. 218. $\sqrt[3]{49}$, $\sqrt[3]{27}$, $\sqrt[3]{512}$.

219. $\sqrt[4]{(a-c^2)}$, $\sqrt[6]{(a^2-c)}$.

220. $\sqrt[4]{5z}$, $\sqrt[4]{(a+1)}$, $\sqrt[6]{(a-1)}$.

221. $\sqrt{p+q}$, $\sqrt[3]{a^3c}$, $\sqrt[6]{(x-1)^2}$.

222. $a^{\frac{4}{3}}$, $\sqrt[n]{2x^3}$. 223. $\sqrt[m]{a^2}$, $\sqrt[n]{c^4}$. 224. $\sqrt[2n]{x^n}$, $\sqrt[4n]{y^4}$, $\sqrt[6n]{z^n}$.

225. $\sqrt[n+1]{a^2x}$, $\sqrt[n-1]{ax^2}$. 226. $\sqrt[m]{a^n}$, $\sqrt[n]{a^m}$. 227. $\sqrt[m]{x^n}$, $\sqrt[n]{y^n}$, $\sqrt[l]{z^m}$.

228. $(m+n)^{1/n}$, $(n+p)^{n/s}$, $(p+q)^{s/2n}$.

Addition and Subtraction of Radicals.

229. $2\sqrt{3} + 3\sqrt{3}$.

230. $5\sqrt{2} - 3\sqrt{2}$.

231. $4\sqrt{5} + 2\sqrt{5} + 7\sqrt{5}$.

232. $4\sqrt{7} + 3\sqrt{7} - 5\sqrt{7}$.

233. $5\sqrt[3]{2} - 3\sqrt[3]{2} + 4\sqrt[3]{2}$.

234. $4\sqrt{2} - \sqrt{18}$.

235. $7\sqrt[3]{4} - \sqrt[3]{32}$.

236. $\sqrt{24} - \sqrt{150} + \sqrt{54}$.

237. $3\sqrt[5]{2} + 4\sqrt[5]{2} - \sqrt[5]{64}$.

238. $\sqrt[3]{500} + \sqrt[3]{108}$.

239. $-\sqrt{405} + \sqrt{605}$.

240. $3\sqrt{2} + \sqrt{8} + 3\sqrt{50}$.

241. $2\sqrt[3]{4} - 5\sqrt[3]{32} + 3\sqrt[3]{108}$.

242. $\sqrt[3]{40} + 2\sqrt[3]{625} - 3\sqrt[3]{320}$.

243. $3\sqrt{2} - 2\sqrt[3]{3}$.

244. $5\sqrt{2} + 4\sqrt{27}$.

245. $\sqrt[3]{729} + \sqrt[6]{27} + 2\sqrt{48}$.

246. $4\sqrt{24} + 2\sqrt{54} - \sqrt{6} + 3\sqrt{96} - 5\sqrt{150}$.

247. $\sqrt[3]{128} + \sqrt[3]{686} + \sqrt[3]{16} - 4\sqrt[3]{250}$.

248. $\sqrt{\frac{3}{4}} + \sqrt{\frac{1}{3}}$.

249. $3\sqrt{\frac{2}{3}} + 7\sqrt{\frac{27}{50}}$.

250. $2\sqrt{3} + 3\sqrt{1\frac{1}{3}} - \sqrt{5\frac{1}{3}}$.

251. $\frac{2}{3}\sqrt[3]{\frac{3}{2}} + \frac{1}{4}\sqrt[3]{\frac{256}{9}}$.

252. $8\sqrt{\frac{3}{4}} + \sqrt{60} - \frac{1}{5}\sqrt{15} + \sqrt{\frac{3}{5}}$.

253. $\sqrt[3]{\frac{9}{2}} + \sqrt[3]{\frac{4}{3}} + \sqrt[3]{\frac{1}{6}}$.

254. $\sqrt{\frac{3}{2}} - \sqrt{\frac{1}{6}} - \sqrt{\frac{32}{3}} + \sqrt{\frac{1}{27}}$.

255. $\frac{1}{\sqrt[3]{2}} - \frac{1}{\sqrt[3]{16}}$.

256. $\frac{\sqrt[3]{72}}{18} - \frac{1}{3}\sqrt[3]{\frac{1}{3}} + 6\sqrt[3]{21\frac{1}{8}}$.

257. $2\sqrt{\frac{5}{8}} + \sqrt{60} - \sqrt{15} + \sqrt{\frac{3}{5}} + \sqrt{\frac{4}{15}}$.

258. $\sqrt[3]{\frac{4}{27}} - \sqrt[3]{\frac{10}{2}} - 3\sqrt[3]{256} + \sqrt[3]{625} + \sqrt[3]{\frac{32}{27}} - \sqrt[3]{\frac{125}{54}}$.

259. $\frac{3}{2}\sqrt[3]{\frac{2}{49}} + .8\sqrt{\frac{8}{3}} - \frac{1}{15}\sqrt{96} + 1.5\sqrt[3]{\frac{7}{4}} - \frac{11}{28}\sqrt[3]{1750} + 8\sqrt{\frac{3}{2}}$.

260. $\left(\frac{1}{20}\right)^{-\frac{1}{2}} + \frac{3}{\sqrt{5}} + 4\sqrt{125} + 7\left(\frac{9}{5}\right)^{\frac{1}{2}} + 3\left(\frac{80}{9}\right)^{-\frac{1}{2}} + \sqrt[4]{\frac{25}{16}}$.

261. $\sqrt{80x} + \sqrt{125x}$.

262. $\sqrt{98x} - \sqrt{50x}$.

263. $\sqrt[3]{8c^3} + \sqrt[3]{125c^3}$.

264. $3\sqrt{5x^2y} - x\sqrt{5y}$.

265. $\sqrt[4]{32a} + 5\sqrt[4]{2a}$.

266. $4p^2\sqrt[3]{pq^3} - 4pq\sqrt{p^3q}$.

267. $5x\sqrt{3y^3} - y\sqrt{48x^3}$.

268. $\sqrt{a^5x} + 2\sqrt{a^3x^3} + \sqrt{ax^5}$.

269. $q\sqrt[3]{8p^6q} - p\sqrt[3]{p^3q^4}$.

270. $\sqrt[4]{32x} - 2\sqrt[3]{40x}$.

271. $\sqrt[3]{x^3} + \frac{1}{2}\sqrt[3]{x^3} - 3\sqrt[3]{27x^3}$.

272. $\frac{1}{2}\sqrt{a^3cx} + \frac{1}{3}\sqrt{4cxz^4}$.

273. $\sqrt{2x^4} + 2\sqrt{2x^3y^3} + \sqrt{2y^4}$.

274. $\sqrt[3]{a^7c} + \sqrt[3]{8a^4c^4} + \sqrt[3]{ac^7}$.

275. $\sqrt[3]{16a + 24} + \sqrt[3]{54a + 81}$.

276. $\sqrt{\frac{2}{x^3}} - \sqrt{\frac{2}{y^3}} + \sqrt{\frac{2}{z^3}}$.

277. $2\sqrt{2a^3} - \sqrt{8a} - \sqrt{2a^3 - 4a^2 + 2a}$.

278. $\sqrt[4]{256z^4} - \sqrt[4]{16z} + \sqrt[4]{81z^5}$.

279. $\sqrt[5]{32x} + \sqrt[5]{c^5x} - \sqrt[5]{x^6}$.

280. $\sqrt{\frac{1}{x}} - \sqrt[4]{\frac{1}{x^3}} - \sqrt[6]{\frac{1}{x^3}}$.

281. $\sqrt[3]{\frac{27a^4x^4}{2c}} - \sqrt[3]{\frac{ax^4}{54c}}$.

282. $(a - x)^2\sqrt{xy} + (a + x)^2\sqrt{xy}$.

283. $(a + x)^2\sqrt{xy} - (a - x)^2\sqrt{xy}$.

284. $\sqrt{2ax^2 - 4ax + 2a} - \sqrt{2ax^2 + 4ax + 2a}$.

285. $(8x^3 - 24x^2 + 18x)^{\frac{1}{2}} - (2x^3 - 12x^2 + 18x)^{\frac{1}{2}}$.

286. $\sqrt{a^3 - a^2c} - \sqrt{ac^2 - c^3} - \sqrt{(a + c)(a^2 - c^2)}$.

287. $2\sqrt{(a^2 + 60an + 75n^2)} - \sqrt{(48a^2 - 72an + 27n^2)}$.

288. $c\sqrt[n]{bc^n} - y\sqrt[n]{by^n}$.

289. $a\sqrt{1 + \left(\frac{b}{a}\right)^{\frac{2}{3}}} + b\sqrt{1 + \left(\frac{a}{b}\right)^{\frac{2}{3}}}$.

290. $\frac{a - c}{a + c} + \sqrt{\frac{a + c}{a - c}} + \sqrt{9(a^2 - c^2)(a + c)^{-2}}$.

291. $\sqrt{\frac{mn}{(m + n)^{-a}}} + \sqrt{\frac{(m - n)^a}{(mn)^{-1}}}$.

292. $\sqrt[m]{2^m a^{mp+3} b^{mn+5}} + \sqrt[m]{3^m a^{3m-mn+3} b^{m+5}} - \sqrt[m]{a^3 b^5 c^{2m}}$.

Multiplication of Radicals.

293. $\sqrt{3} \times \sqrt{3}$.

294. $\sqrt{5} \times \sqrt{20}$.

295. $\sqrt{3} \times \sqrt{40}$.

296. $\sqrt{27} \times \sqrt{3}$.

297. $\sqrt{12} \times \sqrt{27}$.

298. $\sqrt{3} \times \sqrt{18}$.

299. $2\sqrt{3} \times 3\sqrt{8}$.

300. $3\sqrt{12} \times 5\sqrt{24}$.

301. $\sqrt[3]{3} \times 2\sqrt[3]{9}$.

302. $4\sqrt[3]{9} \times \sqrt[3]{15}$.

303. $\sqrt[3]{18} \times 2\sqrt[3]{12}$.

304. $\sqrt[4]{9} \times \sqrt[4]{135}$.

305. $2\sqrt[4]{27} \times 5\sqrt[4]{3}$.

306. $\sqrt[5]{54} \times \sqrt[5]{9}$.

307. $-2\sqrt[5]{8} \times 3\sqrt[5]{4}$.

308. $\sqrt[3]{7} \times \sqrt[3]{-49}$.

309. $\sqrt[5]{-8} \times \sqrt[5]{4}$.

310. $\sqrt[4]{9} \times \sqrt[4]{81} \times \sqrt[4]{729}$.

311. $\sqrt[3]{9} \times \sqrt[3]{-9^4} \times \sqrt[3]{81^2}$.

312. $\sqrt[5]{7^3} \times \sqrt[5]{-112} \times \sqrt[5]{14}$.

313. $\sqrt[5]{125} \times \sqrt[5]{5^7} \times \sqrt[5]{5^6}$.

314. $\sqrt[4]{6} \times \sqrt[4]{12} \times \sqrt[4]{18}$.

315. $\sqrt[3]{16} \times \sqrt[3]{9} \times \sqrt[3]{6}$.

316. $5\sqrt{12} \times 3\sqrt{8} \times 6\sqrt{2} \times \sqrt{3}$.

317. $2\sqrt[5]{2} \times \sqrt[5]{12} \times \sqrt[5]{16} \times \sqrt[5]{96}$.

318. $\sqrt{2\frac{1}{2}} \times \sqrt{3\frac{5}{8}}$.

319. $\frac{2}{3}\sqrt[3]{18} \times \frac{3}{4}\sqrt[3]{3}$.

320. $\frac{1}{2}\sqrt[4]{8} \times \frac{1}{3}\sqrt[4]{160}$.

321. $\sqrt{\frac{5}{7}} \times \sqrt{1\frac{7}{125}}$.

322. $2\sqrt[3]{2\frac{5}{16}} \times \frac{1}{5}\sqrt[3]{\frac{2}{5}}$.

323. $\frac{3\sqrt{11}}{2\sqrt{98}} \times \frac{7}{5}\sqrt{22}$.

324. $12\sqrt[3]{\frac{5}{6}} \times 3\sqrt[3]{\frac{1}{4}} \times 4\sqrt[3]{\frac{2}{5}}$.

325. $\sqrt{2} \times \sqrt[3]{3}$.

326. $\sqrt[3]{4} \times \sqrt{8}$.

327. $5\sqrt[3]{2} \times 2\sqrt{5}$.

328. $\sqrt{5} \times \sqrt[5]{125}$.

329. $4\sqrt[3]{\frac{1}{16}} \times 3\sqrt{8}$.

330. $2\sqrt{1\frac{2}{7}} \times \frac{1}{2}\sqrt[4]{\frac{2}{9}}$

331. $\sqrt{\frac{2}{5}} \times \sqrt[3]{\frac{3}{8}}$.

332. $3\sqrt[3]{\frac{2}{3}} \times 6\sqrt[5]{\frac{1}{2}}$.

333. $\frac{3}{2}\sqrt{\frac{8}{9}} \times \frac{5}{2}\sqrt{\frac{7}{9}} \times \frac{2}{5}\sqrt{\frac{3}{7}}$.

334. $\sqrt{\frac{1}{2}} \times \sqrt[3]{\frac{1}{3}} \times \sqrt[4]{\frac{1}{4}}$.

335. $\sqrt{2} \times \sqrt[3]{4} \times \sqrt[4]{3}$.

336. $\sqrt[5]{54} \times 3\sqrt{6} \times 5\sqrt[3]{2}$.

337. $\sqrt[3]{12} \times \sqrt[4]{216} \times \sqrt[6]{96}$.

338. $3\sqrt[3]{2} \times 2\sqrt{3} \times \sqrt[6]{\frac{1}{2}}$.

339. $\sqrt{3} \times \sqrt[3]{2} \times \sqrt[6]{\frac{1}{6}}$.

340. $3\sqrt{8} \times 2\sqrt[3]{6} \times 3\sqrt[4]{54}$.

341. $4\sqrt[6]{24} \times 3\sqrt[4]{18} \times 2\sqrt{24}$.

342. $\sqrt[3]{12} \times \sqrt[6]{\frac{1}{9}} \times \sqrt[9]{\frac{1}{8}}$.

343. $\sqrt{2} \times \sqrt[3]{3} \times \sqrt[4]{5}$.

344. $2\sqrt[3]{\frac{1}{8}} \times 3\sqrt{\frac{1}{2}} \times \sqrt[4]{\frac{2}{3}}$.

345. $12\sqrt[4]{14} \times \sqrt{\frac{1.5}{7}} \times \sqrt[8]{\frac{49}{800}}$.

346. $\sqrt{10} \times \sqrt[3]{100} \times \sqrt[4]{500}$.

347. $(\sqrt{18} + 2\sqrt{72} - 3\sqrt{8} - \frac{1}{2}\sqrt{128}) \times \sqrt{2}$.

348. $(\sqrt[3]{32} + 3\sqrt[3]{4} - \frac{1}{2}\sqrt[3]{864}) \times \sqrt[3]{2}$.

349. $(9\sqrt{162} + 3\sqrt{50} + 5\sqrt{2} - 3\sqrt{32}) \times \sqrt{2}$.

350. $(3\sqrt[3]{24} + 4\sqrt[3]{81} - 5\sqrt[3]{375} + \frac{1}{2}\sqrt[3]{192}) \times \sqrt[3]{2}$.

351. $(\sqrt[3]{24} + 5\sqrt[3]{375} - \sqrt[3]{81} - 2\sqrt[3]{192}) \times 2\sqrt[3]{9}$.

352. $(\sqrt{48} - 2\sqrt{45} + \sqrt{75} - \sqrt{80} - \sqrt{27}) \times 2\sqrt{5}$.

353. $(\sqrt[3]{4} - \sqrt[3]{\frac{1}{10}} + \sqrt[3]{\frac{1}{2}}) \times \sqrt[3]{4}$.

354. $(\sqrt{5} + \sqrt{\frac{5}{4}} - \sqrt{\frac{1}{5}}) \times \sqrt{3}$.

355. $(3 + 3\sqrt{6} - 5\sqrt{8}) \times \sqrt{6}$.

356. $(\sqrt[3]{9} - 2\sqrt[3]{45} + 5\sqrt[3]{54}) \times \sqrt[3]{3}$.

357. $(4 + \sqrt{2})(4 - \sqrt{2})$.

358. $(\sqrt{7} + \sqrt{6})(\sqrt{7} - \sqrt{6})$.

359. $(2\sqrt{3} + 5)(2\sqrt{3} - 5)$.

360. $(5\sqrt{3} - 4\sqrt{3})(5\sqrt{3} + 4\sqrt{3})$.

361. $(\sqrt{3} + 2)(\sqrt{3} + 2)$.

362. $(\sqrt{5} - 3)(\sqrt{5} - 3)$.

363. $(2\sqrt{3} + 4)(2\sqrt{3} + 4)$.

364. $(3\sqrt{5} - \sqrt{2})(3\sqrt{5} - \sqrt{2})$.

365. $(2\sqrt{6} + 3\sqrt{2})(2\sqrt{6} + 3\sqrt{2})$.

366. $(2 + \sqrt{2})(3 - \sqrt{2})$.

367. $(5 + \sqrt{3})(1 - 2\sqrt{3})$.

368. $(5 - 2\sqrt{3})(4 + 3\sqrt{3})$.

369. $(7\sqrt{2} - 3\sqrt{3})(4\sqrt{2} + \sqrt{3})$.

370. $\left(\sqrt{6 + \sqrt{11}}\right)\left(\sqrt{6 - \sqrt{11}}\right)$.

371. $(\sqrt{3+2\sqrt{2}})(\sqrt{3-2\sqrt{2}})$. 372. $(\sqrt{3-\sqrt{2}})(\sqrt{3+\sqrt{2}})$.

373. $(\sqrt{9+\sqrt{17}})(\sqrt{9-\sqrt{17}})$.

374. $(\sqrt{8-\sqrt{15}})(\sqrt{8+\sqrt{15}})$.

375. $(\sqrt[3]{9+\sqrt{17}})(\sqrt[3]{9-\sqrt{17}})$.

376. $(\sqrt[3]{12+\sqrt{19}})(\sqrt[3]{12-\sqrt{19}})$.

377. $(11\sqrt{3}+2\sqrt{10})(\sqrt{6}-\sqrt{5})$.

378. $(7-\sqrt{3})(\sqrt{2}+\sqrt{5})$.

379. $(3\sqrt{45}-7\sqrt{5})(\sqrt{\frac{8}{5}}+2\sqrt{\frac{42}{5}})$.

380. $(5\sqrt[3]{9}+3\sqrt[3]{25})(\sqrt[3]{3}-\sqrt[3]{5})$.

381. $(\sqrt[4]{27}-\sqrt[4]{2})(2\sqrt[4]{3}+3\sqrt[4]{8})$.

382. $(\sqrt[3]{3}+\sqrt[3]{4})(\sqrt[3]{2}+\sqrt[3]{9})$.

383. $(2\sqrt[4]{2}-\sqrt[4]{12})(3\sqrt[4]{4}-\sqrt[4]{3})$.

384. $(\sqrt[4]{3}+2\sqrt[4]{2})(2\sqrt[4]{3}+\sqrt[4]{2})$. 385. $(\sqrt{3}+\sqrt{2})(\sqrt{\frac{1}{2}}+\sqrt{\frac{1}{3}})$.

386. $(2\sqrt{3}+\frac{2}{5}\sqrt{\frac{3}{5}})(3\sqrt{\frac{5}{2}}-4\sqrt{3})$.

387. $(1-\sqrt{2}+\sqrt{3})(\sqrt{6}-\sqrt{2})$.

388. $(\sqrt{7}+\sqrt[4]{21}+\sqrt{3})(\sqrt[4]{7}-\sqrt[4]{3})$.

389. $(\sqrt[3]{2-\sqrt{3}})(\sqrt{2+\sqrt{3}})$.

390. $(\sqrt{3-2\sqrt{2}})(\sqrt[4]{17+12\sqrt{2}})$.

391. $(\sqrt{2-\sqrt{2}})(\sqrt{5+\sqrt{7}})(\sqrt{2+\sqrt{2}})(\sqrt{5-\sqrt{7}})$.

392. $\sqrt[3]{2-\sqrt{(2+\sqrt{3})}}\times\sqrt[3]{2-\sqrt{(2+\sqrt{3})}}\times\sqrt[3]{(2+\sqrt{3})}$.

393. $(3\sqrt{2}+2\sqrt{3})(2\sqrt{3}-3\sqrt{2})(3\sqrt{3}+2\sqrt{2})$.

394. $(3\sqrt{5}+7\sqrt{2}-2\sqrt{3})(4\sqrt{5}+\sqrt{3})$.

395. $(\sqrt{\frac{5}{16}}-\frac{2}{5}\sqrt{2}+\sqrt{10})(4\sqrt{5}-\sqrt{10})$.

396. $(2\sqrt{\frac{2}{3}} + 3\sqrt{\frac{1}{2}} + 1)(3\sqrt{\frac{1}{3}} - 2\sqrt{\frac{1}{2}})$.

397. $\left(\frac{\sqrt{3}}{2} - \frac{\sqrt{10}}{3} + \frac{2}{\sqrt{3}}\right)\left(\sqrt{6} - \frac{\sqrt{5}}{4}\right)$.

398. $(\sqrt{2} - \sqrt{3} + \sqrt{5})(\sqrt{2} + \sqrt{3} + \sqrt{5})$.

399. $(6\sqrt{6} - 2\sqrt{2} - 8\sqrt{3})(3\sqrt{6} - \sqrt{2} + 4\sqrt{3})$.

400. $(5\sqrt{3} + 3\sqrt{5} + 4\sqrt{7})(5\sqrt{3} + 3\sqrt{5} - 4\sqrt{7})$.

401. $(2\sqrt{2} + 5\sqrt{15} - \sqrt{3})(2\sqrt{2} - 5\sqrt{15} + \sqrt{3})$.

402. $(3\sqrt[3]{875} - 7\sqrt[3]{56} + 2\sqrt[3]{189})(5\sqrt[3]{81} - 7\sqrt[3]{192} + 4\sqrt[3]{648})$.

403. $(4\sqrt[4]{32} + 32\sqrt[4]{2} - 2\sqrt[4]{162})(3\sqrt[4]{162} + 7\sqrt[4]{32} + \sqrt[4]{1250})$.

404. $2\sqrt{\frac{1}{2}} - 3\sqrt{3} + 4\sqrt{8})(3\sqrt{8} + \sqrt{18} - \sqrt{3})$.

405. $.4\sqrt[3]{25} + .2\sqrt[3]{200} - \frac{1}{2}\sqrt[3]{75})(2\sqrt[3]{135} - 5\sqrt[3]{5} - 10\sqrt[3]{15})$.

406. $(2\sqrt{\frac{2}{3}} - 8\sqrt{\frac{3}{8}} + 3\sqrt{\frac{3}{2}})(3\sqrt{\frac{2}{3}} - 12 - \sqrt{6})$.

407. $(6\sqrt{\frac{3}{2}} - 9\sqrt{\frac{5}{8}} + 10\sqrt{\frac{2}{5}})(2\sqrt{\frac{3}{2}} - 3\sqrt{\frac{5}{8}} - 5\sqrt{\frac{2}{5}})$.

408. $(5 + \sqrt[3]{4})(\sqrt{3} + \sqrt{2})$.

409. $(2\sqrt{3} + \sqrt[3]{2})(2\sqrt{3} - \sqrt[3]{4})$.

410. $(\sqrt[6]{2} - 1 + \frac{1}{2}\sqrt[6]{32} - \frac{1}{2}\sqrt[3]{4} + \frac{1}{2}\sqrt{2} - \frac{1}{2}\sqrt[3]{2})(\sqrt[6]{32} + \sqrt[3]{4})$.

411. $\sqrt{a} \times \sqrt{b}$.

412. $2\sqrt{x} \times 3\sqrt{y}$.

413. $a\sqrt{m} \times c\sqrt{n}$.

414. $\sqrt{\frac{a}{x}} \times \sqrt{\frac{x}{a}}$.

415. $3\sqrt{ax} \times \sqrt{ay}$.

416. $4\sqrt{5ac} \times 3\sqrt{20a}$.

417. $\sqrt[3]{16x^3y} \times \sqrt[3]{4xy^2}$.

418. $r^2s\sqrt{rs^2} \times rs^2\sqrt{r^2s}$.

419. $\sqrt{\frac{3pq}{2x}} \times q\sqrt{\frac{pq}{x}}$.

420. $4\sqrt{\frac{cm^5}{p^3}} \times 2\sqrt{\frac{c^3p^5}{m^3}}$.

421. $2\sqrt{ax} \times 3\sqrt[3]{ax}$.

422. $3\sqrt{2ax} \times 3\sqrt[3]{xy}$.

423. $\sqrt{28a} \times \sqrt[4]{42a}$.

424. $\sqrt[3]{12k^2} \times \sqrt{8k^3}$.

425. $\sqrt[5]{9b^2c^4} \times \sqrt{6bc^2}$.

426. $\sqrt[3]{4x} \times \sqrt[4]{2x}$.

427. $\sqrt[7]{2ax} \times \sqrt[5]{2ax}$.

428. $\sqrt{cx^2} \times \sqrt[3]{3c^3x} \times \sqrt[6]{2c^4x^3}$.

429. $x\sqrt[6]{n} \times y\sqrt[3]{n^2} \times z\sqrt{n^3}$.

430. $\sqrt[3]{\frac{x^5}{n^2}} \times \sqrt[5]{\frac{n^3}{x^4}}$.

431. $\sqrt{\frac{p}{q}} \times \sqrt[4]{\frac{p^3}{t}}$.

432. $\sqrt[3]{6a^{\frac{1}{2}}bc^{-1}} \times \sqrt[3]{3^{-1}a^{-\frac{1}{2}}bc^2}$.

433. $\sqrt[m]{x} \times \sqrt[n]{y}$.

434. $a\sqrt[m]{x} \times b\sqrt[n]{y} \times c\sqrt[p]{z}$.

435. $(\sqrt{x+z})(\sqrt[3]{x+z})$.

436. $2\sqrt{f} \times 5c\sqrt[n]{(x+y)}$.

437. $(a+2\sqrt{c})(a-3\sqrt{c})$.

438. $(\sqrt{x}+\sqrt{y})(x\sqrt{y}-y\sqrt{x})$.

439. $\sqrt[5]{\frac{m+n}{m-n}} \times 6\sqrt[3]{m-n}$.

440. $\sqrt{6(a^2-1)} \times \sqrt{\frac{3(a-1)}{2(a+1)}}$.

441. $(7m\sqrt{a^2-c^2})\left(6n\sqrt[3]{\frac{1}{a^2-c^2}}\right)$.

442. $\left(a+\frac{b}{2}-\sqrt{\frac{b^2}{4}-c}\right)\left(a+\frac{b}{2}+\sqrt{\frac{b^2}{4}-c}\right)$.

443. $\left(\sqrt{a-\sqrt{13}}\right)\left(\sqrt{a+\sqrt{13}}\right)$.

444. $(xy+\sqrt{ab})(4-\sqrt{ab})$.

445. $(a-\sqrt{ac}+c)(\sqrt{a}+\sqrt{c})$.

446. $(x^2-x\sqrt{2}+1)(x^2+x\sqrt{2}+1)$.

447. $(a+\sqrt{b}+\sqrt{c})(\sqrt{a}-\sqrt{b}+\sqrt{c})$.

448. $(\sqrt{a}+c\sqrt[3]{x})(\sqrt{a}-c\sqrt[3]{x})$.

449. $(a^2+1)(a^2-a\sqrt{3}+1)(a^2+a\sqrt{3}+1)$.

450. $(\sqrt{x+y}+\sqrt{x-y})(\sqrt{x+y}-\sqrt{x-y})$.

451. $(\sqrt{x}+\sqrt{x-y}+\sqrt{y})(\sqrt{x}-\sqrt{x-y}+\sqrt{y})$

452. $(4\sqrt{3x-7}+5\sqrt{6x+2})(4\sqrt{3x-7}-5\sqrt{6x+2})$.

453. $(\sqrt{x-y})(\sqrt[3]{x+y})(\sqrt[4]{x+y})$.

454. $\left(\frac{ax}{bc}\sqrt{ax}\right)\left(\frac{by}{cd}\sqrt[3]{by}\right)\left(\frac{c^2d}{a}\sqrt[4]{cz}\right)$.

455. $\left(\sqrt{\frac{cp}{c+p}}\right)\left(\sqrt{\frac{c^2-p^2}{c}}\right)(\sqrt{cp-p^2})$.

456. $(\sqrt[a]{x-y})(\sqrt[c]{x+y})$.

457. $(\sqrt[m]{a^{2m-n}b^{5m+1}}+c^{3p})(\sqrt[m]{a^nb^{m-1}}-c^{3p})$.

458. $(\sqrt[m]{a+b})(\sqrt[n]{a+b})(\sqrt[m]{a-b})(\sqrt[n]{a-b})(\sqrt[mn]{(a^2+b^2)^{m+n}})$.

459. $\sqrt{\frac{p^2+q^2}{p+q}}\times\sqrt{\frac{p^2-q^2}{p^2+q^2}}\times\sqrt{\frac{(p+q)^2}{p-q}}$.

460. $\left(\frac{2c-d}{3c+d}\sqrt{\frac{2c^2-cd}{3cd-d^2}}\right)\left(\frac{9c^2-d^2}{4c^2-4cd+d^2}\sqrt{\frac{2cd-d^2}{3c^2-cd}}\right)$.

461. $\left(\frac{a^2-9x^2}{9a^2-4x^2}\sqrt[3]{\frac{9a^2-12ax+4x^2}{a^2+6ax+9x^2}}\right)\left(\sqrt[3]{\frac{9a^2-12ax+4x^2}{4a+12x}}\right)$.

462. $\left(\frac{c+x}{c-x}\sqrt[3]{\frac{c-3x}{x^2}+\frac{3c-x}{c^2}}\right)\left(\sqrt[3]{\frac{c^3-cx^2}{cx^2}}\right)$
$\left(\frac{cx}{c+x}\sqrt[3]{\frac{c^2x+2cx^2+x^3}{c}}\right)$.

463. $\left(\frac{4a^2-4ac+c^2}{27a^3-c^3}\sqrt{\frac{9a^3+3a^2c+ac^2}{2ac-c^2}}\right)$
$\left(\frac{3a-c}{2a-c}\sqrt{\frac{9a^2c+3ac^2+c^3}{2a^2-ac}}\right)$.

Division of Radicals.

464. $\sqrt{54}\div\sqrt{6}$. 465. $\sqrt{48}\div\sqrt{3}$. 466. $\sqrt{108}\div\sqrt{18}$.

467. $\sqrt[3]{56}\div\sqrt[3]{7}$. 468. $\sqrt[3]{500}\div\sqrt[3]{4}$. 469. $\sqrt[4]{80}\div\sqrt[4]{5}$.

470. $\sqrt[4]{243}\div\sqrt[4]{3}$. 471. $6\sqrt{96}\div3\sqrt{6}$. 472. $\sqrt[3]{64}\div\sqrt[3]{8}$.

473. $12\sqrt{75} \div 4\sqrt{3}$. 474. $\sqrt{48} \div 2\sqrt{3}$. 475. $\sqrt{36} \div 2\sqrt{3}$.

476. $\sqrt{54} \div 4\sqrt{2}$. 477. $12\sqrt{32} \div 4\sqrt{288}$.

478. $3\sqrt{8} \div 5\sqrt{27}$. 479. $12\sqrt{75} \div 5\sqrt{3}$.

480. $\sqrt[3]{162} \div 2\sqrt[3]{3}$. 481. $9\sqrt[3]{5} \div 2\sqrt[3]{15}$.

482. $\sqrt{\frac{1}{2}} \div \sqrt{\frac{2}{9}}$. 483. $\sqrt{\frac{8}{11}} \div \sqrt{\frac{3}{2}}$. 484. $\sqrt{\frac{2}{9}} \div \sqrt{\frac{1}{2}}$.

485. $\sqrt{\frac{14}{15}} \div \sqrt{\frac{24}{35}}$. 486. $\sqrt{\frac{52}{35}} \div \sqrt{\frac{17}{65}}$. 487. $3\sqrt{\frac{1}{2}} \div 2\sqrt{\frac{2}{3}}$.

488. $\frac{33}{7}\sqrt{\frac{2}{3}} \div \frac{12}{5}\sqrt{\frac{3}{4}}$. 489. $\frac{8}{7}\sqrt[3]{\frac{2}{3}} \div \frac{5}{2}\sqrt[3]{\frac{9}{4}}$.

490. $\frac{23}{4}\sqrt{\frac{1}{185}} \div \frac{2}{3}\sqrt{\frac{1}{5}}$. 491. $\frac{9}{13}\sqrt{\frac{22}{27}} \div \frac{8}{26}\sqrt{\frac{6}{11}}$.

492. $\sqrt{\frac{8}{45}} \div \sqrt[4]{6\frac{1}{4}}$. 493. $\sqrt[4]{17\frac{7}{9}} \div \sqrt[6]{33\frac{3}{8}}$. 494. $15 \div \sqrt{\frac{3}{5}}$.

495. $21 \div \sqrt{\frac{7}{8}}$. 496. $8 \div \sqrt[3]{\frac{4}{5}}$. 497. $10 \div \sqrt[3]{\frac{5}{4}}$.

498. $\sqrt[5]{160} \div 2$. 499. $\sqrt[4]{567} \div 3$. 500. $\sqrt[3]{297} \div 3$.

501. $\sqrt{1620} \div 9$. 502. $\sqrt[6]{448} \div 4$. 503. $6 \div \sqrt{24}$.

504. $7 \div \sqrt{63}$. 505. $10 \div \sqrt[3]{5}$. 506. $6 \div \sqrt{3}$.

507. $\sqrt[3]{64} \div \sqrt{8}$. 508. $\sqrt[3]{12} \div \sqrt{6}$. 509. $\sqrt{2} \div \sqrt[10]{8}$.

510. $\sqrt{4} \div \sqrt[3]{6}$. 511. $\sqrt[3]{12} \div \sqrt{8}$.

512. $5\sqrt[3]{108} \div 2\sqrt{432}$. 513. $\sqrt[6]{.064} \div \sqrt{10}$.

514. $\sqrt[4]{.0324} \div \sqrt{.5}$. 515. $\sqrt[5]{10.4976} \div 2\sqrt{5}$.

516. $\sqrt[5]{.8} \div 2\sqrt[10]{.0025}$. 517. $\sqrt{.027} \div \sqrt[5]{.0625}$.

518. $\sqrt{\frac{1}{6}} \div \sqrt[3]{\frac{1}{18}}$. 519. $\sqrt{\frac{1}{2}} \div \sqrt[3]{\frac{2}{3}}$. 520. $\sqrt{\frac{10}{8}} \div \sqrt[3]{\frac{5}{2}}$.

521. $\sqrt[12]{\frac{27}{5}} \div \sqrt[4]{\frac{3}{5}}$. 522. $\sqrt[5]{\frac{16}{27}} \div \sqrt[3]{\frac{4}{9}}$.

523. $4\sqrt{\frac{2}{3}} \div 3\sqrt[3]{\frac{3}{2}}$. 524. $\sqrt[4]{\frac{3}{8}} \div \sqrt[3]{\frac{9}{16}}$.

525. $(2\sqrt{8} \times 4\sqrt[4]{\frac{1}{4}}) \div (4\sqrt[3]{\frac{1}{2}} \times 4\sqrt[6]{4})$.

526. $(3\sqrt{8} + 5\sqrt{2}) \div 3\sqrt{2}$. 527. $(3\sqrt{10} - 4\sqrt{15}) \div \sqrt{5}$.

528. $72\sqrt{20} + 32\sqrt{45}) \div 2\sqrt{5}$. 529. $(\sqrt{6} - 3\sqrt[4]{4}) \div \sqrt[4]{2}$.

530. $(\sqrt[6]{3} - 3\sqrt[6]{6}) \div \sqrt[6]{3}$.

531. $(3\sqrt{20} + 2\sqrt{15} - 4\sqrt{5}) \div \sqrt{10}$.

532. $(30\sqrt[3]{4} - 36\sqrt[3]{10} + 30\sqrt[3]{90}) \div 3\sqrt[3]{20}$.

533. $(\sqrt{5} + \sqrt{\frac{5}{4}} - \sqrt{\frac{1}{5}}) \div \sqrt{3}$. **534.** $(\sqrt[3]{4} - \sqrt[3]{\frac{1}{16}} + \sqrt[3]{\frac{1}{2}}) \div \sqrt[3]{4}$.

535. $(3\sqrt{5} + \sqrt{15} - 3 - \sqrt{3}) \div \frac{1}{5}\sqrt{15}$.

536. $(2\sqrt[3]{2} - 3\sqrt[4]{3} + 7\sqrt[6]{5}) \div 5\sqrt{2}$.

Solve the following five examples without rationalizing:

537. $\frac{1}{2}\sqrt{\frac{1}{2}} \div (\sqrt{2} + 3\sqrt{\frac{1}{2}})$. **538.** $\frac{2}{5}\sqrt{\frac{1}{3}} \div (\sqrt{3} + 2\sqrt{\frac{1}{3}})$.

539. $\frac{7}{8}\sqrt{\frac{8}{5}} \div (2\sqrt{5} - 3\sqrt{\frac{1}{5}})$. **540.** $\frac{2}{3}\sqrt[3]{\frac{1}{2}} \div (3\sqrt[3]{2} - 4\sqrt[3]{\frac{1}{4}})$.

541. $11\sqrt[4]{\frac{1}{8}} \div (3\sqrt[4]{2} + 5\sqrt[4]{\frac{1}{8}})$.

542. $4\sqrt{acx} \div 2\sqrt{ax}$.

543. $\sqrt{125a^5bc} \div \sqrt{5a^3b}$.

544. $\sqrt[3]{x^4y^3z} \div \sqrt[3]{x^3y}$.

545. $12\sqrt[4]{a^5x^7} - 3\sqrt[4]{ax}$.

546. $\sqrt[3]{54g^7} \div \sqrt[3]{2g}$.

547. $\sqrt[3]{a^2b^3cd} \div \sqrt[3]{a^2bd}$.

548. $\sqrt[3]{16x^3} \div \sqrt[3]{64x^4}$.

549. $\sqrt[3]{25a^3} \div \sqrt[3]{\frac{1}{5}a}$.

550. $\sqrt[3]{3c} \div \sqrt[3]{\frac{1}{9c^3}}$. **551.** $\sqrt{\frac{5}{x^3}} \div \sqrt{\frac{10}{x}}$. **552.** $\sqrt{\frac{x^3}{z^5}} \div \sqrt{\frac{x}{z^3}}$.

553. $\sqrt[4]{\frac{8x}{3}} \div \sqrt[4]{\frac{27x^5}{2}}$.

554. $\sqrt[4]{\frac{54p^3}{q}} \div \sqrt[4]{\frac{8q^5}{pq}}$.

555. $\sqrt{k} \div \sqrt[3]{k}$. **556.** $\sqrt{d^3} \div \sqrt[3]{d^2}$. **557.** $\sqrt{n} \div \sqrt[4]{n}$.

558. $\sqrt[3]{x} \div \sqrt{a}$.

559. $4\sqrt{x^3} \div 2\sqrt[3]{x}$.

560. $\sqrt[3]{2m^2n^3} \div \sqrt[5]{2m^2n^3}$.

561. $\sqrt{6ef^2} \div \sqrt[5]{9e^2f^4}$.

562. $\sqrt[3]{12y^2} \div \sqrt{8y^3}$.

563. $3\sqrt{a^3c} \div 5\sqrt[3]{a^4c^3}$.

564. $12\sqrt[3]{pq^2} \div 4\sqrt[4]{pq}$.

565. $4\sqrt{a^3} \div 3\sqrt[3]{a^5}$.

566. $\frac{9}{2}\sqrt{x} \div \frac{8}{3}\sqrt[3]{xz}$.

567. $4\sqrt[3]{ax} \div 3\sqrt{xy}$.

568. $\sqrt[3]{a^2x} \div \sqrt{ax}$.

569. $30\sqrt{cx} \div 5\sqrt[3]{c^2x^2}$.

570. $\sqrt{14xy} \div \sqrt[3]{28x^2y^2}$.

571. $\sqrt[3]{3x} \div \sqrt[4]{7x^3}$.

572. $\sqrt[n]{a} \div \sqrt[n]{b}$.

573. $bc\sqrt[n]{ab} \div b\sqrt[n]{a}$.

574. $\sqrt[3]{15a^2b} \div \sqrt[4]{25ab^2}$.

575. $\frac{7}{8}\sqrt[m]{x} \div \frac{5}{11}\sqrt[m]{x}$.

576. $\sqrt[5]{mn^{c-1}p^2} \div \sqrt[5]{\frac{m^3q^{-1}}{p^{c-1}n^{-2}}}$.

577. $\sqrt[5]{\frac{x^py}{z^3w}} \div \sqrt[2q]{\frac{x^{p-1}z^3}{w^5}}$.

RATIONALIZATION.

578. $\frac{1}{\sqrt{2}}$.

579. $\frac{1}{\sqrt[3]{2}}$.

580. $\frac{2}{\sqrt[3]{3}}$.

581. $\frac{2}{\sqrt[4]{2}}$.

582. $\frac{10}{\sqrt{2}}$.

583. $\frac{3\sqrt{2}}{\sqrt{5}}$.

584. $\frac{\sqrt{5}}{\sqrt{3}}$.

585. $\frac{12}{5\sqrt{3}}$.

586. $\frac{3\sqrt{3}}{2\sqrt{2}}$.

587. $\frac{\sqrt{10}}{2\sqrt{5}}$.

588. $\frac{3\sqrt{8}}{\sqrt{6}}$.

589. $\frac{8}{3\sqrt[3]{4}}$.

590. $\frac{12}{\sqrt[4]{3}}$.

591. $\frac{1}{\sqrt[4]{27}}$.

592. $\frac{8}{\sqrt[5]{64}}$.

593. $\frac{3}{\sqrt{12}}$.

594. $\frac{\sqrt{5}}{\sqrt{10}}$.

595. $\frac{5}{\sqrt[5]{8}}$.

596. $\frac{6}{4\sqrt[4]{24}}$.

597. $\frac{10}{\sqrt[3]{5}}$.

598. $\frac{\sqrt{15}}{\sqrt[3]{5}}$.

599. $\frac{\sqrt{21}}{\sqrt[7]{8}}$.

600. $\frac{5\sqrt[3]{40}}{2\sqrt[3]{\frac{1}{25}}}$.

601. $\frac{\frac{1}{3}\sqrt{35}}{\frac{1}{2}\sqrt{\frac{5}{7}}}$.

602. $\frac{\frac{3}{2}\sqrt[3]{3}}{3\sqrt[3]{\frac{3}{2}}}$.

603. $\frac{2\sqrt[3]{6}}{\sqrt{2}}$.

604. $\frac{15\sqrt{24}}{3\sqrt[3]{24}}$.

605. $\frac{3\sqrt{2}}{\sqrt[3]{9}}$.

606. $\frac{12\sqrt[3]{192}}{4\sqrt[5]{729}}$.

607. $\frac{a}{\sqrt{a}}$.

608. $\frac{m}{\sqrt{n}}$.

609. $\sqrt{\frac{x}{y}}$.

610. $\frac{\sqrt{x}}{\sqrt[3]{x}}$.

611. $\frac{\sqrt{n}}{\sqrt[4]{n}}$.

612. $\frac{1}{\sqrt{x^3}}$.

613. $\frac{3c}{\sqrt{a}}$.

614. $\frac{5}{\sqrt{2ax}}$.

615. $\frac{c}{\sqrt[3]{d^2}}$.

616. $\frac{m\sqrt{n}}{p\sqrt{q}}$.

617. $\frac{1}{\sqrt[3]{a^5}}$.

618. $\frac{4a}{\sqrt[3]{3a^2}}$.

619. $\frac{2cd}{\sqrt[5]{16d^3}}$.

620. $\frac{3np^4}{\sqrt[3]{2nq^4}}$.

621. $\frac{\sqrt[3]{c^2}}{\sqrt[4]{c^2}}$.

622. $\frac{2x}{\sqrt[4]{27y^2}}$.

623. $\frac{\sqrt{a+c}}{\sqrt{a-c}}$.

624. $\frac{5}{3\sqrt[x]{y^{2a}}}$.

625. $\frac{3x}{\sqrt[n]{a^c}}$.

626. $\frac{x}{\sqrt[n]{a^{n-2}c^3}}$.

627. $\frac{3z}{\sqrt[n]{a^{2p}c^{3q}}}$.

628. $\frac{2}{5\sqrt[n]{a^c b^c}}$.

629. $\frac{1}{2-\sqrt{3}}$.

630. $\frac{4}{3-\sqrt{5}}$.

631. $\frac{7}{\sqrt{10}+\sqrt{3}}$.

632. $\frac{3}{2\sqrt{3}-2}$.

633. $\frac{4}{5+\sqrt{3}}$.

634. $\frac{6}{4+\sqrt{7}}$.

635. $\frac{2\sqrt{5}}{\sqrt{5}+\sqrt{3}}$.

636. $\frac{1+\sqrt{2}}{2-\sqrt{2}}$.

637. $\frac{3\sqrt{2}-1}{3\sqrt{2}+1}$.

638. $\frac{\sqrt{5}-\sqrt{3}}{\sqrt{5}+\sqrt{3}}$.

639. $\frac{4+\sqrt{5}}{3-\sqrt{5}}$.

640. $\frac{\sqrt{8}+\sqrt{7}}{\sqrt{7}-\sqrt{2}}$.

641. $\frac{3\sqrt{5}-2\sqrt{2}}{2\sqrt{5}-\sqrt{18}}$.

642. $\frac{7\sqrt{5}-3}{\sqrt{10}-\sqrt{2}}$.

643. $\frac{1-4\sqrt{5}}{6+2\sqrt{3}}$.

644. $\frac{2+2\sqrt{2}}{6+5\sqrt{6}}$.

645. $\frac{\sqrt{5}-\sqrt{3}}{\frac{2}{3}\sqrt{\frac{1}{5}}+\frac{1}{5}\sqrt{\frac{1}{3}}}$.

646. $\frac{\sqrt{2}+\sqrt{3}}{\frac{1}{3}\sqrt{\frac{1}{2}}-\frac{1}{2}\sqrt{\frac{1}{3}}}$.

647. $\frac{2+3\sqrt{\frac{2}{3}}}{\frac{1}{3}\sqrt{\frac{1}{2}}-\frac{2}{3}\sqrt{\frac{3}{5}}}$.

648. $\frac{\frac{2}{3}\sqrt{\frac{1}{2}}-\frac{3}{4}\sqrt{\frac{1}{3}}}{\frac{2}{5}\sqrt{\frac{5}{8}}+\frac{1}{2}\sqrt{\frac{1}{5}}}$.

649. $\dfrac{\sqrt{x}-a}{\sqrt{x}+a}$.

650. $\dfrac{\sqrt{b}+\sqrt{c}}{\sqrt{b}-\sqrt{c}}$.

651. $\dfrac{\sqrt{3a}+2x}{\sqrt{3a}-2x}$.

652. $\dfrac{p\sqrt{q}+q\sqrt{p}}{\sqrt{p}+\sqrt{q}}$.

653. $\dfrac{x\sqrt{y}-y\sqrt{x}}{x\sqrt{y}+y\sqrt{x}}$.

654. $\dfrac{m^2}{a+\sqrt{a^2-m^2}}$.

655. $\dfrac{\sqrt{x-4}-2}{\sqrt{x-4}+2}$.

656. $\dfrac{\sqrt{c^2+1}-c}{\sqrt{c^2+1}+c}$.

657. $\dfrac{1+\sqrt{1-2a}}{1-\sqrt{1-2a}}$.

658. $\dfrac{x+\sqrt{x^2+2}}{x-\sqrt{x^2+2}}$.

659. $\dfrac{\sqrt{1+a^2}-\sqrt{1-a^2}}{\sqrt{1+a^2}+\sqrt{1-a^2}}$.

660. $\dfrac{\sqrt{c+2}+\sqrt{c-2}}{\sqrt{c+2}-\sqrt{c-2}}$.

661. $\dfrac{\sqrt{x^2+1}-\sqrt{x^2-1}}{\sqrt{x^2+1}+\sqrt{x^2-1}}$.

662. $\dfrac{\sqrt{x+y}+\sqrt{x-y}}{\sqrt{x+y}-\sqrt{x-y}}$.

663. $\dfrac{\sqrt{n^2+n}+\sqrt{n^2-n}}{\sqrt{n^2+n}-\sqrt{n^2-n}}$.

664. $\dfrac{3\sqrt{a}-2\sqrt{a-1}}{2\sqrt{a}-3\sqrt{a-1}}$.

665. $\dfrac{\sqrt{k}-4\sqrt{k-2}}{2\sqrt{k}+3\sqrt{k-2}}$.

666. $\dfrac{4\sqrt{a-1}+\sqrt{a+1}}{3\sqrt{a-1}-2\sqrt{a+1}}$.

667. $\dfrac{1}{1+\sqrt{2}-\sqrt{3}}$.

668. $\dfrac{1}{\sqrt{2}+\sqrt{3}-\sqrt{5}}$.

669. $\dfrac{4}{\sqrt{3}+\sqrt{2}+1}$.

670. $\dfrac{4}{\sqrt{3}-\sqrt{2}+\sqrt{5}}$.

671. $\dfrac{3}{\sqrt{3}+2\sqrt{5}-\sqrt{2}}$.

672. $\dfrac{23}{4+3\sqrt{2}+5\sqrt{3}}$.

673. $\dfrac{4\sqrt{3}+3}{\sqrt{6}-\sqrt{5}+\sqrt{2}}$.

674. $\dfrac{7\sqrt{18}-3\sqrt{2}}{6\sqrt{3}-2\sqrt{12}+\sqrt{2}}$.

675. $\dfrac{2-\sqrt{3}}{1+\sqrt{2}+\sqrt{3}}$.

676. $\dfrac{\sqrt{x}+\sqrt{y}}{x+\sqrt{xy}+y}$.

677. $\dfrac{1-\sqrt{2}+\sqrt{3}}{1+\sqrt{2}-\sqrt{3}}$.

678. $\dfrac{3-\sqrt{5}-\sqrt{2}}{3+\sqrt{2}+\sqrt{5}}$.

679. $\dfrac{\sqrt{2}-\sqrt{3}-\sqrt{7}}{\sqrt{2}+\sqrt{3}+\sqrt{7}}$.

680. $\dfrac{3\sqrt{2}-2\sqrt{3}+7}{3-2\sqrt{2}+3\sqrt{3}}$.

681. $\dfrac{1}{\sqrt{6}+\sqrt{21}-\sqrt{10}-\sqrt{35}}$.

682. $\dfrac{2\sqrt{3}-3\sqrt{6}}{\sqrt{2}-\sqrt{3}+\sqrt{5}-\sqrt{6}}$.

683. $\dfrac{15}{\sqrt{10}+\sqrt{20}+\sqrt{40}-\sqrt{5}-\sqrt{80}}$.

684. $\dfrac{(\sqrt{2}+\sqrt{5})(\sqrt{5}+\sqrt{3})}{\sqrt{5}+\sqrt{2}+\sqrt{3}}$.

685. $\dfrac{\sqrt[3]{9}-\sqrt[3]{6}+\sqrt[3]{4}}{\sqrt[3]{9}+\sqrt[3]{6}+\sqrt[3]{4}} \times \dfrac{3^{\frac{1}{3}}+2^{\frac{1}{3}}}{3^{\frac{1}{3}}-2^{\frac{1}{3}}}$.

686. $\dfrac{10}{\sqrt[3]{7}-\sqrt[3]{5}}$.

687. $\dfrac{\sqrt{3}}{\sqrt[3]{9}+\sqrt[3]{10}}$.

688. $\dfrac{4}{\sqrt[4]{4}+\sqrt[4]{5}}$.

689. $\dfrac{1}{\sqrt{5}-\sqrt[4]{2}}$.

690. $\dfrac{\sqrt[3]{4}-8\sqrt[3]{72}}{\sqrt[3]{2}-4\sqrt[3]{3}}$.

Extract the square root of

691. $4+2\sqrt{3}$.

692. $7+2\sqrt{10}$.

693. $49+2\sqrt{48}$.

694. $15+2\sqrt{56}$.

695. $4\frac{1}{2}+2\sqrt{2}$.

696. $6-\sqrt{20}$.

697. $5 + \sqrt{24}$. **698.** $7 + 4\sqrt{3}$. **699.** $11 + 4\sqrt{7}$.

700. $8 - 4\sqrt{3}$. **701.** $19 + 4\sqrt{21}$. **702.** $31 - 4\sqrt{55}$.

703. $30 - 10\sqrt{5}$. **704.** $37 - \sqrt{640}$. **705.** $101 - 28\sqrt{13}$.

706. $4 - \sqrt{15}$. **707.** $6 - \sqrt{11}$. **708.** $5 + \sqrt{21}$.

709. $9 - \sqrt{17}$. **710.** $46 - 3\sqrt{20}$. **711.** $28 - 5\sqrt{12}$.

712. $75 - 3\sqrt{96}$. **713.** $4\frac{1}{2} + 2\sqrt{2}$. **714.** $2\frac{1}{4} + \sqrt{5}$.

715. $4\frac{3}{4} - \sqrt{3}$. **716.** $9\frac{2}{5} - 2\sqrt{2}$. **717.** $4\frac{1}{3} - \frac{4}{3}\sqrt{3}$.

718. $\frac{7}{2} - \frac{3}{2}\sqrt{5}$. **719.** $\frac{5}{7} - \frac{1}{7}\sqrt{21}$. **720.** $\frac{3}{3}\frac{5}{6} + \frac{1}{3}\sqrt{6}$.

721. $\frac{7}{1\,2} - \sqrt{\frac{1}{6}}$. **722.** $\frac{1\,1}{1\,6} + \frac{1}{2}\sqrt{\frac{3}{2}}$. **723.** $\frac{1\,1}{3\,6} - \frac{1}{3}\sqrt{\frac{2}{3}}$.

724. $\sqrt{27} - \sqrt{24}$. **725.** $\sqrt{24} + 2\sqrt{12}$. **726.** $4\sqrt{2} + 2\sqrt{6}$.

727. $3\sqrt{7} - 2\sqrt{14}$. **728.** $6x + 2x\sqrt{5}$. **729.** $a - 2\sqrt{a-1}$.

730. $x - 2\sqrt{xy - y^2}$. **731.** $2 - 2\sqrt{1 - a^2}$.

732. $p^2 - 2q\sqrt{p^2 - q^2}$. **733.** $(c+x)^2 - 4(c-x)\sqrt{cx}$.

734. $2y - 4\sqrt{yx - x^2}$. **735.** $m - n - 2\sqrt{m - n - 1}$.

736. $\dfrac{\sqrt{a} + \sqrt{c}}{3\sqrt{c}} - \dfrac{\sqrt{a} - \sqrt{c}}{3\sqrt{a}}$. **737.** $\dfrac{2 + \sqrt{3}}{2 - \sqrt{3}} - \dfrac{2 - \sqrt{3}}{2 + \sqrt{3}}$.

738. $\dfrac{x - y}{\sqrt{x} - \sqrt{y}} - \dfrac{x + y}{\sqrt{x} + \sqrt{y}}$. **739.** $\dfrac{\sqrt{12}}{(1 + \sqrt{2})(\sqrt{6} - \sqrt{3})}$.

740. $\dfrac{\sqrt{2} + \sqrt{45}}{\sqrt{2 + \sqrt{7 - 2\sqrt{10}}}}$.

741. $\dfrac{1}{\sqrt{11 - 2\sqrt{30}}} - \dfrac{3}{\sqrt{7 - 2\sqrt{10}}} - \dfrac{4}{\sqrt{8 + 4\sqrt{3}}}$.

742. $\dfrac{a-1}{a+1}\left\{\dfrac{a-1}{\sqrt{a}-1} + \dfrac{1-a}{a+\sqrt{a}}\right\}$.

743. $\dfrac{\sqrt{m^2+n^2}+\sqrt{m^2-n^2}}{\sqrt{m^2+n^2}-\sqrt{m^2-n^2}}+\dfrac{\sqrt{m^2+n^2}-\sqrt{m^2-n^2}}{\sqrt{m^2+n^2}+\sqrt{m^2-n^2}}$.

744. $\dfrac{2+\sqrt{3}}{\sqrt{2}+\sqrt{2-\sqrt{3}}}-\dfrac{2-\sqrt{3}}{\sqrt{2}-\sqrt{2-\sqrt{3}}}$.

745. $\dfrac{(3+\sqrt{3})(3+\sqrt{5})(\sqrt{5}-2)}{(5-\sqrt{5})(1+\sqrt{3})}$.

IMAGINARY QUANTITIES.

1. $\sqrt{-4}$.
2. $\sqrt{-9}$.
3. $\sqrt{-16}$.
4. $\sqrt{-25}$.
5. $\sqrt{-81}$.
6. $\sqrt{-c^8}$.
7. $\sqrt{-9p^6}$.
8. $\sqrt{-a^2c^4}$.
9. $\sqrt{-(x+y)^2}$.
10. $\sqrt{-49c^{4m}d^{6n}}$.
11. $\sqrt{-\frac{a^2}{4}}$.
12. $\sqrt{-\frac{c^2}{d^4}}$.
13. $\sqrt{\frac{-17x^4}{68y^6}}$.
14. $\sqrt{-12}$.
15. $\sqrt{-a^3}$.
16. $\sqrt{-7x^3}$.
17. $\sqrt[6]{-64}$.
18. $\sqrt[6]{-729}$.
19. $\sqrt[6]{-m^{12}}$.
20. $\sqrt[4]{-(x^2-y^2)^8}$.
21. $\sqrt[6]{-(2a-3x^2)^{12}}$.
22. i^3.
23. $(\sqrt{-a})^4$.
24. i^7.
25. $\frac{1}{i^3}$.
26. $(\sqrt{-1})^6$.
27. i^{29}.
28. $i^{\frac{1}{52}}$.
29. i^{15}.
30. $(\sqrt{-1})^{27}$.
31. $\sqrt[10]{-c^{20}}$.
32. $\sqrt[6]{-a^{18}}$.
33. $\sqrt[4]{-16}$.
34. $\sqrt[4]{-81}$.
35. $\sqrt[4]{-625}$.

Addition and Subtraction of Imaginaries.

36. $\sqrt{-9}+\sqrt{-25}$.
37. $\sqrt{-49}+\sqrt{-121}$.
38. $\sqrt{-4}+\sqrt{-81}$.
39. $\sqrt{-36}-\sqrt{-64}$.
40. $3\sqrt{-81}-2\sqrt{-144}$.
41. $\sqrt{-x}+\sqrt{-y}$.
42. $a\sqrt{-a^3}-\sqrt{-a^5}$.
43. $\sqrt{-25x^6}-\sqrt{49x^6}$.

44. $\sqrt{-(a-c)^2}+\sqrt{-(a+c)^2}$. 45. $\sqrt{-\frac{49}{100}}-\sqrt{-\frac{9}{25}}$.

46. $2\sqrt{-\frac{1}{9}}+4\sqrt{-\frac{1}{144}}$. 47. $\sqrt{-\frac{16}{x^2}}-\sqrt{-\frac{9}{x^2}}$.

48. $\sqrt{-1}-\sqrt{-64}+\sqrt{-49}$.

49. $\sqrt{-9a^2}+\sqrt{-25a^2}-\sqrt{49a^2}$.

50. $3\sqrt{-4}-\sqrt{-25}+4\sqrt{-9}$.

51. $3\sqrt{-c^2}-2\sqrt{-16c^2}+4\sqrt{-9c^2}$.

52. $\sqrt{-a^2}+\sqrt{-b^2}-\sqrt{-c^2}$.

53. $(x+y)\sqrt{-1}+x-y\sqrt{-1}$.

54. $\sqrt{-1+2q-q^2}-\sqrt{-4q^2}$.

55. $\sqrt{-x^4}+\sqrt{-16x^4}+\sqrt{-x^2}-\sqrt{-4x^4}-\sqrt{-81x^2}$.

56. $2\sqrt{-m^5}+2\sqrt{-m^4n}-4\sqrt{-m^3n^3}$.

57. $3x\sqrt{-1}+(2x+4y)\sqrt{-1}-(3x-y\sqrt{-1})+5x\sqrt{-1}$.

58. $\sqrt{-(a-c)^2}+\sqrt{(a^2-2ac+c^2)}+\sqrt{-16a^2c^2}$.

59. $\sqrt{-(p+q)^2}-\sqrt{-(p-q)^2}+\sqrt{-(1+p)^2q}-\sqrt{-4q}+\sqrt{-(1-p)^2q}$.

Multiplication of Imaginaries.

60. $\sqrt{3}\cdot\sqrt{-2}$. 61. $\sqrt{-5}\cdot\sqrt{3}$. 62. $\sqrt{6}\cdot\sqrt{-3}$.

63. $\sqrt{-3}\cdot\sqrt{-12}$. 64. $\sqrt{-a^2}\cdot\sqrt{-b^2}$.

65. $\sqrt{-2x}\cdot\sqrt{-8x}$. 66. $\sqrt{-c}\cdot\sqrt{-c}$.

67. $\sqrt{-x}\cdot\sqrt{-y}$. 68. $\sqrt{-1}\cdot\sqrt{-k}$. 69. $\sqrt{-x^2}\cdot\sqrt{-x}$.

70. $-\sqrt{-6}\cdot\sqrt{-2}$. 71. $-\sqrt{-8}\cdot-\sqrt{-2}$.

72. $\sqrt{-x^4} \cdot \sqrt{x^8}$.

73. $\sqrt{-4n^2} \cdot \sqrt{-n}$.

74. $-\sqrt{-16p^3} \cdot \sqrt{-9p^3}$.

75. $-\sqrt{-8a^3} \cdot -\sqrt{-2a^4}$.

76. $2\sqrt{-9} \cdot 3\sqrt{-4}$.

77. $-2\sqrt{-x} \cdot 7\sqrt{-y}$.

78. $-6\sqrt{-4} \cdot -3\sqrt{-9}$.

79. $\sqrt{-24} \cdot \sqrt{-6} \cdot \sqrt{-2}$.

80. $\sqrt{6} \cdot \sqrt{-5} \cdot \sqrt{-2}$.

81. $\sqrt{-a^2} \cdot \sqrt{-c^2} \cdot \sqrt{-x^2}$.

82. $\sqrt{-3mn} \cdot \sqrt{-3np} \cdot \sqrt{-4mp}$.

83. $\sqrt[6]{-a^6} \cdot \sqrt{-a^3}$.

84. $\sqrt[4]{-81x^4} \cdot \sqrt{-16x^2}$.

85. $\sqrt[3]{-27c^3} \cdot \sqrt[5]{-32c^{10}}$.

86. $-\sqrt[4]{-16x^5} \cdot -\sqrt{-8x^3}$.

87. $i\sqrt{-a^3}$.

88. $i\sqrt{-9} \cdot \sqrt{16}$.

89. $i\sqrt{-25} \cdot i\sqrt{-49}$.

90. $i^2\sqrt{-36} \cdot i\sqrt{-64}$.

91. $i\sqrt{-9} \cdot \sqrt{5-k}$.

92. $\sqrt{a-1} \cdot \sqrt{1-a}$.

93. $\sqrt{m^2-n^2} \cdot \sqrt{n-m}$.

94. $\sqrt{-\frac{4}{9}} \cdot \sqrt{-\frac{9}{16}}$.

95. $\sqrt{-\frac{4}{x^2}} \cdot \sqrt{-\frac{9}{x^2}} \cdot \sqrt{\frac{16}{x^2}}$.

96. $\sqrt{-\frac{4}{36}} \cdot -\sqrt{-\frac{1}{4}} \cdot \sqrt{-\frac{4}{9}}$.

97. $\sqrt{-\frac{16}{25}} \cdot \sqrt{-\frac{4}{5}} \cdot \sqrt{-\frac{5}{9}}$.

98. $\frac{2}{3}\sqrt{\frac{-4a^2}{c^2}} \cdot \frac{3}{4}\sqrt{-\frac{3c}{4x^2}} \cdot \frac{4}{5}\sqrt{-\frac{c}{6a^4}}$.

99. $\sqrt{-2} \cdot \sqrt{-4} \cdot \sqrt{-5} \cdot \sqrt{-8}$.

100. $2\sqrt{-x^3} \cdot x\sqrt{-6} \cdot \sqrt{-y^3} \cdot y\sqrt{-3}$.

101. $-i\sqrt{-a^4x^2} \cdot i\sqrt{-ax^3} \cdot 2\sqrt{-a^3x^5} \cdot -5\sqrt{-a^2x}$.

102. $(x+\sqrt{-1})(x-\sqrt{-1})$. **103.** $(3-\sqrt{-3})(3+\sqrt{-3})$.

104. $(3+2\sqrt{-1})(3-2\sqrt{-1})$.

105. $(3-\sqrt{-2})^2$. **106.** $(1+\sqrt{-3})^2$.

107. $(2-5\sqrt{-1})(3+4\sqrt{-1})$.

108. $(3-2\sqrt{-2})(2+3\sqrt{-2})$.

109. $(4+\sqrt{-7})(8-2\sqrt{-7})$.

110. $(a+ci)(a-ci)$. **111.** $(3-i\sqrt{8})(2+i\sqrt{2})$.

112. $(\sqrt{-2}+3\sqrt{-1})(\sqrt{-2}-3\sqrt{-1})$.

113. $(\sqrt{-a}+\sqrt{-c})(\sqrt{-a}-\sqrt{-c})$.

114. $(3\sqrt{-b}+2\sqrt{-d})(3\sqrt{-b}-2\sqrt{-d})$.

115. $(x\sqrt{-x}+y\sqrt{-y})(x\sqrt{-x}-y\sqrt{-y})$.

116. $(a\sqrt{-a}+b\sqrt{-b})^2$.

117. $(\sqrt{-(m+n)}+\sqrt{-n})(\sqrt{-(m+n)}-\sqrt{-n})$.

118. $(4\sqrt{-x}-6\sqrt{-y})(2\sqrt{-x}+3\sqrt{-y})$.

119. $(\sqrt{-2}-\sqrt{-4})(\sqrt{-2}+\sqrt{-3})$.

120. $(1+\sqrt{-3})^3$. **121.** $(1-\sqrt{-3})^3$.

122. $(2\sqrt{-3}-\sqrt{-1})^3$. **123.** $(\sqrt{-3}+\sqrt{-2})^3$.

124. $(5-\sqrt{-7}-\sqrt{-3})(5+\sqrt{-7}+\sqrt{-3})$.

125. $(\sqrt{-3}+\sqrt{-2}-\sqrt{-1})(\sqrt{-3}+\sqrt{-2}+\sqrt{-1})$.

126. $(\frac{1}{2}+\frac{2}{3}\sqrt{-2})(\frac{1}{2}-\frac{2}{3}\sqrt{-2})$.

127. $\left(a+\frac{\sqrt{-3}}{2}\right)\left(a-\frac{\sqrt{-3}}{2}\right)$.

128. $(\frac{1}{2} - \sqrt{-3})(\sqrt{2} - \sqrt{-\frac{1}{3}})$.

129. $(2\sqrt{-1} + \sqrt{-2})^2 + (\sqrt{-2} - 2\sqrt{-1})^2$.

130. $\left(c - \frac{1+\sqrt{-3}}{2}\right)\left(c - \frac{1-\sqrt{-3}}{2}\right)$.

131. $\left(-\frac{1}{2} + \frac{\sqrt{3}}{2}\sqrt{-1}\right)^3$.

Division of Imaginaries.

132. $\sqrt{-2} \div \sqrt{2}$.

133. $\sqrt{-2} \div \sqrt{-2}$.

134. $\sqrt{2} \div \sqrt{-2}$.

135. $\sqrt{-16} \div \sqrt{-4}$.

136. $\sqrt{-32} \div \sqrt{2}$.

137. $\sqrt{18} \div \sqrt{-2}$.

138. $x \div \sqrt{-1}$.

139. $a \div \sqrt{-a}$.

140. $c \div \sqrt{-c^2}$.

141. $\sqrt{-d^2} \div \sqrt{-d}$.

142. $\sqrt{14} \div \sqrt{-2}$.

143. $\sqrt{-14} \div \sqrt{-2}$.

144. $-\sqrt{-14} \div -\sqrt{2}$.

145. $-\sqrt{14} \div \sqrt{-2}$.

146. $-\sqrt{-14} \div -\sqrt{-2}$.

147. $i \div 1$.

148. $i \div i$.

149. $1 \div i$.

150. $-1 \div i$.

151. $-1 \div -i$.

152. $1 \div i^3$.

153. $1 \div i^{17}$.

154. $-1 \div i^5$.

155. $i \div i^7$.

156. $i^5 \div i$.

157. $-i \div -i^3$.

158. $\sqrt{-x} \div \sqrt{-y}$.

159. $-\sqrt{c} \div \sqrt{-c^2}$.

160. $\sqrt{-mn} \div \sqrt{-n}$.

161. $\sqrt{ax} \div \sqrt{-a}$.

162. $\sqrt{-np} \div \sqrt{-pq}$.

163. $q \div i\sqrt{q}$.

164. $\sqrt{-2x} \div \sqrt{-5x}$.

165. $\sqrt[4]{-52} \div \sqrt[4]{-13}$.

166. $-\sqrt[6]{-56} \div -\sqrt[6]{-7}$.

167. $6\sqrt{3} \div 3\sqrt{-4}$.

168. $6\sqrt{-1} \div -2\sqrt{-9}$.

169. $-5\sqrt{-x} \div 3\sqrt{-4x}$.

170. $-\sqrt{-1} \div -6\sqrt{-3}$.

171. $\sqrt{3} \div \frac{1}{2}\sqrt{-5}$.

172. $(\sqrt{-12} - \sqrt{-15}) \div \sqrt{-3}$.

173. $(\sqrt{16} - \sqrt{8}) \div \sqrt{-2}$.

174. $(2\sqrt{8} - \sqrt{-10}) \div -\sqrt{-2}$.

175. $(\sqrt{-12} - \sqrt{18}) \div \sqrt{-3}$.

176. $(1 + \sqrt{-1}) \div (1 - \sqrt{-1})$.

177. $(5 - \sqrt{-2}) \div (1 + \sqrt{-2})$.

178. $(\sqrt{-3} + \sqrt{-2}) \div (\sqrt{-3} - \sqrt{-2})$.

179. $(5 + i\sqrt{3}) \div (5 - i\sqrt{3})$.

Factors of

180. $x + y$.

181. $c^2 + 5$.

182. $d^2 - 7$.

183. $p^2 + 25$.

184. $a^2 - 6a + 13$.

185. $q^2 + 8q + 17$.

186. $x^2 - 8x + 25$.

187. $m^2 + 6m + 34$.

188. $a^2 - 10a + 31$.

189. $3b^2 + 6b - 3$.

190. $5n^2 + 6n + 2$.

191. $4a^2 - 4ac + 3c^2$.

192. $16d^2 + 8dk + 5k^2$.

193. $c^2 - \frac{2}{3}c + \frac{4}{9}$.

194. $3f^2 + 2f - 14\frac{2}{3}$.

EQUATIONS CONTAINING RADICALS.

1. $\sqrt{x} = 2.$
2. $\sqrt{x-2} = 3.$
3. $5 = 2\sqrt{x} + 1.$
4. $\sqrt{5y+5} = 5.$
5. $11 - 4\sqrt{5x} = 9.$
6. $\sqrt[3]{x} = 3.$
7. $\sqrt[3]{5x-7} = 2.$
8. $\sqrt[3]{2x-3} = -3.$
9. $5 + \sqrt[4]{2x} = 7.$
10. $\sqrt[5]{80x-43} = -3.$
11. $\sqrt{5x-1} = 2\sqrt{x+3}.$
12. $d\sqrt[n]{x} = c.$
13. $a = \sqrt{x} + c.$
14. $\sqrt{x+3b} = 3\sqrt{b}.$
15. $\sqrt[3]{7x+11} = \sqrt[3]{10x-4}.$
16. $\dfrac{\sqrt{x}+5}{\sqrt{x}-3} = 5.$
17. $\sqrt{x+3} = \dfrac{x-1}{\sqrt{x-3}}.$
18. $\dfrac{\sqrt{x}-8}{1-\sqrt{x}} = \dfrac{4}{3}.$
19. $2\sqrt{x+8} = \sqrt{4x^2-12x+32}.$
20. $\sqrt[3]{3x+2} = \sqrt[6]{9x^2+10x-6}.$
21. $\sqrt{x+5} = 5 - \sqrt{x}.$
22. $-x + 7 = \sqrt{x^2-7}.$
23. $\sqrt{x} - \sqrt{5} = \sqrt{x-5}.$
24. $\sqrt{x+n^2} = \sqrt{x} + n.$
25. $\sqrt{x-a} = \sqrt{x} - \sqrt{a}.$
26. $c + x = \sqrt{x^2+5cx-2c^2}.$
27. $\sqrt{d+x} = d + \sqrt{x}.$
28. $x + 11 + \sqrt{x+6} = 5.$
29. $\sqrt{x-7} - \sqrt{x-14} = 1.$
30. $3\sqrt{x-3} + \sqrt{9x+1} = 14.$
31. $\sqrt{x+2a} = \sqrt{2a} + \sqrt{x-2a}.$

32. $\sqrt{cx+a}=\dfrac{2a}{\sqrt{2a}}$.

33. $2\sqrt{x}-\sqrt{a}=\sqrt{4x-c}$.

34. $\sqrt{x}+\sqrt{a+x}=\dfrac{2a}{\sqrt{a+x}}$.

35. $\sqrt{x-\sqrt{x-9}+17}=\sqrt{x-23}$.

36. $\dfrac{\sqrt{x}+3}{\sqrt{x}-8}=\dfrac{\sqrt{x}-18}{\sqrt{x}+4}$.

37. $(\sqrt{x+15})^{\frac{1}{2}}=\sqrt{3}$.

38. $\sqrt{x+8}+\sqrt{3x-2}-\sqrt{8x+8}=0$.

39. $\sqrt{\dfrac{x+3}{x-3}}+\sqrt{\dfrac{x-3}{x+3}}=5$.

40. $\sqrt{cx+ab}+\sqrt{cx-ab}=\sqrt{4cx-2ab}$.

41. $\sqrt{\sqrt{x}+3}-\sqrt{\sqrt{x}-3}=\sqrt{2\sqrt{x}}$.

42. $\sqrt[3]{x+5}=\sqrt[6]{x^2+15x+10}$.

43. $\dfrac{a+x}{\sqrt{a}+\sqrt{x-a}}+\dfrac{a-x}{\sqrt{a}+\sqrt{x-a}}=\sqrt{a}$.

44. $\dfrac{12\sqrt{x}-11}{4\sqrt{x}-4\frac{2}{3}}=\dfrac{6\sqrt{x}+5}{2\sqrt{x}+\frac{2}{3}}$.

45. $\dfrac{2\sqrt{x}-1}{2\sqrt{x}+\frac{1}{3}}=\dfrac{\sqrt{x}-2}{\sqrt{x}-\frac{1}{3}}$.

46. $\sqrt{x+2a}-\sqrt{x+2b}=2\sqrt{x}$.

47. $\sqrt{x-a}-\sqrt{x-b}=\sqrt{b}-\sqrt{a}$.

48. $\sqrt{7+3\sqrt{4+\sqrt{5x}}}=4$.

49. $\sqrt{1\frac{5}{4}+x}=\frac{3}{2}+\sqrt{x}$.

50. $\dfrac{\sqrt{a+x}+\sqrt{x}}{\sqrt{a+x}-\sqrt{x}}=b$.

51. $\dfrac{\sqrt{a+x}+\sqrt{a-x}}{\sqrt{a+x}-\sqrt{a-x}}=c$.

52. $\sqrt[4]{x+10}=\sqrt{\sqrt{x+21}-1}$.

53. $\sqrt[3]{x^3+6x^2+4\sqrt{9x^2+8x}}=x+2.$

54. $\sqrt{\frac{x+1}{x-1}}+\sqrt{\frac{x-1}{x+1}}=a.$

55. $\frac{7-4\sqrt{x-7}}{2\sqrt{x-7}-3}=\frac{1-6\sqrt{x-7}}{5+3\sqrt{x-7}}.$

56. $\sqrt{\sqrt{\sqrt{4x+10}+5}}=3.$

57. $\sqrt{\frac{b}{a+x}}+\sqrt{\frac{c}{a-x}}=\sqrt[4]{\frac{4bc}{a^2-x^2}}.$

58. $\frac{x-c}{\sqrt{c}+\sqrt{x}}=\frac{\sqrt{x}-\sqrt{c}}{3}+2\sqrt{c}.$

59. $\frac{\sqrt{x}-1}{x-1}=1-\frac{\sqrt{x}}{2\sqrt{x}-5}.$

60. $\frac{x+\sqrt{x^2-1}}{x-\sqrt{x^2-1}}+\frac{x-\sqrt{x^2-1}}{x+\sqrt{x^2-1}}=4x(x-1).$

61. $x^{-1}+a^{-1}=\sqrt{a^{-2}+\sqrt{4a^{-2}x^{-2}+9x^{-4}}}.$

62. $\sqrt{1+x+\sqrt{x}}+\sqrt{1+x-\sqrt{x}}=a.$

63. $\frac{1}{2}-\frac{3}{x}=\sqrt{\frac{1}{4}-\frac{1}{x}\sqrt{9-\frac{36}{x}}}.$

64. $\sqrt{(\sqrt{(\sqrt{(\sqrt{(\sqrt{(x+123)}+4)}+5)}+6)}+1)}=2.$

65. $\frac{\sqrt{x+a}}{\sqrt{4x^2-3ax+\frac{2}{3}a}}=\frac{\sqrt{x^2+3ax+2a^2}}{\sqrt{4x^3+8ax^2}}.$

66. $\sqrt{x+\sqrt{x}}-\sqrt{x-\sqrt{x}}=\frac{3}{2}\sqrt{\frac{x}{x+\sqrt{x}}}.$

PURE QUADRATIC EQUATIONS.

1. $x^2 = 81$.
2. $5y^2 = 80$.
3. $9x^2 = 16a^2$.
4. $x^2 = 1.69$.
5. $cy^2 = d^2$.
6. $x = \frac{49}{x}$.
7. $\frac{x}{16} = \frac{4}{x}$.
8. $\frac{2x}{7} = \frac{7}{8x}$.
9. $\frac{3}{4}x^2 = 1.08$.
10. $\frac{x^2 - 1}{2} = 4$.
11. $y^2 + ab = 5y^2$.
12. $x^2 - ab = c$.
13. $3x^2 - a = x^2 + 7a$.
14. $\frac{x^2}{3} + 10 = 1$.
15. $\frac{6}{x^2 - 1} = 2$.
16. $\frac{2}{x^2 + 1} = \frac{1}{13}$.
17. $\frac{3x^2}{8} + 8 = x^2 - 2$.
18. $2(3y^2 - 14) = 3y^2 - 1$.
19. $y^2 - a = c - y^2$.
20. $(x + 1)^2 = 2x + 17$.
21. $mx^2 + n = nx^2 + m$.
22. $4x + 2 = \frac{2x + 16}{x}$.
23. $5x = \frac{20x - 135}{4 - 3x}$.
24. $\left(x + \frac{1}{3}\right)\left(x - \frac{1}{3}\right) = 11$.
25. $\frac{x - 7}{3} = \frac{5}{x + 7}$.
26. $(x+2)(x+3) = 42 + 5x$.
27. $(4 - x)^2 = 4(1 - x)^2$.
28. $2x(3x - 12) = x(4x - 24) + 162$.
29. $(x + 6)(x - 6) = 13 + 2x^2$.

30. $\frac{a-x^2}{c}=\frac{b-x^2}{d}$.

31. $\frac{4}{3x^2}=\frac{5}{2x^2}-\frac{7}{12}$.

32. $x^2-\frac{c^2}{4}-a^2=2c^2-3ac$.

33. $\frac{2a}{x}+\frac{x}{a}=\frac{6a^2-x^2}{ax}$.

34. $(5+x)^2+(5-x)^2=148$

35. $(4x+3)^2+(4x-3)^2=144$.

36. $(x^2+1)(x^2+2)=(x^2+6)(x^2-1)$.

37. $4x-150x^{-1}=x-3x^{-1}$.

38. $\frac{x+3}{3x-1}=\frac{3x+1}{x-3}$.

39. $\frac{x}{4}+\frac{4}{x}=\frac{x}{16}+\frac{16}{x}$.

40. $\frac{4}{7}x^2+\frac{5}{7}=\frac{3}{4}x^2-\frac{105}{28}$.

41. $(x+1)^3-(x-1)^3=728$.

42. $\sqrt{x^2-5}=\frac{2x}{3}$.

43. $\sqrt{2x^2+4x}=x+2$.

44. $\frac{a}{x}+\frac{c}{x}=\frac{2\sqrt{ac}}{x}+x$.

45. $\sqrt{x-3}=\frac{2\sqrt{10}}{\sqrt{x+3}}$.

46. $\sqrt{\frac{c}{y}}+\sqrt{\frac{y}{c}}=\sqrt{\frac{y}{d}}+\sqrt{\frac{d}{y}}$.

47. $3x^{\frac{3}{2}}=12$.

48. $3x^{\frac{2}{3}}=81$.

49. $4x^{\frac{2}{3}}=500$.

50. $2x^{\frac{4}{3}}=50$.

51. $\sqrt{x^3}=2\sqrt{2}$.

52. $5\sqrt[5]{x^4}=20$.

53. $\sqrt{w+m}=\sqrt{w+\sqrt{n^2+w^2}}$.

54. $\sqrt{x+\sqrt{x^2+k^2}}=\sqrt{k+x}$.

55. $\sqrt{x^2+\sqrt{x^4-a^4}}=a$.

56. $\sqrt{\frac{20x^2-9}{4x}}=\sqrt{x}$.

57. $\sqrt{x^2-c^2}=c\sqrt{d-1}$.

58. $15+\sqrt{x^2+17}=24$.

59. $(\sqrt{9+x^2}-2)^2=64$.

60. $\frac{3}{4}\frac{7}{} - \sqrt[3]{2(x^4 + \frac{7}{16})} = \frac{3}{4}\frac{3}{}$. 61. $12 - x^2 : \frac{1}{2}x^2 = 20 : 5$.

62. $\frac{1}{2}x^2 + \frac{1}{4}x^2 - 3 : \frac{1}{4}x^2 - \frac{1}{8}x^2 + 3 = 3 : 1$.

63. $\frac{1}{2}(x^2 - 5)^2 : x^2 - 5 = 2 : 1$.

64. $\sqrt{p^2 + y^2} = \sqrt[4]{q^4 + y^4}$. 65. $\frac{v(10 + v^2)^{\frac{1}{2}}}{\sqrt{v}} = \left(\frac{v^{\frac{1}{2}}}{5v}\right)^{-1}$.

66. $\frac{1}{2}(x^2 - \frac{1}{6}a^2) - \frac{1}{3}(x^2 - \frac{1}{8}a^2) + \frac{1}{4}(x^2 - \frac{1}{10}a^2) = 0$.

67. $\frac{1}{5}(3x^2 - 7) + \frac{1}{9}(25 - 4x^2) = \frac{1}{3}(5x^2 - 14)$.

68. $(3 + x)(1 - x)(5 + x) + (1 + x)(5 - x)(3 - x) = 16$.

69. $x + (a^2 + x^2)^{\frac{1}{2}} = 2a^2(a^2 + x^2)^{-\frac{1}{2}}$.

70. $\frac{a - \sqrt{a^2 - x^2}}{a + \sqrt{a^2 - x^2}} = c$. 71. $\frac{\sqrt{3x^2 + 4} + 2}{\sqrt{3x^2 + 4} - 2} = 3$.

72. $\frac{\sqrt{x^2 + 1} - \sqrt{x^2 - 1}}{\sqrt{x^2 + 1} + \sqrt{x^2 - 1}} = \frac{1}{2}$.

73. $\frac{x^3 + x + 8}{x^3 + 4} + \frac{x^3 + x - 8}{x^3 - 4} = 2$.

74. $\frac{x^2 + x + 1}{x - 1} - \frac{x^2 - x + 1}{x + 1} = 6$.

75. $\frac{x^4 - 3x^2 + 4}{3x^4 + 2x^2 - 4} = \frac{x^2 - 3}{3x^2 + 2}$.

76. $\frac{x + a + 2b}{x + a - 2b} = \frac{b - 2a + 2x}{b + 2a - 2x}$.

77. $\frac{5a + b + x}{a + 5b - x} = \frac{a^2(a + 5b + x)}{b^2(5a + b + x)}$.

78. $\sqrt{1 + x^2} + \sqrt{1 + x^2 + \sqrt{1 - x^2}} = \sqrt{1 - x^2}$.

79. $\frac{1}{m - \sqrt{m^2 - x^2}} - \frac{1}{m + \sqrt{m^2 - x^2}} = \frac{m}{x^2}$.

80. $\dfrac{2}{x^2+\sqrt{2-x^2}}+\dfrac{2}{x-\sqrt{2-x^2}}=x.$

81. $\dfrac{x+5}{x^2-5x}-\dfrac{x-5}{x^2+5x}=\dfrac{5}{x^2-67}.$

82. $\sqrt{\dfrac{x+3}{x-3}}+\sqrt{\dfrac{x-3}{x+3}}=5.$

83. $\dfrac{x+m}{x-m}+\dfrac{x-m}{x+m}=\dfrac{m+n}{m-n}+\dfrac{m-n}{m+n}.$

84. $\dfrac{x+a}{x-a}+\dfrac{x-a}{x+a}=\dfrac{3a+2b}{3a-2b}+\dfrac{3a-2b}{3a+2b}.$

85. $\dfrac{5}{2+x}-\dfrac{5}{x-4}=\dfrac{2-x}{x-4}+\dfrac{2}{x^2-2x-8}.$

AFFECTED QUADRATICS.

1. $x^2 = x$.
2. $x^2 + 15x = 0$.
3. $x^2 - 3x = 4x$.
4. $x^2 - 4x = -4$.
5. $x^2 - 6x + 9 = 0$.
6. $x^2 - 6x + 5 = 0$.
7. $x^2 - 5x + 6 = 0$.
8. $x^2 - 8x = -12$.
9. $x^2 - 8x = -15$.
10. $x^2 - 10x + 16 = 0$.
11. $x^2 + 2x = 8$.
12. $x^2 - 5x + 4 = 0$.
13. $x^2 - 3x = 4$.
14. $x^2 - 3x = 10$.
15. $x^2 + 6x = 7$.
16. $x^2 + x = 30$.
17. $x^2 + 3x = 4$.
18. $x^2 - 4x = 45$.
19. $x^2 - 12x + 35 = 0$.
20. $x^2 + 9x = -8$
21. $x^2 + 120 = -26x$.
22. $x^2 + 84 = -20x$.
23. $x^2 + 20x + 99 = 0$.
24. $x^2 - x = 132$.
25. $x^2 - 35x = 200$.
26. $x^2 - 15x = 154$.
27. $x^2 + 29x + 210 = 0$.
28. $y^2 - 7y + 6 = 0$.
29. $y^2 - 3y = 10$.
30. $y^2 + 5y = 14$.
31. $y^2 - 7y + 10 = 0$.
32. $y^2 - y + 3 = 45$.
33. $y^2 - 12 = 11y$.
34. $y^2 - 3y = 28$.
35. $y^2 - 3y = 130$.
36. $y^2 + 5y = -6$.
37. $5y^2 + 20 = 25y$.
38. $z^2 - 2z = -19$.
39. $z^2 - 2z + 2 = 0$.
40. $z^2 + 13 = 4z$.
41. $z^2 + 12z = 44$.
42. $9w^2 - 12w + 4 = 0$.
43. $9w^2 - 6w = 24$.
44. $9w^2 + 18w = 72$.

45. $4w^2 - 4w = 3$.

46. $4w^2 - 8w = 5$.

47. $4w^2 + 8w = 5$.

48. $36w^2 - 36w = -5$.

49. $64w^2 + 48w = 7$.

50. $2x = 2 - x^2$.

51. $x^2 - 4x + 1 = 0$.

52. $x^2 - 5x + 2 = 0$.

53. $x^2 - x = 1$.

54. $x^2 - 2x = 4$.

55. $x^2 + 5x = 10$.

56. $x^2 + 2x = 6$.

57. $x^2 - 7x = 7$.

58. $2u^2 - 9u = -4$.

59. $5u^2 + 8 = 22u$.

60. $6u^2 + 5u - 56 = 0$.

61. $3u^2 + 5u = 42$.

62. $5u^2 + 4u = 273$.

63. $10u^2 - 8u = 312$.

64. $2u^2 + 11u = -5$.

65. $5u^2 + 7u = 7$.

66. $y^2 - \frac{5}{6}y = \frac{1}{6}$.

67. $y^2 - \frac{2}{3}y = 1\frac{3}{8}$.

68. $y^2 - 5\frac{3}{4}y = 18$.

69. $y^2 + \frac{5}{8}y - \frac{2}{8} = 0$.

70. $y^2 + 1 = 2\frac{1}{6}y$.

71. $y^2 - \frac{5}{6}y = 1$.

72. $y^2 - \frac{2}{3}y - \frac{4}{8} = 0$.

73. $y^2 + \frac{2}{5}y = 3$.

74. $y^2 - \frac{7}{9}y = 310$.

75. $x - \frac{10}{x} = 3$.

76. $x^2 - \frac{x}{8} = \frac{3}{16}$.

77. $\frac{x^2}{12} + \frac{20}{3} = 2x$.

78. $\frac{1}{2}x^2 - \frac{1}{3}x = \frac{5}{8}$.

79. $\frac{x^2}{4} - \frac{3x}{8} = \frac{5}{2}$.

80. $x - 9\frac{1}{2} = \frac{5}{x}$.

81. $3x = \frac{6}{x} + 10\frac{1}{2}$.

82. $\frac{x}{2} + \frac{2}{x} = \frac{x}{4} + \frac{3}{2}$.

83. $\frac{1}{8x^2} - \frac{13}{24x} = -\frac{1}{2}$.

84. $\frac{(x-12)^2}{6} - \frac{x}{9} + \frac{x(x-9)}{18} = \frac{(x-14)^2}{2} + 5$.

85. $\frac{2x+5}{5} - \frac{x-4}{3} = \frac{2x(x-7)}{6} - \frac{7x}{10}$.

86. $\frac{(x-8)(x-10)}{4} - \frac{(x-12)^2}{8} = \frac{x(x-15)}{5} + \frac{x}{10}$.

87. $\frac{2x^2}{3} - \frac{x}{2} - \frac{16}{15} = \frac{34x^2}{69} - \frac{76x}{115} + \frac{5}{6}$.

88. $w = \frac{8}{w-2}$.

89. $w - \frac{20}{w-5} = 6$.

90. $\frac{6}{w+1} + \frac{2}{w} = 3$.

91. $\frac{36}{w+2} = \frac{w}{2} + 4$.

92. $\frac{w}{w+60} = \frac{7}{3w-5}$.

93. $\frac{8w}{w+2} - 6 = \frac{20}{3w}$.

94. $\frac{21}{5-w} - \frac{w}{7} = \frac{23}{7}$.

95. $\frac{w+12}{w} + \frac{w}{w+12} = \frac{78}{15}$.

96. $\frac{w^2-5w}{w+3} = w - 3 + w^{-1}$.

97. $\frac{4}{z+1} + 1 = \frac{10}{z+2}$.

98. $\frac{3}{1+2z} - \frac{6}{1-2z} = -5$.

99. $\frac{10}{z-3} + \frac{12}{z+4} = 3$.

100. $\frac{5}{3z+2} - \frac{3}{z-1} = \frac{1}{2}$.

101. $\frac{3}{z+2} + \frac{2}{7-2z} = \frac{13}{5}$.

102. $\frac{2}{6z-1} + \frac{2}{2+3z} = \frac{7}{6}$.

103. $\frac{5}{6(z-2)} - \frac{7}{3(z-1)} = \frac{1}{4}$.

104. $\frac{u}{u+8} = \frac{u+3}{2u+1}$.

105. $\frac{2u-7}{u-1} = \frac{u+1}{3u+3}$.

106. $\frac{4u-1}{u+.5} = \frac{6u}{u+2.5}$.

107. $\frac{u+.5}{u-.5} = \frac{2u+1}{u+2.5}$.

108. $\dfrac{2u+1}{u+.5}=\dfrac{4u-2}{3u-3.5}$.

109. $\dfrac{3u-\frac{1}{5}}{2u+1}=\dfrac{2u+\frac{1}{5}}{4u+\frac{1}{5}}$.

110. $\dfrac{3u-1}{4u+\frac{1}{3}}=\dfrac{2u+\frac{2}{3}}{5u+2\frac{2}{3}}$.

111. $\dfrac{3u+\frac{1}{3}}{2u-\frac{1}{3}}=\dfrac{6u+\frac{2}{3}}{2u+\frac{8}{3}}$.

112. $\dfrac{1}{y+6}+8y^{-1}=\dfrac{3}{y+2}$.

113. $\dfrac{2}{y-5}+\dfrac{7}{y}=\dfrac{42}{2y+7}$.

114. $\dfrac{5}{y-2}-\dfrac{4}{y}=\dfrac{3}{y+6}$.

115. $\dfrac{5}{y+2}+\dfrac{3}{y}=\dfrac{14}{y+4}$.

116. $\dfrac{3x}{x-2}+\dfrac{4}{2-x}=\dfrac{4}{x+3}$.

117. $\dfrac{6x}{5x-6}-\dfrac{4}{2-3x}=\dfrac{16}{3x-2}$.

118. $\dfrac{3}{1-2x}=\dfrac{15x}{x+2}-\dfrac{8}{2x-1}$.

119. $\dfrac{12x}{3x-1}=\dfrac{2x-3}{2}-\dfrac{14}{5(1-3x)}$.

120. $\dfrac{1}{w^2-1}-\dfrac{1}{1-w}=\dfrac{7}{8}-\dfrac{1}{w+1}$.

121. $\dfrac{1}{w^2-3w}-\dfrac{1}{9-w^2}=\dfrac{13}{16w}$.

122. $\dfrac{1}{w^2-4}-\dfrac{3}{2-w}=1+\dfrac{1}{3w+6}$.

123. $\dfrac{15}{3w-1}-\dfrac{3}{4}=\dfrac{220}{9w^2-1}-\dfrac{11}{3w+1}$.

124. $\dfrac{5}{u+1}-\dfrac{10}{u+10}=\dfrac{2}{3u-3}$.

125. $\dfrac{1}{u+1}-\dfrac{2}{1-u}=\dfrac{13}{4u-1}$.

126. $\dfrac{3}{u-1}+\dfrac{5}{u-2}=\dfrac{6}{u-3}$.

127. $\dfrac{3}{5-u}+\dfrac{2}{4-u}=\dfrac{8}{u+2}$.

128. $\dfrac{y+3}{2}+\dfrac{16-2y}{2y-5}=\dfrac{26}{5}$.

129. $\dfrac{1}{y^2-4y+3}-6=\dfrac{2}{y-1}-\dfrac{3}{y-3}$.

130. $\dfrac{3y-7}{y}+\dfrac{4y-10}{y+5}=\dfrac{7}{2}$.

131. $\dfrac{y}{2y-1}+\dfrac{8-y}{y+5}=\dfrac{13}{6}$.

132. $\dfrac{y+3}{y-2}+\dfrac{5y-3}{5y-6}=\dfrac{22}{3}$.

133. $\dfrac{3y+1}{13y-15}-\dfrac{2y-7}{2y-8}=\dfrac{5}{2}$.

134. $\dfrac{17y-3}{11y+15}+\dfrac{13y+3}{5y-7}=6\frac{1}{4}$.

135. $\dfrac{v+2}{v+3}-\dfrac{v+4}{v+5}=-\dfrac{14}{v+3}$.

136. $\dfrac{3v+2}{3v-2}+\dfrac{3v-2}{3v+2}=\dfrac{15v+11}{3v+2}$.

137. $\dfrac{4v+67}{40v^2-36}+\dfrac{v}{30v^2-27}=\dfrac{2}{3}$.

138. $\dfrac{9v+1}{9v-3v^2}=\dfrac{v}{21-7v}-\dfrac{v+3}{21v}$.

139. $\dfrac{4v-3}{3v-7}=3+\dfrac{2v-3}{v-1}$.

140. $\dfrac{3v-6}{5-v}=\dfrac{7}{2}-\dfrac{11-2v}{2(5-2v)}$.

141. $\dfrac{v+1}{v-1}+\dfrac{v+2}{v-2}=\dfrac{2v+13}{v+1}$.

142. $3 - \frac{1}{x+2} - \frac{3}{4x+6} = \frac{5}{2x^2+x-6}$.

143. $1 + \frac{7x}{3x+1} + \frac{2x^2}{21x^2+10x+1} = 0$.

144. $\frac{x}{x^2-2x-15} - \frac{7\frac{1}{2}}{x^2+2x-35} = \frac{1}{x^2+10x+21}$.

145. $\frac{4x+6}{2x^2-5x-12} + \frac{32x+3}{4x^2+8x+3} = \frac{9x-2}{2x^2-7x-4}$.

146. $\frac{z}{z+1} + \frac{z+1}{z+2} = \frac{z-2}{z-1} + \frac{z-1}{z}$.

147. $\frac{z}{z-3} - \frac{z-3}{z} + \frac{z}{z+3} - \frac{z+3}{z} = \frac{2}{3}$.

148. $\frac{z-2}{z+2} - \frac{z-3}{z+3} = \frac{z+4}{z-4} - \frac{z+2}{z-2}$.

149. $\sqrt{w^5} + \sqrt{w^3} = 6\sqrt{w}$.

150. $3w + \sqrt{w} = 5\sqrt{4w}$.

151. $w - \sqrt{w+1} = 0$.

152. $\sqrt{w+2} = w$.

153. $\sqrt{w+20} = w$.

154. $3w + 2\sqrt{w} = 1$.

155. $7w - 52 = \sqrt{2w}$.

156. $2w - 5\sqrt{w} = 3$.

157. $2w + 1 = \sqrt{6w+3}$.

158. $w + 2\sqrt{w-5} = 5$.

159. $w - 5 = \sqrt{w-3}$.

160. $w - 1 + \sqrt{w+5} = 0$.

161. $8\sqrt{w} - 8w = \frac{3}{2}$.

162. $(4w+5)^{\frac{1}{2}}(7w+1)^{\frac{1}{2}} = 30$.

163. $w + 2\sqrt{3w} + 1 = 0$.

164. $3w = 5 + \sqrt{30w-71}$.

165. $w + \frac{2}{3}\sqrt{w+4} = 7$.

166. $7w - 33 = \sqrt{3w-11}$.

167. $5y = 2\sqrt{3y^2-y+15}$.

168. $7\sqrt{y} = 3\sqrt{y^2+3y-59}$.

169. $\sqrt{y} + \sqrt{5y+1} = 5.$

170. $\sqrt{y+1} + \dfrac{1}{y+1} = 2.$

171. $\dfrac{y-1}{\sqrt{y}+1} = 4 + \dfrac{\sqrt{y}-1}{2}.$

172. $(5 - \sqrt{y})^2 = 2(7 + \sqrt{y}).$

173. $\sqrt{2u-1} - \sqrt{u-1} = 1.$

174. $\sqrt{u-1} + \sqrt{3u+3} = 4.$

175. $\sqrt{u+2} + \sqrt{3u+4} = 8.$

176. $\sqrt{2u+3} - \sqrt{u-7} = 3.$

177. $\sqrt{1+4u} - \sqrt{1-4u} = 4\sqrt{u}.$

178. $\sqrt{u-1} + \sqrt{2u-1} = \sqrt{5u}.$

179. $\sqrt{u+3} + \sqrt{u+8} = 5\sqrt{u}.$

180. $\sqrt{2u+4} - \sqrt{\dfrac{u}{2}+6} = \sqrt{\dfrac{u}{6}}.$

181. $\sqrt{3u+4} + \sqrt{2u+1} = \sqrt{11u+5}.$

182. $\sqrt{3-4u} + \sqrt{5u+2} = \sqrt{u+5}.$

183. $\sqrt{2u+9} + \sqrt{3u-15} = \sqrt{7u+8}.$

184. $\sqrt{2u+7} + \sqrt{3u-18} = \sqrt{7u+1}.$

185. $\sqrt{2u-3} + \sqrt{4u+1} = \sqrt{6u+28}.$

186. $\dfrac{\sqrt{x^2-16}}{\sqrt{x-3}} + \sqrt{x+3} = \dfrac{7}{\sqrt{x-3}}.$

187. $\sqrt{2x^2+7} - \sqrt{x^2-x+1} = 1.$

188. $\sqrt{4x^2+6x-9} - 2\sqrt{x^2-x-4} = 3.$

189. $\dfrac{1}{z+\sqrt{2-z^2}}+\dfrac{1}{z-\sqrt{2-z^2}}=\dfrac{z}{2}$.

190. $\dfrac{1}{1+z+z^2}+\dfrac{2}{\sqrt{1+z+z^2}}=3$.

191. $\dfrac{z^2+1}{z^2-1}=z+\dfrac{\sqrt{6}}{\sqrt{z}}$.

192. $\sqrt{\sqrt{\sqrt{9z+3}-1}}=\sqrt[4]{z}$.

193. $\dfrac{z+\sqrt{z^2-9}}{z-\sqrt{z^2-9}}=(z-2)^2$.

194. $\sqrt{2z^2+9z+9}+\sqrt{2z^2+7z+5}=\sqrt{2}$.

195. $\sqrt[3]{\dfrac{z+1}{4z-1}}+\sqrt[3]{\dfrac{4z-1}{z+1}}=\dfrac{13}{6}$.

196. $x^2+3p^2=4px$.

197. $x^2-3qx+2q^2=0$.

198. $x^2+8cx=9c^2$.

199. $x^2-4dx=9e^2-4d^2$.

200. $x^2+2ax=b^2+2ab$.

201. $x^2+2ac=c^2+2ax$.

202. $x^2+2ax=2a+1$.

203. $4x^2+m^2=n^2-4mx$.

204. $x^2+5tx=14t^2$.

205. $9x^2-6ax=b^2-a^2$.

206. $x^2-a=1+ax$.

207. $x^2-2ax=4b^2-a^2$.

208. $2x^2-ax=a^2$.

209. $y^2-2(m-n)y=4mn$.

210. $y^2-(a+c)y=-ac$.

211. $y^2-ab=ay-by$.

212. $my-y^2=2m-2y$.

213. $y^2-3py+6pq=2qy$.

214. $y^2-2ey=16e-8y$.

215. $y^2-2by+2cy+c^2=2bc$.

216. $y^2-4cy+3c^2=4cd-2dy-d^2$.

217. $\dfrac{4}{z^2-2az+a^2}=\dfrac{1}{4}$.

218. $z^2-\dfrac{z}{n}=\dfrac{3}{4n^2}$.

219. $z^2-\dfrac{a^2+c^2}{ac}z=-1$.

220. $z^2-\dfrac{5cz}{3}+\dfrac{c}{4}=0$.

221. $z^2 - \frac{az}{b} - \frac{bz}{a} + 1 = 0.$ 222. $z + \frac{mn}{z} = m + n.$

223. $\frac{z}{q} + \frac{q}{z} = \frac{5}{2}.$ 224. $\frac{z}{p} + \frac{p}{z} = \frac{p}{q} + \frac{q}{p}.$

225. $(a-1)^2x^2 - a(x+b) = ax(a-2) - bx.$

226. $mpx^2 - npx + mqx - nq = 0.$

227. $(x+s)(3x-t) = (3t-s)x - (s-t)^2 - st.$

228. $\frac{y}{a-y} - \frac{a-y}{y} = \frac{a}{y}.$ 229. $\frac{ay^2 - by + c}{my^2 - ny + p} = \frac{c}{p}.$

230. $\frac{4b}{a} + \frac{a-4b}{y-2b} = \frac{a+4b}{y+2b}.$

231. $\frac{3a}{y+a} + \frac{2a}{y+2a} - \frac{a}{y+3a} = \frac{4a}{y}.$

232. $\left(\frac{z-a}{z+a}\right)^2 + 12 = 7\left(\frac{z-a}{z+a}\right).$

233. $\frac{z-2h+2k}{hz} = \frac{7k-4h}{hz-kz} - \frac{4k-z}{h^2-hk}.$

234. $\frac{z}{m^2-4n^2} - \frac{1}{m+2n} = \frac{z-2m+19n}{mz-2nz} + \frac{z-5n}{mz+2nz}.$

235. $\sqrt{a-x} + \sqrt{b-x} = \frac{b}{\sqrt{b-x}}.$

236. $\sqrt{a+x} - \sqrt{a-x} = \sqrt{2x}.$

237. $\sqrt{x+a^2} - \sqrt{x-2a^2} = \sqrt{2x-5a^2}.$

238. $\sqrt{x-a} + \sqrt{b-x} = \sqrt{b-a}.$

239. $\sqrt{ax+b^2} + \sqrt{bx+a^2} = a - b.$

240. $\frac{x+1}{\sqrt{x}} = \frac{a+1}{\sqrt{a}}.$ 241. $\sqrt{x-12ab} = \frac{9a^2-b^2}{\sqrt{x}}.$

242. $\sqrt{a+x}+\sqrt{a-x}=\dfrac{12a}{5\sqrt{a+x}}$.

243. $\sqrt{x+a}-\sqrt{x+b}=\sqrt{2x}$.

244. $\sqrt{a-x}+\sqrt{b-x}=\sqrt{a+b-2x}$.

245. $\dfrac{a-\sqrt{2ax-x^2}}{a+\sqrt{2ax-x^2}}=\dfrac{x}{a-x}$.

246. $\dfrac{1}{c-\sqrt{c^2-x^2}}-\dfrac{1}{c+\sqrt{c^2-x^2}}=\dfrac{c}{x^2}$.

247. $\dfrac{x+\sqrt{12a-x}}{x-\sqrt{12a-x}}=\dfrac{\sqrt{a}+1}{\sqrt{a}-1}$.

248. $\dfrac{x}{\sqrt{x}+\sqrt{a-x}}+\dfrac{x}{\sqrt{x}-\sqrt{a-x}}=\dfrac{b}{\sqrt{x}}$.

249. $\dfrac{\sqrt{a^2+x^2}+\sqrt{a^2-x^2}}{\sqrt{a^2+x^2}-\sqrt{a^2-x^2}}=\dfrac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}$.

250. $\dfrac{x^2}{\sqrt{a}+\sqrt{b}}-(a^{\frac{1}{2}}-b^{\frac{1}{2}})x=\dfrac{1}{(ab^2)^{-\frac{1}{2}}+(a^2b)^{-\frac{1}{2}}}$.

251. $x^{\frac{2}{3}}=4$. 252. $x^{\frac{5}{3}}=32$. 253. $x^{2/c}=3$.

254. $x^{\frac{3}{2}}=27$. 255. $x^3-1=0$. 256. $x^4-1=0$.

257. $x^3+1=0$. 258. $x^6-1=0$.

259. $x^3=(2c-x)^3$. 260. $3ax^{m/n}=2ax^{m/n}+5c$.

261. $x-\dfrac{7}{x^3}=\dfrac{x}{8}$. 262. $12x^{-\frac{3}{2}}=\dfrac{2}{x^{\frac{3}{2}}}+\dfrac{10}{9}$.

263. $y^4=10y^2-9$. 264. $y^4+7y^2=44$.

265. $y^6-56=y^3$. 266. $y^3=9y^{\frac{3}{2}}-8$.

267. $3y + 2\sqrt{y} = 1.$

268. $y^{-1} + 3 = 4y^{-\frac{1}{2}}.$

269. $y^{-\frac{1}{2}} + 8 = 9y^{-\frac{3}{4}}.$

270. $6y^{-2} = 5y^{-1} - 1.$

271. $y^{\frac{1}{3}} - \frac{6}{y^{\frac{1}{3}}} + 1 = 0.$

272. $y^{1/n} + 13y^{1/2n} = 14.$

273. $y^{2/n} - 5y^{1/n} = -6.$

274. $y^{2n} + 4y^{n} = 5.$

275. $3y^{4n/3} + 2y^{2n/3} = 16.$

276. $(z-3)^2 + 2(z-3) = 3.$

277. $(z+2)^2 + z + 2 = 20.$

278. $\sqrt{z^2+7} + z^2 + 7 = 20.$

279. $z + 5 - \sqrt{z+5} = 6.$

280. $(z^2+1)^2 + 4(z^2+1) = 140.$

281. $z + 3 + \sqrt[4]{z+3} = 6.$

282. $(2z+6)^{\frac{1}{2}} + (2z+6)^{\frac{1}{4}} = 6.$

283. $\left(z + \frac{1}{z}\right)^2 + 4\left(z + \frac{1}{z}\right) = 12.$

284. $\left(\frac{12}{z} - 1\right)^2 + 8\left(\frac{12}{z} - 1\right) = 33.$

285. $(z^3+17)^{\frac{1}{2}} - 35(z^3+17)^{\frac{1}{4}} = -216.$

286. $(2z+5)^{-5} + 31(2z+5)^{-\frac{5}{2}} = 32.$

287. $x^2 - 3x + 6 + 2\sqrt{x^2 - 3x + 6} = 24.$

288. $x^2 + 2x + 3 = \sqrt{x^2 + 2x + 9}.$

289. $x^2 + 6\sqrt{x^2 - 2x + 5} = 2x + 11.$

290. $\sqrt{2x^2 - 3x + 5} + 2x^2 - 3x = 1.$

291. $2x^2 - 4x + 3\sqrt{x^2 - 2x + 6} = 15.$

292. $2x^2 - 2x + 6\sqrt{x^2 - x + 7} = 22.$

293. $2x^2 + 7x - 31 = x + \sqrt{x^2 + 3x + 7}.$

294. $2x^2 - x + 14 - 11\sqrt{2x^2 - 3x + 16} + 30 = 2(x - 1)$.

295. $\sqrt{3x^2 - 7x + 9} + 3(x^2 + 3) = 7(x + 8)$.

296. $x^2 - 3x + 7\sqrt{11x - 2x^2 + 2} = \frac{5x}{2} + 21$.

297. $9x - 4x^2 + (4x^2 - 9x + 11)^{\frac{1}{2}} = 5$.

298. $5x - 7x^2 + 8\sqrt{7x^2 - 5x + 1} = 8$.

299. $\frac{w^2 + w}{2} + \frac{2}{w^2 + w} = 2$.

300. $\sqrt{\frac{3w - 4}{w - 5}} + \sqrt{\frac{w - 5}{3w - 4}} = \frac{5}{2}$.

301. $\frac{w^2 + 1}{w - 1} - \frac{4(w - 1)}{w^2 + 1} = \frac{21}{5}$.

302. $\frac{w^2 - c^2}{w^2 + c^2} + \frac{w^2 + c^2}{w^2 - c^2} = \frac{34}{15}$.

303. $\frac{a + w}{a - w} - \frac{a - w}{a + w} = \frac{35}{6}$.

304. $w^2 - 2w + \frac{6}{w^2 - 2w - 6} = 11$.

305. $\frac{w^2 + w + 1}{w^2 - w + 2} + \frac{w^2 - w + 2}{w^2 + w + 1} = \frac{13}{6}$.

306. $(x^2 + x + 1)(x^2 + x + 2) = 12$.

307. $(x^2 + x + 3)(x^2 + x + 5) = 35$.

308. $(x^2 + 3x - 4)(x^2 + 3x + 2) + 8 = 0$.

309. $\frac{1}{x^2 + x - 4} + \frac{1}{x^2 + x - 6} - \frac{1}{x^2 + x - 8} = 0$.

310. $2^{x+1} + 4^x = 8$.

311. $2^{2x} - 2^{x+2} = 32$.

312. $3^{x+2} - 9^x + 486 = 0$.

313. $4^x - 3 \cdot 2^{x+3} + 80 = 0$.

314. $3^{2x} - 2 \cdot 3^{x+2} + 81 = 0.$

315. $2^x \cdot 2^{x^2+1} = 2.$

316. $2 \cdot 4^{3\sqrt{x}} = 2^{3x-6}.$

317. $\left(\frac{4^{3x/2}}{8}\right)^x = 2^{3x+9}.$

318. $\frac{a^{88x^2} \cdot a^{13x}}{3} = 1.$

319. $a^{35x^2} \cdot (a^2)^{37x} = \frac{1}{a^{35}}.$

320. $x\sqrt{\frac{6}{x} - x} = \frac{1 + x^2}{\sqrt{x}}.$

321. $(ax^{2n} + bx^n + c)^{2m} \pm (ex^n + f)^{2m} = 0.$

322. $(ax^{2n} + bx^n + c)^{2m} \pm (dx^{2n} + ex^n)^{2m} = 0.$

323. $(4a^3 - 9cd^2)x^3 + (4a^2c^2 + 4abd^2)x + (ac^2 + bd^2)^2 = 0.$

324. $\frac{x^2 + 2x + 2}{x + 1} + \frac{x^2 + 8x + 20}{x + 4} = \frac{x^2 + 4x + 6}{x + 2} + \frac{x^2 + 6x + 12}{x + 3}.$

325. $\frac{7}{\sqrt{x-6} + 4} + \frac{12}{\sqrt{x-6} + 9} + \frac{1}{\sqrt{x-6} - 4} + \frac{6}{\sqrt{x-6} - 9} = 0.$

326. $\frac{123 + 41\sqrt{x}}{5\sqrt{x} - x} = \frac{4(5\sqrt{x} + x)}{3 - \sqrt{x}} - \frac{2x^2}{(5\sqrt{x} - x)(3 - \sqrt{x})}.$

327. $32a^{2m}c^{n-1} + 4a^{m+3}c^{n-1}(ac^3 - 2)x = a^7c^{n+2}x^2.$

328. $\frac{\sqrt{x^2+1} + \sqrt{x^2-1}}{\sqrt{x^2+1} - \sqrt{x^2-1}} + \frac{\sqrt{x^2+1} - \sqrt{x^2-1}}{\sqrt{x^2+1} + \sqrt{x^2-1}} = 4\sqrt{x^2-1}.$

329. $\frac{1}{5}\left\{\frac{(x+1)(x-3)}{(x+2)(x-4)}\right\} + \frac{1}{9}\left\{\frac{(x+3)(x-5)}{(x+4)(x-6)}\right\} - \frac{2}{13}\left\{\frac{(x+5)(x-7)}{(x+6)(x-8)}\right\} = \frac{92}{585}.$

330. $x^4\left(1+\frac{1}{3x}\right)^2-(3x^2+x)=70.$

331. $x^2+x+1+\frac{1}{x}+\frac{1}{x^2}=0.$

332. $x-3=\frac{3+4\sqrt{x}}{x}.$

333. $\sqrt{5x^2-2x+6}-\sqrt{5x^2+2x-3}=1.$

334. $\frac{x}{14}-\frac{30}{7x^2}+\frac{12+\frac{1}{2}x}{3x}=\frac{7}{2x^2}+\frac{5}{x}.$

Simultaneous Equations.

335. $2x^2-3y=20,\ x^2+5y=36.$

336. $x^2+y^2=625,\ x^2-y^2=175.$

337. $x^2+xy=6,\ x+y=3.$

338. $x-y=4,\ x^2-y^2=32.$

339. $x+y=6,\ x^2-y^2=24.$

340. $x^2-y^2=-\frac{1}{4},\ x+y=\frac{5}{2}.$

341. $\frac{x}{y}=2,\ xy=8.$

342. $x+xy=15,\ xy-x=8.$

343. $x^2-xy=25,\ x-y=5.$

344. $x^2+y^2=a,\ x^2-y^2=c.$

345. $xy=54,\ 3x=2y.$

346. $x+y=13,\ xy=12.$

347. $x+y=\frac{7}{6},\ xy=\frac{1}{3}.$

348. $x+y=-1,\ x^3+y^3=25.$

349. $xy=8,\ 3x-y=10.$

350. $4xy=12,\ 4x+y=13.$

351. $x-2y=2,\ x^2+4y^2=100.$

352. $x^2-y^2=16,\ 2x+y=13.$

353. $x+y=5,\ x^2y+xy^2=30.$

354. $x^2+xy=-6,\ xy+y^2=15.$

355. $x^2 + xy = 20,\ xy + y^2 = 5.$

356. $x(x+y) = x,\ y(x-y) = -1.$

357. $x - y = a,\ xy = b^2.$ 358. $x + y = a,\ xy = c^2.$

359. $x + y = p,\ x^2 + y^2 = q^2.$

360. $x^2 - xy = 3,\ y^2 - xy = -2.$

361. $49x^2 - 9y^2 = 187,\ 21x - 9y = 51.$

362. $xy = 80,\ \frac{x}{y} = 5.$ 363. $x - y = 11,\ \frac{6}{x} = \frac{y}{10}.$

364. $\frac{x^2}{2} + 3y^2 = 140,\ 5x^2 - \frac{y^2}{3} = 308.$

365. $\frac{x}{3} + \frac{y}{4} = 5,\ \frac{3}{x} + \frac{4}{y} = \frac{5}{6}.$ 366. $\frac{1}{x} + \frac{1}{y} = \frac{1}{2},\ \frac{1}{x^2} + \frac{1}{y^2} = \frac{5}{36}.$

367. $\frac{x}{y} + \frac{y}{x} = 2.9,\ 2x - 3y = 4.$

368. $\frac{1}{6}x^2 + \frac{1}{4}y^2 = 60,\ \frac{1}{6}x + \frac{1}{4}y = 5.$

369. $\frac{1}{16}x^2 + y^2 = 40,\ \frac{1}{2}x - y = 2.$

370. $.01x + .5y = 2,\ .65x - .25y = 3.$

371. $x^2y^{-1} + y^2x^{-1} = 9,\ x^{-1} + y^{-1} = \frac{3}{4}.$

372. $.1y + .125x = y - x,\ y - .5x = .75xy - 3x.$

373. $.3x + .125y = 3x - y,\ 3x - .5y = 2.25xy + 3y.$

374. $x^2 + xy + y^2 = 13x,\ x^2 - xy + y^2 = 7x.$

375. $x^4 - y^4 = 369,\ x^2 - y^2 = 9.$

376. $x^4 + y^4 = x,\ x^2 + y^2 = 1.$ 377. $x^3 - y^3 = 28,\ x - y = 2.$

378. $x^3 + y^3 = 65,\ x + y = 5.$

379. $2x^3 - 3y^3 = 1512,\ 3x = 2y + 6.$

380. $x + y = 4,\ x^5 - y^5 = (x - y)^5.$

381. $x - y = 37,\ x^{\frac{1}{2}} - y^{\frac{1}{2}} = 1.$

382. $x^2 + y^2 = 91,\ x^2 - xy + y^2 = 13.$

383. $x^2 + xy + y^2 = 175,\ x^2 - y^2 = 875.$

384. $x^{\frac{1}{2}} + y^{\frac{1}{2}} = 6,\ x^{\frac{3}{2}} + y^{\frac{3}{2}} = 126.$

385. $x^{\frac{1}{2}} + y^{\frac{1}{2}} = 5,\ x^{\frac{3}{2}} + y^{\frac{3}{2}} = 13.$

386. $\frac{1}{x^2} + \frac{1}{y^2} = 91,\ \frac{1}{x} + \frac{1}{y} = 7.$

387. $x^4 + y^2 = 25,\ x^2 + y = 7.$

388. $x^4 + y^4 = 97,\ x + y = 1.$

389. $x^4 + y^4 = 272,\ x - y = 2.$

390. $16x^4 - 81y^4 = -2465,\ 2x - 3y = -1.$

391. $x^5 - 32y^5 = 211,\ x - 2y = 1.$

392. $x^5 - y^5 = 242,\ x - y = 2.$

393. $x^5 - y^5 = \frac{211}{32},\ x - y = \frac{1}{2}.$

394. $\sqrt[3]{x} + \sqrt[3]{y} = 3,\ \sqrt[3]{xy} = 2.$

395. $x^3 + y^3 = 152,\ x^2y + xy^2 = 120.$

396. $x^3 - y^3 = 335,\ xy^2 - x^2y = -70.$

397. $xy^2 + xy = 24,\ xy^3 + x = 56.$

398. $x - y = \sqrt{x} + \sqrt{y},\ x^{\frac{3}{2}} - y^{\frac{3}{2}} = 61.$

399. $x^2 - xy + y^2 = 7,\ x^4 + x^2y^2 + y^4 = 133.$

400. $x + \sqrt{xy} + y = 14,\ x^2 + xy + y^2 = 84.$

401. $x^2 + xy + y^2 = 27,\ x^2y^2 + xy = 1806.$

402. $x^2 - xy = a^2 + b^2,\ xy - y^2 = 2ab.$

403. $x^2 + xy = a^2 + ab,\ y^2 + yx = b^2 + ab.$

404. $x^2 + xy = 4x - 2$, $y^2 + xy = 4y - 1$.

405. $x^3 - y^3 = 24$, $x^3 - x^2y - xy^2 + y^3 = 96$.

406. $x^3 - y^3 = 3c^2 + 3c + 1$, $x - y = 1$.

407. $x + y + x^2 + y^2 = 162$, $x - y + x^2 - y^2 = -102$.

408. $(x^2 + y^2)(x + y) = 671$, $(x^2 - y^2)(x - y) = 11$.

409. $x^2 = x + y$, $y^2 = 3y - x$.

410. $(x^2 - y^2)(x + y) = 200$, $x^2 - y^2 + x + y = 30$.

411. $\frac{x^2}{y^2} + \frac{y^2}{x^2} + \frac{x}{y} + \frac{y}{x} = \frac{27}{4}$, $x^2 + y^2 = 20$.

412. $(x^2 - xy + y^2)\sqrt{x^2 + y^2} = 65$, $(x^2 + xy + y^2)\sqrt{x^2 + y^2} = 185$.

413. $x^2 + xy + y^2 = 7a^2 - 13ab + 7b^2$, $x^2 - xy + y^2 = 3a^2 - 3ab + 3b^2$.

414. $(x - 7)^4 + (y - 3)^4 = 257$, $x - y + 1 = 0$.

415. $\sqrt{x^2 - y^2} - \sqrt{x^2 + y^2} = -2$, $\sqrt{x + y} - \sqrt{x - y} = 1.5$.

416. $x^2 + xy = 3y^2 - xy$, $4x^2 + 5y^2 = 9$.

417. $5y^2 + 2xy = 16$, $3xy + 10 = 4x^2$.

418. $3x^2 + 2xy + y^2 = 4x^2 - xy - 3y^2 = 8$.

419. $53x^2 - 128xy + 64y^2 = 5 = 26x^2 - 62xy + 32y^2$.

420. $2x^2 - 3xy + 5y^2 = 64$, $3x^2 + xy + y^2 = 36$.

MISCELLANEOUS.

421. $\frac{x + y}{x - y} + \frac{x - y}{x + y} = \frac{10}{3}$, $x^2 + y^2 = 20$.

422. $(3x + 4y)(7x - 2y) - 7x + 2y = 136$,
$(3x + 4y)(7x - 2y) + 3x + 4y = 162$.

423. $\sqrt{\frac{3x}{x + y}} + \sqrt{\frac{x + y}{3x}} = 2$, $xy - (x + y) = 54$.

424. $x + \sqrt{x} + y + \sqrt{y} = 18, (x + \sqrt{x})(y + \sqrt{y}) = 72.$

425. $x^2 - 2xy + 3y^2 + 5y - 4x = 33,\ 6xy - 12y - 9y^2 + 4x - x^2 = -45.$

426. $(x + y)xy = 70,\ (x^2 + y^2)x^2y^2 = 2900.$

427. $x - y + \sqrt{\frac{x-y}{x+y}} = \frac{30}{x+y},\ x^2 + y^2 = 313.$

428. $x^2 + \frac{x^4}{y^2} + y^2 = 91,\ x + \frac{x^2}{y} + y = 12.$

429. $x + y = 1072,\ x^{\frac{1}{2}} + y^{\frac{1}{2}} = 6.$

430. $x + y = 4,\ 41(x^5 + y^5) = 122(x^4 + y^4).$

431. $(x^2 - xy + y^2)(x^2 + y^2) = 91,\ (x^4 + x^2y^2 + y^4) = 133.$

432. $(x + y)(2x^2 + 2y^2) = 15xy,\ (4x^4 - 4y^4)(x^2 - y^2) = 45x^2y^2.$

433. $x^3 + xy^2 = ay,\ x^2y + y^3 = bx.$

434. $\frac{\sqrt[3]{x+y} - \sqrt[3]{x-y}}{\sqrt[3]{x+y} + \sqrt[3]{x-y}} = 3,\ x^2 + y^2 = 75.$

435. $\frac{y}{x} + \frac{3x}{x+y} = \frac{x^2 - y^2}{y},\ \frac{x}{y} - \frac{x+y}{x} = \frac{y}{x}.$

436. $(x + y)(xy + 1) = 18xy,\ (x^2 + y^2)(x^2y^2 + 1) = 208x^2y^2.$

437. $\sqrt{y} - \sqrt{a - x} = \sqrt{y - x},\ 2\sqrt{y - x} + 2\sqrt{a - x} = 5\sqrt{a - x}.$

438. $x + y = 2,\ 13(x^5 + y^5) = 121(x^3 + y^3).$

439. $x^2 - b^2 = y^2 - a^2,\ (x + y)^2 = y^2 + b^2 - (a - b)^2.$

440. $x^m y^n = (\tfrac{3}{2})^{m-n},\ x^n y^m = (\tfrac{2}{3})^{m-n}.$

441. $(x^2 + y^2)(x^3 + y^3) = 45,\ x + y = 3.$

442. $x + \frac{2}{3}y = 11,\ x^3 + 2x^2y + \frac{4}{3}xy^2 + \frac{8}{27}y^3 = 1331.$

443. $\sqrt{5\sqrt{x} + 5\sqrt{y}} + \sqrt{y} = 10 - \sqrt{x},\ \sqrt{x^5} + \sqrt{y^5} = 275.$

444. $\frac{a}{y+4b}=\frac{2b}{x-y}$, $\frac{1}{(b-a)x}=\frac{3}{(a+b)y}-\frac{1}{a^2-b^2}$.

445. $x^2+y=19$, $x+y^2=13$.

446. $\frac{5x^3}{3y}-10x+\frac{20x}{3y}=\frac{1600}{3xy}-\frac{15xy-20x}{x}$, $x^2-y^2=16$.

447. $(x+y)^3+x+y=30$, $x-y=1$.

448. $xy=48$, $xz=72$, $yz=96$.

449. $xyz=105$, $35x=3yz$, $7xy=15z$.

450. $yz=a^2$, $xz=b^2$, $xy=c^2$.

451. $x+y+z=20$, $xy+xz=75$, $yz=56$.

452. $x(x+y+z)=54$, $y(x+y+z)=108$, $z(x+y+z)=162$.

453. $x+y+z=k$, $x^2+y^2+z^2=k^2$, $x^3+y^3+z^3=k^3$.

454. $(x+z)(y-z)=22$, $(x-y)(x+z)=33$,
$(x-y)(y-z)=6$.

455. $x\sqrt{y}=a$, $x\sqrt{z}=b$, $y\sqrt{z}-c$.

456. $2x+2y=3xy$, $3y+2z=2yz$, $3x=4xz-5z$.

457. $xy^2z^3=108$, $yz^3=18x$, $2z=3x^2y$.

458. $xy\div z=8$, $xz\div y=18$, $yz\div x=\frac{8}{2}$.

459. $\frac{xyz}{x+y}=-8$; $\frac{xyz}{y+z}=24$; $\frac{xyz}{x+z}=12$.

460. $(2+x)(z-3)=30$, $(5-y)z=6$, $xy=32$.

461. $x(y+z)=90$, $y(x+z)=84$, $z(x+y)=48$.

462. $\frac{x^2+y^2}{xyz}=\frac{5}{6}$, $\frac{x^2+z^2}{xyz}=\frac{5}{3}$, $\frac{y^2+z^2}{xyz}=\frac{13}{6}$.

463. $x+y-2z=-9$, $3x+2y+z=9$, $x^2+y^2+z^2=30$.

464. $\frac{y+z}{x} = \frac{x+z}{b} = \frac{x+y}{c} = 2xy.$

465. $yz = np,\ nx + my = mn,\ px + mz = mp.$

466. $x + y + z = \frac{7}{2},\ \frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{7}{2},\ xyz = 1.$

467. $x^2 + y^2 + z^2 = \frac{449}{144},\ x + y + z = \frac{35}{12},\ 2y^2 = 9xz.$

468. $xy = a(x + y),\ xz = b(x + z),\ yz = c(y + z).$

469. $xy + 3xz - 5yz = 37,\ 2xy - 6xz - 7yz = 50,$
$5xy - 13xz - 20yz = 121.$

470. $x^2 - yz = c,\ y^2 - xz = d,\ z^2 - xy = e.$

471. $xyz = 105,\ xyv = 135,\ xzv = 189,\ yzv = 315.$

472. $xy = z,\ yz = v,\ xv = a,\ yv = bx.$

473. $x^2y^2z^2w = 12,\ x^2y^2zw^2 = 8,\ x^2yz^2w^2 = 1,\ xy^2z^2w^2 = \frac{4}{3}.$

Problems Involving Simultaneous Quadratics.

474. The sum of the squares of two numbers is 58, and the difference of their squares is 40. What are the numbers?

475. The difference of two numbers is 8 and the difference of their squares is 144. What are the numbers?

476. The sum of two numbers is 19 and the sum of their squares is 205. What are the numbers?

477. The sum of two numbers is 16 and the difference of their squares is 96. What are the numbers?

478. Find two numbers whose sum is 32 and whose product is 192.

479. The product of two numbers is 750, and their quotient is $3\frac{1}{3}$. Find them.

480. The sum of two numbers is 52 and the square root of their product is 24. What are the numbers?

481. Divide 20 into two parts such that the square of their product shall be 9,216.

482. The difference of two numbers is 405, and the difference of their square roots is 9. What are the numbers?

483. The sum of two numbers is a, and the sum of their reciprocals is b. What are the numbers?

484. On what day of a year which is not a leap year are the "days past" and the "days to come" of a calendar consecutive squares? Note that the "days past" for any given date include that day.

485. The sum of two numbers is 8, and the sum of their cubes is 152. Find them.

486. The sum of the cubes of two numbers is 35, and the sum of their ninth powers is 20,195. What are the numbers?

487. The sum of the fourth powers of two numbers is 1,921, and the sum of their squares is 61. Required the numbers.

488. The sum of two numbers is 7 and the sum of their fourth powers is 641. What are the numbers?

489. The sum of two numbers is 280, and the sum of their cube roots is 10. What are the numbers?

490. The product of the sum and difference of two numbers is a, and the quotient of the sum divided by their difference is b. What are the numbers?

491. Find two numbers whose sum, whose product, and the difference of whose squares are all equal.

294. $2x^2 - x + 14 - 11\sqrt{2x^2 - 3x + 16} + 30 = 2(x - 1)$.

295. $\sqrt{3x^2 - 7x + 9} + 3(x^2 + 3) = 7(x + 8)$.

296. $x^2 - 3x + 7\sqrt{11x - 2x^2 + 2} = \frac{5x}{2} + 21$.

297. $9x - 4x^2 + (4x^2 - 9x + 11)^{\frac{1}{2}} = 5$.

298. $5x - 7x^2 + 8\sqrt{7x^2 - 5x + 1} = 8$.

299. $\frac{w^2 + w}{2} + \frac{2}{w^2 + w} = 2$.

300. $\sqrt{\frac{3w - 4}{w - 5}} + \sqrt{\frac{w - 5}{3w - 4}} = \frac{5}{2}$.

301. $\frac{w^2 + 1}{w - 1} - \frac{4(w - 1)}{w^2 + 1} = \frac{21}{5}$.

302. $\frac{w^2 - c^2}{w^2 + c^2} + \frac{w^2 + c^2}{w^2 - c^2} = \frac{34}{15}$.

303. $\frac{a + w}{a - w} - \frac{a - w}{a + w} = \frac{35}{6}$.

304. $w^2 - 2w + \frac{6}{w^2 - 2w - 6} = 11$.

305. $\frac{w^2 + w + 1}{w^2 - w + 2} + \frac{w^2 - w + 2}{w^2 + w + 1} = \frac{13}{6}$.

306. $(x^2 + x + 1)(x^2 + x + 2) = 12$.

307. $(x^2 + x + 3)(x^2 + x + 5) = 35$.

308. $(x^2 + 3x - 4)(x^2 + 3x + 2) + 8 = 0$.

309. $\frac{1}{x^2 + x - 4} + \frac{1}{x^2 + x - 6} - \frac{1}{x^2 + x - 8} = 0$.

310. $2^{x+1} + 4^x = 8$.

311. $2^{2x} - 2^{x+2} = 32$.

312. $3^{x+2} - 9^x + 486 = 0$.

313. $4^x - 3 \cdot 2^{x+3} + 80 = 0$.

314. $3^{2x} - 2 \cdot 3^{x+2} + 81 = 0.$

315. $2^x \cdot 2^{x^2+1} = 2.$

316. $2 \cdot 4^{3\sqrt{x}} = 2^{3x-8}.$

317. $\left(\frac{4^{3x/2}}{8}\right)^x = 2^{3x+9}.$

318. $\frac{a^{88x^2} \cdot a^{13x}}{3} = 1.$

319. $a^{35x^2} \cdot (a^2)^{37x} = \frac{1}{a^{35}}.$

320. $x\sqrt{\frac{6}{x} - x} = \frac{1 + x^2}{\sqrt{x}}.$

321. $(ax^{2n} + bx^n + c)^{2m} \pm (ex^n + f)^{2m} = 0.$

322. $(ax^{2n} + bx^n + c)^{2m} \pm (dx^{2n} + ex^n)^{2m} = 0.$

323. $(4a^2 - 9cd^2)x^2 + (4a^2c^2 + 4abd^2)x + (ac^2 + bd^2)^2 = 0.$

324. $\frac{x^2 + 2x + 2}{x + 1} + \frac{x^2 + 8x + 20}{x + 4} = \frac{x^2 + 4x + 6}{x + 2} + \frac{x^2 + 6x + 12}{x + 3}.$

325. $\frac{7}{\sqrt{x-6} + 4} + \frac{12}{\sqrt{x-6} + 9} + \frac{1}{\sqrt{x-6} - 4} + \frac{6}{\sqrt{x-6} - 9} = 0.$

326. $\frac{123 + 41\sqrt{x}}{5\sqrt{x} - x} = \frac{4(5\sqrt{x} + x)}{3 - \sqrt{x}} - \frac{2x^2}{(5\sqrt{x} - x)(3 - \sqrt{x})}.$

327. $32a^{2m}c^{m-1} + 4a^{m+3}c^{m-1}(ac^3 - 2)x = a^7c^{m+2}x^2.$

328. $\frac{\sqrt{x^2+1} + \sqrt{x^2-1}}{\sqrt{x^2+1} - \sqrt{x^2-1}} + \frac{\sqrt{x^2+1} - \sqrt{x^2-1}}{\sqrt{x^2+1} + \sqrt{x^2-1}} = 4\sqrt{x^2-1}.$

329. $\frac{1}{5}\left\{\frac{(x+1)(x-3)}{(x+2)(x-4)}\right\} + \frac{1}{9}\left\{\frac{(x+3)(x-5)}{(x+4)(x-6)}\right\} - \frac{2}{13}\left\{\frac{(x+5)(x-7)}{(x+6)(x-8)}\right\} = \frac{92}{585}.$

330. $x^4\left(1+\frac{1}{3x}\right)^2-(3x^2+x)=70.$

331. $x^3+x+1+\frac{1}{x}+\frac{1}{x^3}=0.$

332. $x-3=\frac{3+4\sqrt{x}}{x}.$

333. $\sqrt{5x^2-2x+6}-\sqrt{5x^2+2x-3}=1.$

334. $\frac{x}{14}-\frac{30}{7x^2}+\frac{12+\frac{1}{2}x}{3x}=\frac{7}{2x^2}+\frac{5}{x}.$

Simultaneous Equations.

335. $2x^2-3y=20,\ x^2+5y=36.$

336. $x^2+y^2=625,\ x^2-y^2=175.$

337. $x^2+xy=6,\ x+y=3.$

338. $x-y=4,\ x^2-y^2=32.$

339. $x+y=6,\ x^2-y^2=24.$

340. $x^2-y^2=-\frac{1}{4},\ x+y=\frac{5}{2}.$

341. $\frac{x}{y}=2,\ xy=8.$

342. $x+xy=15,\ xy-x=8.$

343. $x^2-xy=25,\ x-y=5.$

344. $x^2+y^2=a,\ x^2-y^2=c.$

345. $xy=54,\ 3x=2y.$

346. $x+y=13,\ xy=12.$

347. $x+y=\frac{7}{6},\ xy=\frac{1}{3}.$

348. $x+y=-1,\ x^3+y^3=25.$

349. $xy=8,\ 3x-y=10.$

350. $4xy=12,\ 4x+y=13.$

351. $x-2y=2,\ x^3+4y^3=100.$

352. $x^3-y^3=16,\ 2x+y=13.$

353. $x+y=5,\ x^2y+xy^2=30.$

354. $x^3+xy=-6,\ xy+y^2=15.$

355. $x^2 + xy = 20,\ xy + y^2 = 5.$

356. $x(x + y) = x,\ y(x - y) = -1.$

357. $x - y = a,\ xy = b^2.$ 358. $x + y = a,\ xy = c^2.$

359. $x + y = p,\ x^2 + y^2 = q^2.$

360. $x^2 - xy = 3,\ y^2 - xy = -2.$

361. $49x^2 - 9y^2 = 187,\ 21x - 9y = 51.$

362. $xy = 80,\ \frac{x}{y} = 5.$ 363. $x - y = 11,\ \frac{6}{x} = \frac{y}{10}.$

364. $\frac{x^2}{2} + 3y^2 = 140,\ 5x^2 - \frac{y^2}{3} = 308.$

365. $\frac{x}{3} + \frac{y}{4} = 5,\ \frac{3}{x} + \frac{4}{y} = \frac{5}{6}.$ 366. $\frac{1}{x} + \frac{1}{y} = \frac{1}{2},\ \frac{1}{x^2} + \frac{1}{y^2} = \frac{5}{36}.$

367. $\frac{x}{y} + \frac{y}{x} = 2.9,\ 2x - 3y = 4.$

368. $\frac{1}{6}x^2 + \frac{1}{4}y^2 = 60,\ \frac{1}{6}x + \frac{1}{4}y = 5.$

369. $\frac{1}{16}x^2 + y^2 = 40,\ \frac{1}{2}x - y = 2.$

370. $.01x + .5y = 2,\ .65x - .25y = 3.$

371. $x^2y^{-1} + y^2x^{-1} = 9,\ x^{-1} + y^{-1} = \frac{3}{4}.$

372. $.1y + .125x = y - x,\ y - .5x = .75xy - 3x.$

373. $.3x + .125y = 3x - y,\ 3x - .5y = 2.25xy + 3y.$

374. $x^2 + xy + y^2 = 13x,\ x^2 - xy + y^2 = 7x.$

375. $x^4 - y^4 = 369,\ x^2 - y^2 = 9.$

376. $x^4 + y^4 = x,\ x^3 + y^3 = 1.$ 377. $x^3 - y^3 = 26,\ x - y = 2.$

378. $x^3 + y^3 = 65,\ x + y = 5.$

379. $27x^3 - 8y^3 = 1512,\ 3x = 2y + 6.$

380. $x + y = 4,\ x^5 + y^5 = (x + y)^5.$

381. $x - y = 37,\ x^{\frac{1}{3}} - y^{\frac{1}{3}} = 1.$

382. $x^2 + y^2 = 91,\ x^2 - xy + y^2 = 13.$

383. $x^2 + xy + y^2 = 175,\ x^3 - y^3 = 875.$

384. $x^{\frac{1}{2}} + y^{\frac{1}{2}} = 6,\ x^{\frac{3}{2}} + y^{\frac{3}{2}} = 126.$

385. $x^{\frac{1}{2}} + y^{\frac{1}{2}} = 5,\ x^{\frac{3}{2}} + y^{\frac{3}{2}} = 13.$

386. $\frac{1}{x^2} + \frac{1}{y^2} = 91,\ \frac{1}{x} + \frac{1}{y} = 7.$

387. $x^4 + y^2 = 25,\ x^2 + y = 7.$

388. $x^4 + y^4 = 97,\ x + y = 1.$

389. $x^4 + y^4 = 272,\ x - y = 2.$

390. $16x^4 - 81y^4 = -2465,\ 2x - 3y = -1.$

391. $x^5 - 32y^5 = 211,\ x - 2y = 1.$

392. $x^5 - y^5 = 242,\ x - y = 2.$

393. $x^5 - y^5 = \frac{211}{32},\ x - y = \frac{1}{2}.$

394. $\sqrt[3]{x} + \sqrt[3]{y} = 3,\ \sqrt[3]{xy} = 2.$

395. $x^3 + y^3 = 152,\ x^2y + xy^2 = 120.$

396. $x^3 - y^3 = 335,\ xy^2 - x^2y = -70.$

397. $xy^3 + xy = 24,\ xy^3 + x = 56.$

398. $x - y = \sqrt{x} + \sqrt{y},\ x^{\frac{3}{2}} - y^{\frac{3}{2}} = 61.$

399. $x^2 - xy + y^2 = 7,\ x^4 + x^2y^2 + y^4 = 133.$

400. $x + \sqrt{xy} + y = 14,\ x^2 + xy + y^2 = 84.$

401. $x^2 + xy + y^2 = 27,\ x^2y^2 + xy = 1806.$

402. $x^2 - xy = a^2 + b^2,\ xy - y^2 = 2ab.$

403. $x^2 + xy = a^2 + ab,\ y^2 + yx = b^2 + ab.$

404. $x^2 + xy = 4x - 2,\ y^2 + xy = 4y - 1.$

405. $x^3 - y^3 = 24,\ x^3 - x^2y - xy^2 + y^3 = 96.$

406. $x^3 - y^3 = 3c^2 + 3c + 1,\ x - y = 1.$

407. $x + y + x^2 + y^2 = 162,\ x - y + x^2 - y^2 = -102.$

408. $(x^2 + y^2)(x + y) = 671,\ (x^2 - y^2)(x - y) = 11.$

409. $x^2 = x + y,\ y^2 = 3y - x.$

410. $(x^2 - y^2)(x + y) = 200,\ x^2 - y^2 + x + y = 30.$

411. $\dfrac{x^2}{y^2} + \dfrac{y^2}{x^2} + \dfrac{x}{y} + \dfrac{y}{x} = \dfrac{27}{4},\ x^2 + y^2 = 20.$

412. $(x^2 - xy + y^2)\sqrt{x^2 + y^2} = 65,\ (x^2 + xy + y^2)\sqrt{x^2 + y^2} = 185.$

413. $x^2 + xy + y^2 = 7a^2 - 13ab + 7b^2,\ x^2 - xy + y^2 = 3a^2 - 3ab + 3b^2.$

414. $(x - 7)^4 + (y - 3)^4 = 257,\ x - y + 1 = 0.$

415. $\sqrt{x^2 - y^2} - \sqrt{x^2 + y^2} = -2,\ \sqrt{x + y} - \sqrt{x - y} = 1.5.$

416. $x^2 + xy = 3y^2 - xy,\ 4x^2 + 5y^2 = 9.$

417. $5y^2 + 2xy = 16,\ 3xy + 10 = 4x^2.$

418. $3x^2 + 2xy + y^2 = 4x^2 - xy - 3y^2 = 8.$

419. $53x^2 - 128xy + 64y^2 = 5 = 26x^2 - 62xy + 32y^2.$

420. $2x^2 - 3xy + 5y^2 = 64,\ 3x^2 + xy + y^2 = 36.$

MISCELLANEOUS.

421. $\dfrac{x + y}{x - y} + \dfrac{x - y}{x + y} = \dfrac{10}{3},\ x^2 + y^2 = 20.$

422. $(3x + 4y)(7x - 2y) - 7x + 2y = 136,$
$(3x + 4y)(7x - 2y) + 3x + 4y = 162.$

423. $\sqrt{\dfrac{3x}{x + y}} + \sqrt{\dfrac{x + y}{3x}} = 2,\ xy - (x + y) = 54.$

424. $x + \sqrt{x} + y + \sqrt{y} = 18, (x + \sqrt{x})(y + \sqrt{y}) = 72.$

425. $x^2 - 2xy + 3y^2 + 5y - 4x = 33, 6xy - 12y - 9y^2 + 4x - x^2 = -45.$

426. $(x + y)xy = 70, (x^2 + y^2)x^2y^2 = 2900.$

427. $x - y + \sqrt{\frac{x-y}{x+y}} = \frac{30}{x+y}, x^2 + y^2 = 313.$

428. $x^2 + \frac{x^4}{y^2} + y^2 = 91, x + \frac{x^2}{y} + y = 12.$

429. $x + y = 1072, x^{\frac{1}{2}} + y^{\frac{1}{2}} = 6.$

430. $x + y = 4, 41(x^5 + y^5) = 122(x^4 + y^4).$

431. $(x^2 - xy + y^2)(x^2 + y^2) = 91, (x^4 + x^2y^2 + y^4) = 133.$

432. $(x + y)(2x^2 + 2y^2) = 15xy, (4x^4 - 4y^4)(x^2 - y^2) = 45x^2y^2.$

433. $x^2 + xy^2 = ay, x^2y + y^2 = bx.$

434. $\frac{\sqrt[3]{x+y} - \sqrt[3]{x-y}}{\sqrt[3]{x+y} + \sqrt[3]{x-y}} = 3, x^2 + y^2 = 75.$

435. $\frac{y}{x} + \frac{3x}{x+y} = \frac{x^2 - y^2}{y}, \frac{x}{y} - \frac{x+y}{x} = \frac{y}{x}.$

436. $(x + y)(xy + 1) = 18xy, (x^2 + y^2)(x^2y^2 + 1) = 208x^2y^2.$

437. $\sqrt{y} - \sqrt{a - x} = \sqrt{y - x}, 2\sqrt{y - x} + 2\sqrt{a - x} = 5\sqrt{a - x}.$

438. $x + y = 2, 13(x^5 + y^5) = 121(x^3 + y^3).$

439. $x^2 - b^2 = y^2 - a^2, (x + y)^2 = y^2 + b^2 - (a - b)^2.$

440. $x^m y^n = (\frac{3}{2})^{m-n}, x^n y^m = (\frac{2}{3})^{m-n}.$

441. $(x^3 + y^3)(x^5 + y^5) = 45, x + y = 3.$

442. $x + \frac{2}{3}y = 11, x^3 + 2x^2y + \frac{4}{3}xy^2 + \frac{8}{27}y^3 = 1331.$

443. $\sqrt{5\sqrt{x} + 5\sqrt{y}} + \sqrt{y} = 10 - \sqrt{x}, \sqrt{x^5} + \sqrt{y^5} = 275.$

444. $\frac{a}{y+4b}=\frac{2b}{x-y}$, $\frac{1}{(b-a)x}=\frac{3}{(a+b)y}-\frac{1}{a^2-b^2}$.

445. $x^2+y=19$, $x+y^2=13$.

446. $\frac{5x^3}{3y}-10x+\frac{20x}{3y}=\frac{1600}{3xy}-\frac{15xy-20x}{x}$, $x^2-y^2=16$.

447. $(x+y)^3+x+y=30$, $x-y=1$.

448. $xy=48$, $xz=72$, $yz=96$.

449. $xyz=105$, $35x=3yz$, $7xy=15z$.

450. $yz=a^2$, $xz=b^2$, $xy=c^2$.

451. $x+y+z=20$, $xy+xz=75$, $yz=56$.

452. $x(x+y+z)=54$, $y(x+y+z)=108$, $z(x+y+z)=162$.

453. $x+y+z=k$, $x^2+y^2+z^2=k^2$, $x^3+y^3+z^3=k^3$.

454. $(x+z)(y-z)=22$, $(x-y)(x+z)=33$,
$(x-y)(y-z)=6$.

455. $x\sqrt{y}=a$, $x\sqrt{z}=b$, $y\sqrt{z}-c$.

456. $2x+2y=3xy$, $3y+2z=2yz$, $3x=4xz-5z$.

457. $xy^2z^3=108$, $yz^2=18x$, $2z=3x^2y$.

458. $xy\div z=8$, $xz\div y=18$, $yz\div x=\frac{9}{2}$.

459. $\frac{xyz}{x+y}=-8$; $\frac{xyz}{y+z}=24$; $\frac{xyz}{x+z}=12$.

460. $(2+x)(z-3)=30$, $(5-y)z=6$, $xy=32$.

461. $x(y+z)=90$, $y(x+z)=84$, $z(x+y)=48$.

462. $\frac{x^2+y^2}{xyz}=\frac{5}{6}$, $\frac{x^2+z^2}{xyz}=\frac{5}{3}$, $\frac{y^2+z^2}{xyz}=\frac{13}{6}$.

463. $x+y-2z=-9$, $3x+2y+z=9$, $x^2+y^2+z^2=30$.

106. Insert six arithmetical means between $a + b$ and $8a - 13b$.

107. Insert between 0 and 6 a number of arithmetical means so that the sum of the series shall be 39.

108. How many positive integers of three digits are there which are divisible by 9? Find the sum.

109. What value of x will make the arithmetical mean between $x^{\frac{1}{2}}$ and $x^{\frac{1}{4}}$ equal to 6?

110. A man wishes to have his horse shod, but considers \$2 per shoe too high a price. The blacksmith then offers to take 1 cent for the first nail, 3 for the second, 5 for the third, and so on, to which the man agrees. Each shoe has 8 nails. How much does the man save by this bargain?

111. On a certain day the temperature rose $\frac{1}{2}°$ hourly from 5 to 11 a. m., and the average temperature for that period was 8°. What was the temperature at 8 a. m.?

112. Insert nine arithmetical means between $\frac{3}{2}$ and -11.

113. x, $x + y$, $x + 2y$, etc.; $x - 3y$, $x - y$, $x + y$, $x + 3y$; $x - 2y$, $x - y$, x, $x + y$, $x + 2y$, are sometimes used to represent the terms of a series. What is the common difference in each case, and when would one form be preferable to another?

To how many terms do the following series extend and what is the sum of the series in each case?

114. $2\frac{1}{2}a$, $8a \cdots 72.3a$.

115. $c + 254$, $\cdots c + 2$, $c - 2$.

116. $x - 1$, $\cdots 137(1 - x)$, $139(1 - x)$.

117. If a body falls a feet the first second, $3a$ the second, $5a$ the third, and so on, how far will it fall during the nth second?

118. If a body falls p feet the first second, $3p$ the second, $5p$ the third and so on, how far will it fall during t seconds?

119. A goes 1 mile the first day, 2 the second, 3 the third, and so on. B starts a days later and travels b miles per day. How long will A travel before he is overtaken by B?

120. In what case would B never overtake A, in the preceding example?

121. A travels 1 mile the first day, 2 the second, 3 the third and so on. Five days later B starts out and travels 12 miles a day. How long will A travel before he is overtaken by B? Illustrate the answer by a diagram.

122. Divide 1 into four parts in arithmetical progression, the sum of whose cubes shall be $\frac{1}{10}$.

123. How many arithmetical means are inserted between $\frac{3}{5}$ and $-\frac{9}{7}$ when the sum of the first two is $\frac{9}{35}$?

124. The nineteenth term of an arithmetical progression is $9x - 2y$, and the thirty-first is $31x - 8y$. Find the sum of the first thirteen terms.

125. When m means are inserted between two given terms, those two terms and the means make a series of how many terms? What are a and l in the new series?

126. The sum of four terms of an arithmetical progression is $10\frac{1}{2}$, and the sum of their squares is $35\frac{3}{8}$. Find the terms.

127. A man began on New Year's day, 1902, to put into the bank 1 cent, on the next day 2 cents, on the third day 3 cents, and so on. On what date will he have put by \$100?

128. A load of 100 fence posts was laid down at one corner of a field. If, leaving one post at the corner, a man should carry the rest one by one, to lay them down in a straight line 3 yards apart, how far must he walk to accomplish his task?

129. Find four quantities in arithmetical progression such that the product of the extremes shall be 45, and of the means 77.

130. Find the sum of $n + 2$ terms of the series a, $(na + c)/(n + 1)$, $([n - 1]a + 2c)/(n + 1)$, etc.

131. If the nth term is $(3n - 1)/6$, and $S = n/12(3n+1)$, what is the series?

132. Letting m stand for the number of means to be inserted between two extremes, *i. e.*, between the first and last terms, derive the formula for d.

133. If a, b, c and d are in arithmetical progression, prove that $bc - ad$ is positive.

134. Prove that the differences between the squares of consecutive numbers are in arithmetical progression and that d equals 2.

135. Prove that the sum of n consecutive odd integers beginning with 1 is n^2.

136. Prove that the sum of the $(p+q)$th and the $(p - q)$th terms of an arithmetical progression is equal to twice the pth term.

137. With any three of the elements of an arithmetical progression given, derive the formulæ for the two not given.

GEOMETRICAL PROGRESSION.

1. By means of what two formulæ can all questions in geometrical progression be solved?

2. How may the formula for S be made applicable to an infinite decreasing series?

Find l and S in the following:

3. 4, 8, 16 to seven terms.
4. 8, 4, 2 to ten terms.
5. 2, 6, 18 to six terms.
6. 3, 1, $\frac{1}{3}$ to seven terms.
7. 2, -2^2 to eleven terms.
8. $\frac{2}{5}$, $\frac{1}{2}$ to four terms.
9. $\frac{2}{5}$, $-\frac{1}{5}$, $\frac{1}{10}$ to seven terms.
10. $-\frac{5}{2}$, -5, -10 to ten terms.
11. 16.2, 5.4 to seven terms.
12. 3, $\frac{3}{2}$, $\frac{3}{4}$ to seven terms.
13. x, 1, $1/x$ to thirty terms.
14. 1, 4, 16 to q terms.
15. x, x^3, x^5 to t terms.
16. a, ax^2, ax^4 to thirteen terms.
17. a, $a(1+x)$, $a(1+x)^2$ to eight terms.
18. x, x/y, x/y^2 to n terms.
19. $a(1+c)^{n-1}$, $a(1+c)^{n-2}$, $a(1+c)^{n-3}$ to n terms.

Find the sum of the series to infinity in the following:

20. $1+\frac{1}{2}+\frac{1}{4}$.
21. $10-9+8.1$
22. $5\frac{1}{3}+1\frac{1}{8}+\frac{1}{8}$.
23. $-\frac{5}{8}-\frac{10}{8}-\frac{20}{27}$.
24. $.9+.03+.001$.
25. $\frac{2}{3}+(\frac{2}{3})^2+(\frac{2}{3})^3$.

26. .777. **27.** $\frac{1}{2} - \frac{1}{8} + \frac{2}{9}$. **28.** $-\frac{1}{8} - \frac{1}{18} - \frac{2}{81}$.

29. .1212. **30.** .544. **31.** $1 + c + c^2$.

32. $1 + \frac{1}{a} + \frac{1}{a^2}$. **33.** $2x^3 - .25x$.

34. $p + q + \frac{q^2}{p}$. **35.** $a - b + \frac{b^2}{a}$.

36. $1 + \frac{n-1}{n} + \frac{n-2}{n}$. **37.** $\sqrt{\frac{3}{2}} + \sqrt{\frac{2}{3}}$.

38. $\sqrt{.2} + \frac{1}{25}\sqrt{5}$. **39.** $\frac{\sqrt{2}+1}{\sqrt{2}-1} + \frac{1}{\sqrt{2}-2} + \frac{1}{2}$.

In the following examples r, n and l are given to find a and S.

40. 2, 12, 6,144. **41.** 5, 7, 62,500. **42.** 2, 4, 10.

43. 5, 8, 78,125. **44.** 4, 9, 1024.

45. -2, 10, -256. **46.** $\frac{1}{3}$, 5, $\frac{2}{27}$.

In the following examples r, n and S are given to find a and l.

47. 2, 7, 635. **48.** 3, 6, 1,820. **49.** 8, 6, 149,796.

50. -3, 7, 2,916. **51.** $\frac{2}{3}$, 5, $\frac{211}{27}$.

52. 10, 7, 3,333,333. **53.** $\frac{1}{3}$, 8, $\frac{65609}{6561}$.

In the following examples a, n and l are given to find r and S.

54. 2, 10, 24. **55.** 3, 6, 6,144. **56.** 3, 8, 49,152.

57. 36, 7, $\frac{4}{81}$. **58.** $74\frac{2}{3}$, 6, $2\frac{1}{3}$. **59.** 3, 6, $-\frac{3}{1024}$.

In the following examples a, r and l are given to find n and S:

(NOTE: Finding the value of n in these examples results in exponential equations which, usually, can be solved only by the use of logarithms. They are inserted here for such students as can use logarithms.)

60. 7, 10, 700. **61.** 1, 2, 512. **62.** 1, -3, 729.

63. 5, -2, -640. **64.** 979.2, .5, 15.3.

65. 2, $-\frac{1}{2}$, $-\frac{1}{256}$. **66.** $-\frac{2}{3}$, $-\frac{1}{2}$, $-\frac{1}{96}$.

67. 3, $\sqrt{2}$, $192\sqrt{2}$.

In the following examples a, l and S are given to find r and n:

68. 29, 17,500, 109,368. **69.** 10, 1,280, 2,550.

70. 1, 4,096, 5,461. **71.** 3,125, 5, 3,905.

72. 13, .00013, 14.44443. **73.** 2, $10\frac{1}{8}$, $\frac{211}{8}$.

74. 3, $\frac{1}{248}$, $\frac{547}{248}$.

In the following examples a, r and S are given to find n and l.

75. 32, 5, 4,992. **76.** 3, 3, 29,523. **77.** 5, -2, -425.

78. $\frac{1}{64}$, 4, $21,845\frac{21}{64}$. **79.** $\frac{1}{2}$, -3, -91.

80. $\frac{2}{5}$, $-\frac{1}{2}$, $\frac{21}{80}$. **81.** $\frac{3}{4}$, $-\frac{1}{8}$, $\frac{91}{162}$.

In the following examples l, r and S are given, to find a and n.

82. 1,296, 6, 1,555. **83.** 192, 2, 360.

84. 3,125, 5, 3,905. **85.** -128, 2, -255.

86. $-256, -2, 1\frac{2}{64}$.

87. $\frac{32}{729}, -\frac{2}{3}, \frac{461}{1458}$.

88. $243\sqrt{3}, \sqrt{3}, 364(\sqrt{3}+1)$.

89. Letting m represent the number of means to be inserted between two given terms, what is the formula for m when one mean is inserted?

90. What is the formula for r when more than one mean is inserted?

By means of the formula in example 89 solve the following examples:

Find the geometric mean between

91. $\frac{a}{c}$ and $\frac{c}{a}$.

92. 12 and 108.

93. a and $\sqrt{a}$.

94. $p\sqrt{q}$ and $q\sqrt{p}$.

95. $\sqrt{x}$ and $\sqrt{3}x$.

96. $9+4\sqrt{5}$ and $9-4\sqrt{5}$.

97. $x^2+2xy+y^2$ and $x^2-2xy+y^2$.

98. $\frac{6a^2+3ac}{4ac-2c^2}$ and $\frac{4ac+2c^2}{6a^2-3ac}$.

99. $9a^2+12a+4$ and $4a^2-12a+9$.

By means of the formula in example 90 solve the following examples:

Between the following terms insert the number of terms indicated:

100. $31 \cdots 496$, 3 terms.

101. $a \cdots b$, 3 terms.

102. $1 \cdots 1{,}024$, 4 terms.

103. $4 \cdots -\frac{1}{8}$, 4 terms.

104. $-\frac{2}{5} \cdots -\frac{125}{8}$, 3 terms.

105. $4 \cdots \frac{125}{16}$, 2 terms.

106. $\frac{9}{2} \cdots \frac{32}{81}$, 5 terms. **107.** $\frac{1}{9} \cdots 9$, 7 terms.

108. $\frac{2.278125a^3}{3x^2}$ and $\frac{16x^3}{5a^2}$, 4 terms.

PROBLEMS.

109. If $2c$, x, $1/2c$ are in geometrical progression, what is the value of x?

110. If $2c$, x, $1/2c$ are in arithmetical progression, what is the value of x?

111. What is necessary in order that x, y, z may be in geometrical progression?

112. What is the sum of the series to $2n$ terms of $1 - 2 + 3 - 4 \cdots$?

113. What is the sum of the series to $2n$ terms of $1 - 3 + 5 - 7 \cdots$?

114. What is the sum of the series of $3, 6 \cdots 3(n-1)$, $3n$?

115. The sum of the first $(7+n)$ numbers is 153. Find n.

116. What is the sum of $\sqrt{\frac{8}{2}}$, $\frac{1}{8}$, $\sqrt{2}$ to infinity?

117. The fourth term of a geometrical progression is $\pm \frac{4}{3}$, and the seventh term is $\frac{32}{81}$. Determine the series.

118. The third term of a geometrical progression is 1 and the sixth term is $-\frac{1}{8}$. Find the tenth term.

119. Find the amount at compound interest of $\$p$ for n years at r per cent.

120. The fourth term is .016 and the eighth term is .0000256, in a geometrical progression. Find the first term.

121. If the arithmetical mean between two numbers is $1 + x^2$ and the geometrical mean is $1 - x^2$, what are the numbers?

122. The sum of an infinite series in geometrical progression is 12, and the second term is 27 times the fifth term. Find the series.

123. The sum of four numbers in geometrical progression is equal to their ratio plus 1, and the first term is $\frac{1}{10}$. What are the numbers.

124. The difference between two numbers is 48, and the arithmetical mean exceeds the geometrical mean by 18. Find the numbers.

125. Find four numbers in geometrical progression such that the sum of the first and third is k, and the sum of the second and fourth is p.

126. There are four numbers, the first three of which form an arithmetical progression and the last three a geometrical progression. The sum of the first and third is 2, and of the second and fourth is 37. What are the numbers?

127. A cask contains 240 gallons of wine. The first day of January half is drawn out; the next day half the rest; the third day half of what then remains, and so on. How much will have been drawn out altogether on January 31?

128. Given a square whose side is $2a$. The middle points of its adjacent sides are joined by lines forming a second square inscribed within the first. In the same manner a third square is inscribed within the second, and so on indefinitely.

Find the sum of the perimeters of all the squares.

129. With any three of the elements of a geometrical progression given derive the formulæ for each of the two not given.

130. x, xy, xy^2 ; x, $\sqrt{xy}$, y ; x^2/y, x, y, y^2/x are forms used in solving questions in geometrical progression. What is the ratio in each and when would one be preferable to another?

131. Prove that the reciprocals of the terms of a geometrical progression also form a geometrical progression.

132. What are the properties of a series of products formed by multiplying each term of a geometrical progression by the next term following it?

133. Show that, if all the terms of a geometrical progression be multiplied by the same number, the resulting series will form a geometrical progression.

134. Prove that the series of sums or remainders formed by adding or subtracting each term of a geometrical progression to or from the term next following it is also a geometrical progression. What is the common ratio?

135. Prove that every term of the series 1, 2, 4, $\cdots$ is greater by one than the sum of all that precede it.

136. Show that the product of the two extremes of a geometrical progression is equal to the product of any two terms equally distant from these two extremes.

137. If a, b, c are in geometrical progression, show that $(3a + 7b)/(5a - 7b) = (3b + 7c)/(5b - 7c)$.

138. If a, b, c are in arithmetical progression, and a, b', c are in geometrical progression, show that $b/b' = (a + c)/2\sqrt{ac}$.

139. Show that the arithmetical mean between a and l is greater than the geometrical mean.

140. Prove that the product of the $(p + q)$th term of a geometrical progression and the $(p - q)$th term is equal to the square of the pth term.

SOME SPECIAL METHODS OF SOLVING EQUATIONS.

The following methods of solving equations are applicable to many examples in this book. They are given here as suggestions to the pupil to exercise his ingenuity in seeking the simplest and shortest methods of solving examples. They are based upon two underlying principles; first, constant reduction, by means of division, and cancellation, and abstaining from multiplying large quantities, unless it is necessary; second, simplification by rearrangement of terms, and classification under some well known principle, which is usually effected by means of some addition, subtraction or multiplication.

A fraction with a compound numerator is always the sum or difference of two or more simple fractions, into which it may be resolved at will.

1. $$\frac{b^2 + x^2}{bx} = c + \frac{x}{b}.$$

Resolve the first fraction and express

$$\frac{b}{x} + \frac{x}{b} = c + \frac{x}{b},$$

whence $x = b/c$.

2. $$\frac{1}{n} + \frac{n}{n+x} = \frac{n+x}{nx}.$$

Write thus:

$$\frac{1}{n} + \frac{n}{n+x} = \frac{1}{x} + \frac{1}{n},$$

whence $x = n/(n-1)$.

3. $$\frac{m(p^2x + x^2)}{px} = mqx + \frac{mx^2}{p}.$$

Divide through by m, then express

$$p + \frac{x^2}{p} = qx + \frac{x^2}{p},$$

whence $x = p/q$.

Many fractional equations, otherwise involved and often difficult, are readily solved by inspection, if the fractions be first reduced to mixed numbers.

4. $$\frac{x(3x + 2)}{3x - 1} = x + 1 + \frac{1}{2x}. \quad (1)$$

Expanding the first numerator and dividing by the denominator:

$$x + 1 + \frac{1}{3x - 1} = x + 1 + \frac{1}{2x}, \quad (2)$$

whence $1/(3x - 1) = 1/2x$, and $x = 1$.

5. $$\frac{x}{x - 1} - \frac{x - 1}{x - 2} = \frac{x - 10}{x - 11} - \frac{x - 11}{x - 12}. \quad (1)$$

Dividing each numerator by its denominator:

$$1 + \frac{1}{x - 1} - 1 - \frac{1}{x - 2} = 1 + \frac{1}{x - 11} - 1 - \frac{1}{x - 12}. \quad (2)$$

Uniting the terms of each member separately:

$$\frac{-1}{x^2 + 3x + 2} = \frac{-1}{x^2 - 23x + 132}, \quad (3)$$

whence $20x = 130$, and $x = 6\frac{1}{2}$.

6. $$\frac{x - 3}{x - 5} + \frac{x + 4}{x + 2} = \frac{x - 5}{x - 7} + \frac{x + 6}{x + 4}. \quad (1)$$

Dividing each numerator by its denominator :

$$1+\frac{2}{x-5}+1+\frac{2}{x+2}=1+\frac{2}{x-7}+1+\frac{2}{x+4}. \quad (2)$$

Cancelling and dividing by 2 :

$$\frac{1}{x-5}+\frac{1}{x+2}=\frac{1}{x-7}+\frac{1}{x+4}. \quad (3)$$

Adding :

$$\frac{2x-3}{x^2-3x-10}=\frac{2x-3}{x^2-3x-28}. \quad (4)$$

If, now, we divide by $2x-3$, the equation is destroyed, which shows that $2x-3$ is a zero factor, therefore $2x-3=0$, and $x=\frac{3}{2}$.

The solution can often be much simplified by expressing the multiplication between large quantities instead of performing it.

7. $$pqx=qy-2p. \quad (1)$$

$$p^2y+\frac{n(q^3-p^3)}{pq}=\frac{2p^3}{q}+q^2x. \quad (2)$$

From (1)

$$x=\frac{y}{p}-\frac{2}{q}. \quad (3)$$

Substituting this in (2)

$$p^2y+\frac{n(q^3-p^3)}{pq}=\frac{2p^3y}{q}+\frac{q^2y}{p}-2q^2. \quad (4)$$

Transposing and uniting (4),

$$\frac{(p^3-q^3)y}{p}=\frac{2(p^3-q^3)}{q}+\frac{n(p^3-q^3)}{pq}. \quad (5)$$

Adding the second member and factoring,

$$\left(\frac{p^3 - q^3}{p}\right) y = \left(\frac{p^3 - q^3}{p}\right)\left(\frac{n + 2p}{q}\right). \qquad (6)$$

Dividing by $(p^3 - q^3)/p$, $y = (n + 2p)/q$.

8.

$$\frac{x}{2} + \frac{y}{3} + \frac{z}{4} = 218. \qquad (1)$$

$$\frac{x}{3} + \frac{y}{4} + \frac{z}{5} = 164. \qquad (2)$$

$$\frac{x}{4} + \frac{y}{5} + \frac{z}{6} = 132. \qquad (3)$$

Dividing (1) by 2 and subtracting from (3),

$$\frac{y}{30} + \frac{z}{24} = 23. \qquad (4)$$

Dividing (2) by 4, and (3) by 3 and subtracting,

$$\frac{y}{240} + \frac{z}{180} = 3, \qquad (5)$$

$$\frac{y}{240} + \frac{z}{8.24} = \frac{23}{8}, \text{ (4) divided by 8.} \qquad (6)$$

Subtracting (5) from (6)

$$\frac{z}{8 \cdot 24} - \frac{z}{180} = \frac{1}{24}. \qquad (7)$$

Cancelling,

$$\frac{z}{8 \cdot \cancel{24}} - \frac{z}{\cancel{180}\ 45} = \frac{1}{\cancel{24}},$$

$$\frac{z}{360} = 1, \text{ and } z = 360\ ;\ y = 240\ ;\ x = 96.$$

The principles of proportion frequently afford the easiest means of solution.

9. $$\frac{x^3+x+8}{x^3+4} = 2 - \frac{x^3+x-8}{x^3-4}. \qquad (1)$$

Reducing the second member,

$$\frac{x^3+x+8}{x^3+4} = \frac{x^3-x}{x^3-4}. \qquad (2)$$

Writing as a proportion,

$$x^3+x+8 : x^3+4 = x^3-x : x^3-4. \qquad (3)$$

By composition and division,

$$2x^3+x+12 : x+4 = 2x^3-x-4 : 4-x, \qquad (4)$$

By alternation,

$$2x^3+x+12 : 2x^3-x-4 = x+4 : 4-x, \qquad (5)$$

By composition and division,

$$4x^3+8 : 2x+16 = 8 : -2x, \qquad (6)$$

Reducing and alternating,

$$x^3+2 : 2 = x+8 : -x, \qquad (7)$$

By composition and division,

$$x^3+4 : x^3 = 8 : 2x+8, \qquad (8)$$

By composition and division,

$$2x^3+4 : 4 = 2x+16 : 2x, \qquad (9)$$

Multiply extremes and means,

$$4x^4+8x = 8x+16, \quad x = \pm 2.$$

The special advantage of this method is that the work can all be done mentally.

10. $$\frac{\sqrt{x+7}-x}{\sqrt{x+7}+x}=\frac{\sqrt{2x-2}-3}{\sqrt{2x+2}+3}. \quad (1)$$

If $a/b = c/d$, then $(a+b)/(a-b) = (c+d)/(c-d)$,

$$\therefore \frac{\sqrt{x+7}}{\sqrt{x}}=\frac{\sqrt{2x-2}}{3}, \quad (2)$$

$$\frac{x+7}{x}=\frac{2x-2}{9}, \quad (3)$$

$$2x^2-11x-63=0, \quad (4)$$

$$(2x+7)(x-9)=0, \quad (5)$$

$$x=9, \quad \text{or} \quad -\tfrac{7}{2}.$$

Reciprocal equations can be arranged as quadratics as follows :

11. $$x^4+x^3-4x^2+x+1=0. \quad (1)$$

Dividing by x^2

$$x^2+x-4+\frac{1}{x}+\frac{1}{x^2}=0. \quad (2)$$

Adding 6 to both members

$$x^2+2+\frac{1}{x^2}+x+\frac{1}{x}=6. \quad (3)$$

That is,

$$\left(x+\frac{1}{x}\right)^2+\left(x+\frac{1}{x}\right)=6, \quad (4)$$

which is readily solved.

12. $$4x^4-10x^3+12x^2-10x+4=0. \quad (1)$$

Dividing by x^2

$$4x^2-10x+12-\frac{10}{x}+\frac{4}{x^2}=0. \quad (2)$$

Arranging terms

$$4x^2 + \frac{4}{x^2} - \left(10x + \frac{10}{x}\right) + 12 = 0. \qquad (3)$$

or

$$4\left(x^2 + \frac{1}{x^2}\right) - 10\left(x + \frac{1}{x}\right) + 12 = 0. \qquad (4)$$

Put $y = x + 1/x$, then $y^2 = x^2 + 1/x^2 + 2$. Substituting in (4),

$$4(y^2 - 2) - 10y + 12 = 0, \qquad (5)$$

$$4y^2 - 8 - 10y + 12 = 0, \qquad (6)$$

$$2y^2 - 5y + 2 = 0, \qquad (7)$$

$$(y - 2)(2y - 1) = 0, \qquad (8)$$

$$y = 2, \quad \text{or} \quad \tfrac{1}{2};$$

$$x = 1, \quad \text{or} \quad 1, \quad \text{or} \quad \tfrac{1}{4} \pm \tfrac{1}{4}i\sqrt{31}.$$

13.
$$x^4 + y^4 = 706, \qquad (1)$$

$$x + y = 8, \qquad (2)$$

Such examples are usually solved by substituting other letters for x and y, or by raising $x + y$ to the fourth power and subtracting, but the prettiest and easiest solution is to raise x and y to the fourth power, add that to (1), divide the sum by 2, and extract the square root of the quotient. The result is then easily compared with the square of $x + y$.

14.
$$\frac{1}{x^2 + x + 2} + \frac{1}{x^2 + x + 4} = \frac{6}{x^2 + x + 8}. \qquad (1)$$

Put $y = x^2 + x + 3$. Substituting

$$\frac{1}{y - 1} + \frac{1}{y + 1} = \frac{6}{y + 5}. \qquad (2)$$

Adding,

$$\frac{y}{y^2-1}=\frac{3}{y+5}. \tag{3}$$

$$2y^2-5y-3=0, \tag{4}$$

$$(2y+1)(y-3)=0, \tag{5}$$

$$y=3, \quad \text{or} \quad -\tfrac{1}{2},$$

Whence $x=0,\ -1,\ -\frac{1}{2}\pm i\sqrt{13}$.

15. $$\frac{4x^2-x+1}{x^2-4x-1}+\frac{3x^2-12x-3}{4x^2-x+1}+4=0, \tag{1}$$

A moment's consideration will show that, if the numerator of the second fraction be factored, the fraction will appear as a reciprocal of the first, therefore put $(4x^2-x+1)/(x^2-4x-1)=y$, then substituting

$$y+\frac{3}{y}+4=0, \tag{2}$$

whence $y=-3$, or -1 and $x=2$, or 1, or $-\frac{1}{7}$.

16. $$\frac{3}{x+y+z}+\frac{2}{3x-y}+\frac{4}{z-x}=3. \tag{1}$$

$$\frac{9}{x+y+z}+\frac{3}{3x-y}+\frac{6}{z-x}=5. \tag{2}$$

$$\frac{12}{x+y+z}+\frac{2}{3x-y}-\frac{2}{z-x}=1. \tag{3}$$

Put $3/(x+y+z)=u$; $1/(3x-y)=v$; $2/(z-x)=w$.

The solution is now easy, though the process is necessarily long.

17. $$x^2+3xy=27, \tag{1}$$

$$y^2+2xy=16. \tag{2}$$

Let $x = vy$, then substituting

$$v^3y^3 + 3vy^2 = 27. \quad (3)$$

$$2vy^2 + y^2 = 16. \quad (4)$$

$$y^2 = \frac{27}{v^3 + 3v}. \quad (5)$$

$$y^2 = \frac{16}{2v + 1}. \quad (6)$$

Whence

$$16v^3 - 6v - 27 = 0. \quad (7)$$

$$(8v + 9)(2v - 3) = 0. \quad (8)$$

$$v = \tfrac{3}{2}, \text{ or } -\tfrac{9}{8},$$

and

$$x = \pm 3, \text{ or } \pm \tfrac{9}{5}\sqrt{-5}.$$

$$y = \pm 2, \text{ or } \pm \tfrac{8}{5}\sqrt{-5}.$$

18. $$x^4 + y^4 - x^2 - y^2 = 84. \quad (1)$$

$$x^2 + x^2y^2 + y^2 = 49. \quad (2)$$

Arrange (1) as follows:

$$(x^2 + y^2)^2 - 2x^2y^2 - (x^2 + y^2) = 84. \quad (3)$$

Arrange (2) as follows:

$$(x^2 + y^2) + x^2y^2 = 49. \quad (4)$$

Put $x^2 + y^2 = u$, $x^2y^2 = v$. Substituting in (3) and (4),

$$u^2 - 2v - u = 84. \quad (5)$$

$$u + v = 49. \quad (6)$$

Whence $u = 13$, or -14, $v = 36$, or 63, and $x = 3$, or -2, etc., and $y = 2$, or -3, etc. There are eight pairs of roots.

19. $$x^2 + y^2 + x + y = 48, \quad (1)$$

$$x^2 + 3xy + y^2 = 76. \tag{2}$$

Let $x = u + v$, and $y = u - v$, then substituting

$$u^2 + v^2 + u = 24, \tag{3}$$

$$5u^2 - v^2 = 76, \tag{4}$$

whence

$$6u^2 + u - 100 = 0, \tag{5}$$

and $u = 4$, or $-\frac{25}{6}$; and $v = \pm 2$, or $\pm\frac{1}{6}i\sqrt{2111}$ and $x = 6$, or 2, or $-4i\sqrt{2111}$, or $-\frac{13}{3}i\sqrt{2111}$; and $y = 2$, or 6, or $-\frac{13}{3}\sqrt{2111}$, or $-4i\sqrt{2111}$.

20. $$4x^2 - 10\sqrt{x} = 5x + 24. \tag{1}$$

Arrange as follows:

$$4x^2 - 4x + 1 = x + 10\sqrt{x} + 25. \tag{2}$$

Extracting the square root of both members,

$$2x - 1 = \sqrt{x} + 5, \tag{3}$$

$$4x - 2\sqrt{x} - 12 = 0, \tag{4}$$

$$(2\sqrt{x} - 4)(2\sqrt{x} + 3) = 0, \tag{5}$$

$$x = 4, \quad \text{or} \quad \tfrac{9}{4}.$$

21. $$3x^2 - \frac{54}{x} + 45 = 18x + \frac{120}{x^2}. \tag{1}$$

Freeing, dividing by 3, and arranging terms,

$$x^4 - 6x^3 + 15x^2 - 18x - 40 = 0. \tag{2}$$

Extracting the square root of (2) to two places and expressing that

$$(x^2 - 3x)^2 + 6(x^2 - 3x) = 40, \tag{3}$$

which is easily solved as a quadratic, and $x = \frac{3}{2} \pm \frac{1}{2}\sqrt{31}$.

But a better way is to extract the root so far as possible,

$$x^4 - 6x^3 + 15x^2 - 18x - 40 \;\big|\; x^2 - 3x + 3 \qquad (1)$$

$$\begin{array}{r|l} x^4 & \\ \hline 2x^2 - 3x & -6x^3 + 15x^2 \\ & -6x^3 + 9x^2 \\ \hline 2x^2 - 6x + 3 & 6x^2 - 18x - 40 \\ & 6x^2 - 18x + 9 \\ \hline & -49 \end{array}$$

The expression lacks 49 of being a perfect square, therefore add 49 to both sides of the equation, and then extract the square root, *i. e.*,

$$x^4 - 6x^3 + 15x^2 - 18x + 9 = 49, \qquad (2)$$

whence

$$x^2 - 3x + 3 = \pm 7, \qquad (3)$$

and

$$x = \tfrac{3}{2} \pm \tfrac{1}{2}\sqrt{31}.$$

22.

$$x^2 + y = 11, \qquad (1)$$

$$x + y^2 = 7. \qquad (2)$$

Arrange (1) thus,

$$x^2 - 9 = 2 - y. \qquad (3)$$

Arrange (2) thus,

$$3 - x = y^2 - 4, \qquad (4)$$

$$x - 3 = (2 + y)(2 - y), \qquad (5)$$

i. e.,

$$2 - y = \frac{x - 3}{2 + y}, \qquad (6)$$

Substituting (6) in (3)

$$x^2 - 9 = \frac{x - 3}{y + 2}, \qquad (7)$$

Let $y + 2 = a$, then

$$x^2 - 9 = \frac{x - 3}{a}, \qquad (8)$$

Separating fraction into two parts, aud rearranging terms,

$$x^2 - \frac{x}{a} = 9 - \frac{3}{a}, \tag{9}$$

Completing the square,

$$x^2 - \frac{x}{a} + \frac{1}{4a^2} = 9 - \frac{3}{a} + \frac{1}{4a^2}. \tag{10}$$

Extracting the square root,

$$x - \frac{1}{2a} = 3 - \frac{1}{2a}. \tag{11}$$

Whence $x = 3$, $y = 2$.

(NOTE: This solution is given as an example of what rearrangement of terms can accomplish. The student will appreciate its value, if he will try to solve the example in any other way. It is not, however, a full solution, because the example really becomes a biquadratic equation, hence x and y should each have four values as follows: $x = 3$, or $3.584+$, or $-3.779+$, or $-2.805+$, $y = 2$, or $-1.848+$, or $-3.283+$, or $3.131+$.)

23.

$$2x^2 - xy + y^2 = 28. \tag{1}$$

$$x^2 + 2xy + 2y^2 = 40. \tag{2}$$

Multiplying (1) by 10 and (2) by 7, and subtracting

$$13x^2 - 24xy - 4y^2 = 0. \tag{3}$$

$$(13x + 2y)(x - 2y) = 0. \tag{4}$$

$$x = -\frac{2y}{13} \text{ or } 2y. \tag{5}$$

Substituting these values in (1) we obtain

$$x = \pm 4, \text{ or } \pm \tfrac{2}{29}\sqrt{29},$$

$$y = \pm 2, \text{ or } \mp \tfrac{13}{29}\sqrt{29}.$$

If (6) were not easily solved in this way, then we could divide (3) through by y^2,

$$13\left(\frac{x}{y}\right)^2 - 24\left(\frac{x}{y}\right) = 4,$$

and solve for x/y.

24. $$\sqrt{5x^2 - 2x + 6} - \sqrt{5x^2 + 2x - 3} = 1, \qquad (1)$$

Put

$$\sqrt{5x^2 - 2x + 6} = \sqrt{A},$$

and

$$\sqrt{5x^2 + 2x - 3} = \sqrt{B},$$

then $A - B = 9 - 4x$.

Now

$$\sqrt{A} + \sqrt{B} = \frac{A - B}{\sqrt{A} - \sqrt{B}},$$

that is,

$$\sqrt{5x^2 - 2x + 6} + \sqrt{5x^2 + 2x - 3} = \frac{9 - 4x}{\sqrt{5x^2 - 2x + 6} - \sqrt{5x^2 + 2x - 3}}, \qquad (2)$$

Substituting for the denominator its value in equation (1),

$$\sqrt{5x^2 - 2x + 6} + \sqrt{5x^2 + 2x - 3} = 9 - 4x. \qquad (3)$$

Adding

$$\begin{array}{rr} \sqrt{5x^2 - 2x + 6} - \sqrt{5x^2 + 2x - 3} = 1. & (1) \\ \hline \sqrt{5x^2 - 2x + 6} = 5 - 2x. & (4) \end{array}$$

Squaring

$$5x^2 - 2x + 6 = 25 - 20x + 4x^2. \qquad (5)$$

$$x^2 + 18x - 19 = 0. \qquad (6)$$

$$(x + 19)(x - 1) = 0,$$

$$x = 1, \quad \text{or} \quad -19.$$

The following theorem is a very useful one at times: If a/b, c/d, e/f, etc., be all equal to each other, then will each be equal to

$$\frac{pa + qc + re \cdots}{pb + qd + rf \cdots}.$$

25. $$\frac{qy + pz}{a} = \frac{nz + qx}{b} = \frac{px + ny}{c} = k. \qquad (1)$$

Multiplying both terms of the first fraction by $-n$, of the second fraction by p, and of the third fraction by q,

$$\frac{-nqy - npz + npz + pqx + pqx + qny}{-na + pb + qc} = k. \qquad (2)$$

Combining terms

$$\frac{2pqx}{-na + pb + qc} = k, \qquad (3)$$

$$x = \frac{k(-na + pb + qc)}{2pq}.$$

Multiplying both terms of the first fraction by n, of the second fraction by $-p$, and of the third fraction by q,

$$\frac{nqy + npz - npz - pqx + pqx + nqy}{na - pb + qc} = k. \qquad (4)$$

Combining terms

$$\frac{2nqy}{na - pb + qc} = k, \qquad (5)$$

$$y = \frac{k(na - pb + qc)}{2nq}.$$

Multiplying both terms of the first fraction by n, of the second fraction by p, and of the third fraction by $-q$,

$$\frac{nqy + npz + npz + pqx - pqx - nqy}{na + pb - qc} = k. \qquad (6)$$

Combining terms,

$$\frac{2npz}{na + pb - qc} = k, \tag{7}$$

$$z = \frac{k(na + pb - qc)}{2np}.$$

Note that the terms occur in cyclic order, therefore, in practice, after the first answer is obtained, the others can be written by inspection.

Such methods might be continued indefinitely, but enough have been given to enable the pupil to judge their value in comparison with the ordinary methods.

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